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THE GEOMETRIC MODELING OF COMPLICATED TRAJECTORIES OF MANIPULATING HYDROCLEANING DEVICES

Developing a model of manipulating hydraulic cleaning tool trajectory using local spline functions is one of the research objectives. It is important to emphasize that the above-mentioned functions do not require pre-analytical determination of spline coefficients and ensure a simple technical performance.

Keywords: *hydraulic cleaning tools, a complicated trajectory, the manipulator, spline functions, positioning, the degree of trajectory smoothness, nodal points in space.*

1. Introduction.

One of the ways to higher up the efficiency of machining, such as hydraulic cleaning of aircraft engine components, is the development of simple and plausible methods to assess the accuracy of positioning of hydraulic cleaning tools (HCT) during the processing [4].

Cleaning of complex surfaces (holes, channels, grooves, etc.) of engine frame parts includes their previous installation, which is proportionally associated with the dimensional coordinate system of the machine, and the subsequent positioning of HCT concerning the structural elements that are processed [3].

When using high-precision electromechanical devices to clean the frame components with several functionally related holes, which have a high precision placement of axes, HCT should be in the exact position towards the coordinate axes of these elements.

If high-precision devices of movement and the complex control system of axis accuracy are used to position manipulating HCT to make the HCT axis match the axis of the hole, the hydraulic cleaning machine design can be complicated in general and its reliability can be reduced [5].

For this class of machines it is necessary to develop simple and reliable methods of HCT positioning based on the spline function to describe their trajectory.

2. Problem to be solved on the analysis of previous researches.

The HCT movement trajectory is usually given in the Cartesian coordinate system and the position and orientation are specified by the matrix *H(t)* [2] (Fig. 1):

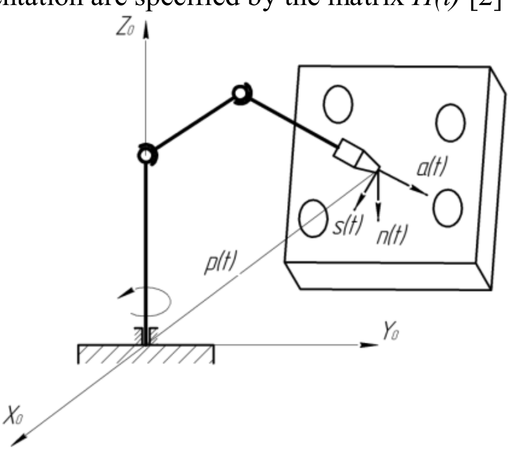

Fig. 1. Schematic diagram of the surface hole cleaningwith manipulating HCT

$$H(t) = \begin{vmatrix} n(t) & s(t) & a(t) & p(t) \\ 0 & 0 & 0 & 1 \end{vmatrix} \quad (1)$$

where p – a vector of the HCT center position in the base coordinate system $X_0Y_0Z_0$ which is associated with constant HCT; n, s, a – unit vectors that form a coordinate system associated with the HCT.

It is necessary to find the equation of motion $q_i(t)$ for all $i = 1, 2, \ldots, s$ units of HCT compounds using the matrix $H(t)$ by solving an inverse kinematics problem. Splines with the coefficients in the interval $[t_i, t_{i+1}]$ determined by several values of the coordinates g_i are called 'local'. These coefficients do not depend on the values g_i all along the grid [1]. A spline that passes through the specified nodes g_i, is called 'interpolating'. A spline that approximates the given sequence, but does not pass through all nodes g_i is called 'approximating'. Let $S^I_{m,\nu}\left(S^A_{m,\nu}\right)$ be an interpolating (approximating) spline of degree m, and defect ν: $\Delta_i = g_i - g_{ki-1}$, $\Delta_i^2 = \Delta_{i+1} + \Delta_i$, $\Delta_i^3 = \Delta_{i+1}^2 + \Delta_i^2$, $\Delta_i^4 = \Delta_{i+1}^3 + \Delta_i^3$ – central difference of the first, second, third and fourth order.

Determining the coefficients a_{ji} of local splines $S_{m,\nu}$, and the nature of a spline (interpolating or approximating), its degree m and defect ν, all of that should be performed according to the objectives. The movement of the manipulator can be specified only by the values of generalized coordinates g_i, or generalized coordinates g_i and velocities $\dot{g}_i$, or coordinates g_i, velocities $\dot{g}_i$ and acceleration $\ddot{g}_i$. In various cases it is necessary to provide different degrees of the trajectory smoothness, in other words we need to fulfill one of the following conditions:

$$g(t) \in C^0[a, b], \quad (2)$$
$$g(t) \in C^1[a, b], \quad (3)$$
$$g(t) \in C^2[a, b], \quad (4)$$
$$g(t) \in C^3[a, b], \quad (5)$$

with given initial $(g_0, \dot{g}_0, \ddot{g}_0)$ and final $(g_n, \dot{g}_n, \ddot{g}_n)$ conditions.

The splines of degree m defect ratio ν are defined as follows [1]:

$$S_{m,\nu}(t) = \sum_{j=0}^{m} a_{ji}(t - t_i)^j, i = 0, 1, \ldots, n;$$
$$S_{m,\nu}(t) \in C^{m-\nu}[a, b]. \quad (6)$$

Some local splines are constructed depending on the specified $g_i, \dot{g}_i, \ddot{g}_i$ and the degree of trajectory smoothness. Let us consider the construction of splines like that for the uniform grid Δ_n.

To construct a spline that interpolates the given values g_i, the two following equations to determine the coefficients of this spline ought to be constructed:

$$a_{0i} = g_i, \quad (7)$$
$$\sum_{j=0}^{m} a_{ji} = g_{i+1}, \quad (8)$$

It is necessary to take $m = 1$ for the system of equations (7) and (8) to have a unique solution. Then we are going to have the required spline of the first degree $S^I_{1,0} = a_{0i} + a_{1i}t$, where $a_{0i} = g_i$, $a_{1i} = \Delta_{i+1}$.

The next step is to set the condition of continuous velocity all along the interval Δ_n:

$$\dot{S}^i(t_{i+1}) = \dot{S}^{i+1}(t_i), \quad (9)$$

where $S^i(t) = \sum_{j=0}^{m} a_{ji}\, t^j$.

It is necessary to fulfill the condition $S^i(t_{i+1}) = S^{i+1}(t_i)$ to make the spline exist. (10)

The spline should not be lower than the second degree to fulfill the condition (9). However, at most $2n$ coefficients can be calculated from the system of equations (9) and (10). If we find $\dot{g}_i = \Delta_i$ acceptable, then we can rewrite equation (9) as:

$$\dot{S}\,(t_i) = \Delta_i, \tag{11}$$

$$\dot{S}\,(t_{i+1}) = \Delta_{i+1}, \tag{12}$$

Taking into consideration that the given spline $S_{2,1}$ is local, we can determine its coefficients by the values of g_{i-1}, g_i, g_{i+1}. Let us assume

$$a_{0i} = k_1 g_{i+1} + k_2 g_i + k_3 g_{i-1}, \tag{13}$$

where k_1, k_2, k_3 are some constants.

From equations (11), (12) we can calculate $a_{1i} = \Delta_i, a_{2i} = 0{,}5\,\Delta_i^2$. If we put a_{1i}, a_{2i}, a_{0i} into equation (10), then we have:

$$(k_1 + 0{,}5)g_{i+1} + k_2 g_i + (k_3 - 0{,}5)g_{i-1} = k_1 g_{i+2} + k_2 g_{i+1} + k_3 g_i. \tag{14}$$

The expression (14) becomes an identity if the coefficients are equal in both parts of the equation under the following condition: $g_{i-1}, g_i, g_{i+1}, g_{i+2}$. Then $k_1 = 0,\ k_2 = 0{,}5,\ k_3 = 0{,}5$, end $a_{0i} = 0{,}5(g_i + g_{i-1})$. The resulted spline $S_{2,1}^A$ is an approximating one.

Constructing an interpolating spline that ensures the trajectory passing through the set of nodes with the given value of derivatives $\dot{g}_i$ in the nodes, the system of equations (7), (8) ought to be added with the equations:

$$a_{1i} = \dot{g}_i, \tag{15}$$

$$\sum_{i=1}^{m} j a_{ji} = g_{i+1} \tag{16}$$

Taking into account that the resulted system has $4n$ equations, we can construct a third degree spline of defect 2 (under condition that $m = 3$):

$$a_{0i} = g_i;\ a_{1i} = \dot{g}_i;\ a_{2i} = 3\Delta_{i+1} - \dot{g}_{i+1} + 2\dot{g}_i;\ a_{3i} = -2\Delta_{i+1} + \dot{g}_{i+1} + \dot{g}_i \tag{17}$$

If the value of derivatives at the nodal points is not determined, the interpolating spline $S_{3,1}^I$ can be constructed in accordance with the condition (11). Then we have $a_{0i} = g_i;\ a_{1i} = \Delta_i;\ a_{2i} = 2\Delta_i^2;\ a_{3i} = -\Delta_i^2$ from the expressions of the equation (17).

The next step is to construct a spline that ensures continuity of the second derivative of the trajectory:

$$\ddot{S}^{i+1}(t_i) = \ddot{S}^i(t_{i+1}). \tag{18}$$

Obviously, in this case the conditions (9) and (10) are fulfilled. The degree of the spline, which ensures fulfillment of the condition (18) should not be lower than the third. We can obtain the fourth equation under the following condition:

$$\ddot{S}^i(t_i) = \Delta_i^2 \tag{19}$$

From the equations (18) and (19) we have $a_{2i} = 0{,}5\Delta_i^2;\ \ a_{3i} = \frac{1}{6}\Delta_i^3$. Taking into account $a_{0i} = k_1 g_{i+1} + k_2 g_i + k_3 g_{i-1}$ and substituting $a_{1i},\ a_{2i},\ a_{3i}$ in the equation (9), we obtain $0{,}5g_{i+2} + (k_1 - 0{,}5)g_{i+1} + (k_2 - 0{,}5)g_i + (k_3 - 0{,}5)g_{i-1} = k_1 g_{i+2} + k_2 g_{i+1} + k_3 g_i$, where $k_1 = 0{,}5; k_2 = 0; k_3 = -0{,}5$ and $a_{1i} = 0{,}5(g_{i+1} - g_{i-1})$. In an analogical manner we calculate $a_{0i} = \frac{1}{6}(g_{i+1} + 4g_i + g_{i-1})$ from the equation (10). The resulted spline $S_{3,1}^A$ is an approximating one.

The next task is to construct an interpolating spline that ensures a movement with the set speed $\dot{g}_i$ and acceleration $\ddot{g}_i$ in the nodal points. If we need to determine the coefficients of this spline the equations (7), (8), (15) and (16) can be added with the next ones:

$$2a_{2i} = \ddot{g}_i, \tag{20}$$

$$\sum_{i=2}^{m} j(j-1)a_{ji} = \ddot{g}_{i+1}. \tag{21}$$

As a result, the coefficients a_{ji} of the fifth degree spline of defect 3 have been obtained: $a_{0i} = g_i$, $a_{1i} = \dot{g}_i$, $a_{2i} = 0{,}5\ddot{g}_i$, $a_{3i} = 0{,}5\ddot{g}_{i+1} - 1{,}5\ddot{g}_i - 4\dot{g}_{i+1} - 6\dot{g}_i + 10\Delta_{i+1}$, $a_{4i} = -\ddot{g}_{i+1} + 1{,}5\ddot{g}_i + 7\dot{g}_{i+1} + 8\dot{g}_i - 15\Delta_{i+1}$, $a_{5i} = 0{,}5\ddot{g}_{i+1} - 0{,}5\ddot{g}_i - 3\dot{g}_{i+1} - 3\dot{g}_i + 6\Delta_{i+1}$.

Using the obtained spline $S_{5,3}^I$ is appropriate when links of the manipulator are equipped with acceleration sensors that generate the value $\ddot{g}_i$. If the manipulating device is equipped only with position and velocity sensors it is appropriate to take that $\ddot{g}_i = \dot{g}_{i+1} - \dot{g}_i$. Then:

$$a_{2i} = 0{,}5(\dot{g}_{i+1} - \dot{g}_{i+1})$$
$$a_{3i} = 0{,}5\dot{g}_{i+2} - 6\dot{g}_{i+1} - 4{,}5\dot{g}_i + 10\Delta_{i+1}$$
$$a_{4i} = -\dot{g}_{i+2} + 9{,}5\dot{g}_{i+1} + 6{,}5\dot{g}_i - 15\Delta_{i+1}$$
$$a_{5i} = 0{,}5\dot{g}_{i+2} - 4\dot{g}_{i+1} - 2{,}5\dot{g}_i + 6\Delta_{i+1}$$

3. Conclusion

Approximating splines should be used when it is necessary to only ensure continuity of programmed movement or its derivatives. Under this condition the degree of an approximating spline is lower than the degree of an interpolating spline having the same degree of trajectory smoothness.

The results show that the proposed method of a trajectory formation ensures a smooth movement of HCT and their impactless contact at a point of positioning.

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ГЕОМЕТРИЧНЕ МОДЕЛЮВАННЯ ТРАЄКТОРІЇ СКЛАДНОГО РУХУ МАНІПУЛЯЦІЙНИХ ГІДРООЧИЩУВАЛЬНИХ ОРГАНІВ

Виконано моделювання траєкторії складного руху маніпуляційних гідроочищувальних органів з використанням локальних сплайн-функцій. Важливо підкреслити, що вищезгадані функції не вимагають попереднього аналітичного визначення коефіцієнтів сплайну та забезпечують просте технічне виконання.

Ключові слова: гідроструминний виконавчий орган, складна траєкторія, маніпулятор, сплайн – функції, позиціонування, ступінь плавності руху, вузлові точки простору.

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