

## INFLUENCE OF TANGENTIAL DISPLACEMENTS ON THE MEASUREMENT OF ELASTIC PROPERTIES BY DEPTH-SENSING INDENTATION

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*Elastic indentation of a half-space by a non-ideal Berkovich indenter is simulated by means of a boundary element method. Paper accounts for tangential displacements which are usually neglected in analysis of indentation data. A simple expression is derived for the impact of the tangential displacements on the values of the reduced Young's modulus determined due to the Oliver-Pharr technique.*

**Introduction.** Depth-sensing indentation is a widely used technique in the study of mechanical properties of materials [1]. It yields information about hardness and elastic modulus and is also applicable for determination of yield stress and strain hardening exponent. Depth-sensing measurements at penetration depths of tens or hundreds of nanometers are referred to as nanoindentation [2; 3], is particularly well suited to the characterization of coated and other surface-engineered systems [4–6].

One of the more commonly used methods for extracting hardness and elastic modulus from nanoindentation load–displacement data is that of Oliver and Pharr [7–9]. It has an advantage that difficult measurements of the contact area at the nano-scale are not required, since it is calculated from the contact depth. Thus, the accuracy of the Oliver-Pharr technique depends on how well it predicts the contact area. The most possible factors distorting the value of the contact area are the roughness of contacting surfaces [10], non-ideal shape of indenters [11; 12], pile-up or sink-in [9; 13] and tangential displacements [14–16]. In the present investigation we refine on the Oliver-Pharr method by allowance for the non-ideal shape of the indenter tip (rounding) and for the tangential displacements.

The Oliver-Pharr method does not account for the tangential dis-

placements. The method is based on the Bulychev-Alekhin-Shorshorov (BASH) relation [8; 17; 18] which is restricted to frictionless contact between elastic bodies and smooth surfaces and considers only the normal displacements on the surface of solids. Neglect of the tangential displacements leads to the incompatibility of strains in the area around the contact [14; 15]. Moreover, the tangential displacements themselves at the boundary of the contact region can achieve approximately 22 % of the indentation depth depending on the Poisson's ratio of the elastic half-space [14; 15]. Therefore, accounting for the tangential displacements demands a particular investigation.

The objective of the study is to specify how much impact the tangential displacement effects have on nanoindentation studies of material properties. To attain the goal we use the mathematical model of elastic contact represented in the previous paper [19] and expand the BASH relation for the tangential displacements. The model concerns an especially important case of shallow indentation (usually less than 100 nm) where the tip rounding is on the same order as the indentation depth. It considers the indentation of half-spaced samples by the rigid Berkovich indenter and accounts for the tangential displacements on the surface of the sample.

**Short description of the model.** We use the mathematical model of a unilateral contact between the Berkovich rigid indenter and an elastic half-space (sample). The indenter with the equation of the surface  $x_3 = -f(x_1, x_2)$  is pressed by the force  $P$  to a boundary of the contacting sample (see Fig. 1, *a*). The sample is considered as a positive half-space  $x_3 \geq 0$ . The origin  $O$  of Cartesian coordinates,  $x_1, x_2, x_3$  is put at the single point of the initial contact between the indenter and the sample. The contact region  $S$  is an orthogonal projection of the contact between the sample and the lateral surface of the indenter on the plane  $x_3 = 0$  after deformation. The tip of the blunted indenter is simulated as a smooth surface (the homogeneous functions with the degree 2)

$$f(M) = \frac{r^2}{2R},$$

where  $R$  is the radius describing the shape of the blunted indenter tip. It accounts for the asymmetry of the Berkovich indenter, Fig. 1, *b*.

$$r = \sqrt{x_1^2 + x_2^2}.$$

Method of non-linear integral boundary equations (NIBEs) [20] was applied to formulate the contact problem. The numerical solution of NIBEs was carried out by means of the boundary element method. The final formula for the load-displacement diagram is

$$P(h) = \frac{\sqrt{2d}}{\lambda} \cdot P_0(h) \cdot h^{3/2}, \quad (1)$$

here function  $P_0(h)$  is the dimensionless compression force, it was obtained numerically from the solution of NIBEs at different values of the mutual approach  $h$ ;  $d$  is the bluntness of the indenter tip. Parameter  $\lambda$  is defined through the reduced Young's modulus  $E^*$ ,  $\lambda = \frac{1}{\pi E^*}$ ,  $\frac{1}{E^*} = \frac{1-\nu_s^2}{E_s} + \frac{1-\nu_i^2}{E_i}$  (subscripts denote the parameters of the sample "s" and of the indenter "i"). For more details about the model and derivation of solution of NBIEs the reader is referred to [19].

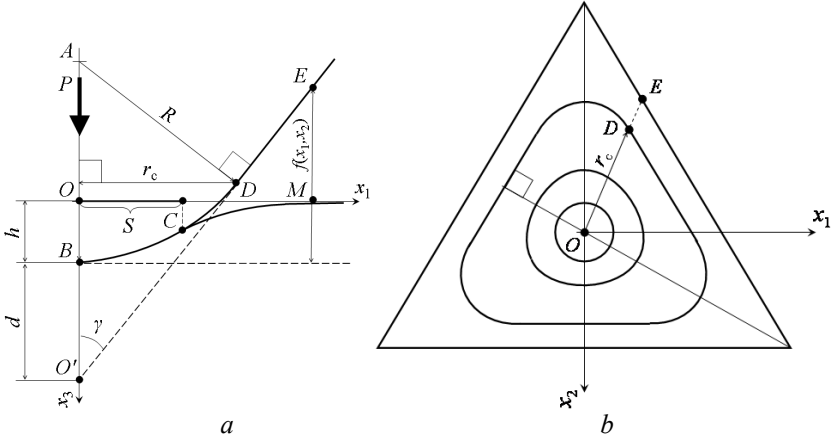


Fig. 1. (a) Geometry of the simulated blunted indenter,  $BCDE$ , and of the ideal Berkovich indenter,  $O'DE$ . The segment  $BD$  is the arc of the circle with the centre  $A$  and radius  $R$ ;  $d$  is the bluntness of the indenter tip.  $OB$  is the displacement of the indenter, which causes the contact  $BC$  with the sample. (b) Cross-section of the simulated blunted indenter. The contour lines correspond to various positions of  $D$

### Effect of tangential displacements on the nanoindentation study of the reduced Young's modulus.

Expression for the dimensionless compression force is given in the following form

$$P_0(h) \approx P_0 + b(\nu_s) \cdot \sqrt{\frac{h}{d}}, \quad (2)$$

where  $P_0 = 1.277$  is a constant,  $b(\nu_s)$  is a function depending on the Poisson's ratio of the sample material,  $b(0.5) = 0$ . The effect of tangential displacements is associated with the second term in (2). If the tangential displacements in the model are neglected, then the dimensionless compression force is constant  $P_0(h) = P_0$ , regardless of the value of Poisson's ratio  $\nu_s$ . So, expressions for the load calculated with allowance for the tangential displacements  $P_{TD}(h)$  and neglecting them  $P_{noTD}(h)$  can be derived from (1) and (2)

$$\begin{aligned} P_{TD}(h) &\approx \frac{\sqrt{2d}}{\lambda} \cdot P_0 \cdot h^{3/2} + \frac{\sqrt{2}}{\lambda} \cdot b(\nu_s) \cdot h^2, \\ P_{noTD}(h) &\approx \frac{\sqrt{2d}}{\lambda} \cdot P_0 \cdot h^{3/2}, \end{aligned} \quad (3)$$

Following the technique of Oliver and Pharr we need an equation for the contact stiffness. The reduced Young's modulus is determined from the contact stiffness  $\mathbf{S}$  at the beginning of unloading and the projected contact area  $A$  using BASH relation [8; 17; 18]

$$\mathbf{S} = \frac{2\beta}{\sqrt{\pi}} E^* \sqrt{A},$$

where  $\beta$  is a constant that depends on the geometry of the indenter (1.034 for a Berkovich indenter and about 1 for the tip bluntness). As was mentioned in introduction, the BASH relation neglects the tangential displacements. We can derive from (3) a refined relation for the contact stiffness  $\mathbf{S}_{TD}$  that accounts for the tangential displacements

$$\mathbf{S}_{TD} = \frac{dP_{TD}(h)}{dh} \approx \sqrt{2\pi} E^* \cdot \left( \frac{3}{2} \cdot \sqrt{d} \cdot P_0 \cdot h^{1/2} + 2 \cdot b(\nu_s) \cdot h \right). \quad (4)$$

Tangential displacements influence the reduced modulus  $E^*$  by means of the contact stiffness only because the contact area is no longer explicitly presented in (4). The contact stiffness that neglects the tangential displacements is

$$\mathbf{S}_{noTD} = \frac{dP_{noTD}(h)}{dh} \approx \frac{3}{2} \sqrt{2\pi} E^* \cdot \sqrt{d} \cdot P_0 \cdot h^{1/2}. \quad (5)$$

Let  $E_{TD}^*$  denotes the reduced modulus determined with allowance for the tangential displacements (4) and  $E_{noTD}^*$  denotes the modulus determined neglecting them (5). Relation between  $E_{TD}^*$  and  $E_{noTD}^*$  can be found from the comparison of  $\mathbf{S}_{TD}$  with  $\mathbf{S}_{noTD}$ . The contact stiffness is evaluated at the beginning of unloading  $h = h_{\max}$  [8], see Fig. 2. We should also account for the final displacement  $h_f$  after complete unloading [8]. Thus we set  $h = h_{\max} - h_f$  to compare the contact stiffness in (4) and (5):

$$\frac{E_{TD}^*}{E_{noTD}^*} = \frac{\frac{3}{2} \cdot \sqrt{d} \cdot P_0 \cdot (h_{\max} - h_f)^{1/2}}{\frac{3}{2} \cdot \sqrt{d} \cdot P_0 \cdot (h_{\max} - h_f)^{1/2} + 2 \cdot b(v_s) \cdot (h_{\max} - h_f)} \approx 1 - \frac{4}{3} \cdot \frac{b(v_s)}{P_0} \cdot \sqrt{\frac{h_{\max} - h_f}{d}} \approx 1 - b(v_s) \cdot \sqrt{\frac{h_{\max} - h_f}{d}}$$

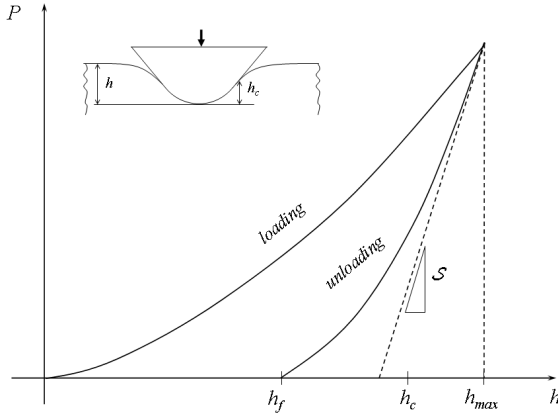


Fig. 2. A schematic load-displacement curve of a nanoindentation test.  $h_{\max}$  – maximal displacement,  $h_f$  – final displacement,  $h_c$  – contact depth,  $S$  – contact stiffness

Here we used that  $b(\nu_s) \ll P_0 = 1.277$ , see table. The model was developed for a rigid indenter. Therefore the effect of tangential displacements on determination of the Young's modulus and of the reduced Young's modulus is the same:

$$\frac{E_{s,TD}}{E_{s,no TD}} = \frac{E_{TD}^*}{E_{no TD}^*} \approx 1 - b(\nu_s) \cdot \sqrt{\frac{h_{\max} - h_f}{d}}. \quad (6)$$

Furthermore, we investigated the case of shallow indentation,  $h_{\max} - h_f \approx d$ . Therefore (6) reduces to:

$$\frac{E_{s,TD}}{E_{s,no TD}} = \frac{E_{TD}^*}{E_{no TD}^*} \approx 1 - b(\nu_s). \quad (7)$$

As follows from (6) and (7), the models neglecting tangential displacements overestimate the reduced and the Young's modulus. For a wide range of materials the error in determination of the elastic modulus is about 4 % (see table 1). For materials with the Poisson's ratio less than 0.2 the error approaches to 6 %.

*Table*

**The values of the parameters  $b$  [19] depending on the Poisson's ratio of the sample**

$\nu_s$	$10^2 \cdot b$
0	$6.17 \pm 0.08$
0.1	$5.56 \pm 0.08$
0.2	$4.63 \pm 0.07$
0.3	$3.38 \pm 0.05$
0.4	$2.03 \pm 0.05$
0.5	0

A difficulty of the Oliver-Pharr method is the estimation of the contact area [13; 18]. To avoid this, we propose to find the unknown parameters ( $d$  or  $E^* = 1/\pi\lambda$ ) by fitting the function  $P(h)$  in (1) to the elastic part of the indentation curves.

### Conclusions

A simple expression is provided for the impact of the tangential displacements on nanoindentation studies of the reduced Young's modulus. Neglecting tangential displacements one overestimates the

Young's modulus up to 6 % depending on the Poisson's ratio of the sample.

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