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A NEURON STOCHASTIC SIGMOID FIRING FUNCTION MODEL CONSTRUCTED ON THE MULTI-OPTIONAL FUNCTIONS ENTROPY CONDITIONAL OPTIMALITY DOCTRINE

It is made an attempt to propose some appropriate models of the friction and wear processes that could happen in the considered structural elements of the engineering units. The uncertainty measure in the given consideration is the entropy of the special hybrid-optional effectiveness functions. Such kind of entropy originates from the Jaynes' principle, being adapted to the subjective entropy maximum principle, with the implementation possibilities to the applicable fields of aviation industry as that follows the readings of the references. The paper theoretically considers the possibility of the multi-optional hybrid functions entropy conditional optimization doctrine applicability with the purpose of discovering substantiated reason for the special hybrid-optional effectiveness functions existence, as well as the reasons for the formula optimality. The neuron model activation function, or a squashing function, of a sigmoid type function like logistic function, formula is obtained with taking into account the degree of uncertainty for a certain type hybrid-optional effectiveness functions. With the help of the variational principle it is shown the optimality of the formula. The evolution of the proposed at this paper approach from the subjective analysis to the hybrid multi-optional functions optimization doctrine implies the use of the hybrid multi-optional functions, as an objectively existing characteristic of a phenomenon, instead of the subjectively preferred, by a human, functions, since no one chooses the objectively existential reality. The approach has a significance of a plausible explanation for the phenomena stipulated by multi-optionality. Theoretical speculations are illustrated with a plotted diagram presenting conducted numerical simulations.

Keywords: *tribology; friction; wear; optimization; entropy doctrine; multi-optionality; hybrid optional function; neuron activation; sigmoid function.*

Introduction. A few centuries past as the Great Swiss Mathematician Leonhard Euler said about that, in everything we see around us, we may notice some features or attributes of a certain maximum or minimum [1, p. 15]. Now we are going to try to apply that attitude to the scientific areas of friction [2], functional coatings applications for strengthening and restoration of aviation products [3], reliability, maintainability and risk [4], gearings with increased teeth wear resistance [5], and others, possibly be modeled with the neural networking [6, 7].

In all enumerated spheres [2-5] and more there are some processes observed which can be described with the neural networking formalisms [6, 7].

Neurobiological analogy used at neural networks input-output mapping implies neuron models developing [6]. Activation functions $\varphi(v)$ are being considered as mathematical dependences upon the induced local fields or activation potentials v of the neurons [6, pp. 43-48, (1.1)-(1.14)].

There is a stochastic neuron model which on the contrary to the deterministic model means that an input signal transformation into the corresponding output signal does not predetermined with the probability equaled "one" for all values of the input signal [6, p. 48].

State of the problem. The working hypothesis is the postulate of a certain functions entropy conditional optimum [1, 6]. The Jaynes' principle of a maximum-entropy and its methods proposed in references [8-10] on statistical physics have been successfully applied to theoretical models constructing in both the active systems theory [1] and neural networking [6, pp. 629-632, § 10.3].

Entropy research through all the science has a growing trend according to the study of reference [11]. Canonical view distributions of subjective individual preferences, taking into account a characteristic of the human-being decision making behavior [1] in an explicit function, have the same view as the probability of a choice [12, 13]. Although the latter use an axiom from [14, 15]; while in [1, 16, 17] it is derived in the style of a theorem proof following the preceding accepted postulate of optimality.

Some explanations of the maximum entropy principle pertaining with the socio-economical systems and psychological aspects (on an example of the light and shadow proportions of the shadow economy) are given in the latest papers [16, 17].

The popularity of the maximum entropy principle is based, apparently, upon the good evidences of the resulted expressions.

Evolution of the subjective entropy maximum principle [1] resulted in emergence of the multi-optional hybrid functions entropy conditional optimality doctrine initiated and developed in [18-27].

Problem statement. In accordance with [6, p. 48] for some applications it better to use stochastic neural network models than the determined ones. The activation function then has a probabilistic interpretation. Probability $P(v)$ is described with the sigmoid function of the following kind, [6, p. 48, (1.15)]:

$$P(v)=\frac{1}{1+\exp\left(-\frac{v}{T}\right)}, \tag{1}$$

where T is an analogue of the temperature used to control the noise levels, thus, uncertainty degree switching.

In such models a neuron can be in one of two states: + 1 or – 1. The decision about the neuron state switching is made with respect to the probability of the event [6, p. 48]:

$$x=\begin{cases}+1, \textit{ with probabilit y } P(v);\\ -1, \textit{ with probabilit y } 1-P(v).\end{cases} \tag{2}$$

Here symbol x means the neuron state. It is important to notice that T does not describe the physical temperature of the neural network either biological or artificial. Parameter T controls thermal fluctuations presenting the synaptic noise effect. Also remarkable that, if parameter T tends to zero, then the stochastic neuron described with the expression of (1) takes the determined form (without the noise) of the McCalloch-Pitts neuron [6, p. 48].

Purpose of the paper. The presented paper is aimed at the sigmoid probability of a neuron firing functions $P(v)$ modeling on the basis of the developed doctrine about the multi-optional functions entropy conditional optimality.

Problem setting. In order to reveal the optimality of equation (1) [6], it is applied the prototype model of subjective analysis [1] and optional functions doctrine [18-27]:

$$\Phi_h=-\sum_{i=1}^{n}h_i(v_i)\ln h_i(v_i)-\frac{1}{T}\sum_{i=1}^{n}h_i(v_i)v_i+\gamma\left(\sum_{i=1}^{n}h_i(v_i)-1\right), \tag{3}$$

where $h_i(v_i)$ is the hybrid multi-optional function (objective fundamental value of the process) deemed to be relevant to the induced local field or activation potential v_i; $-1/T$ is the other essential parameter, the sign "minus" in front of it ensures higher values of the hybrid multi-optional function $h_i(v_i)$ distributed for the lower values of the induced local field v_i at $T > 0$; γ is the coefficient for the normalizing condition.

On the condition of the hybrid multi-optional function $h_i(v_i)$ optimality for (3)

$$h_i(v_i) = \frac{\exp(-v_i/T)}{\sum_{j=1}^{n} \exp(-v_j/T)}. \tag{4}$$

For any two induced local fields or activation potentials [6, p. 43, (1.3)] v_1 and v_2, at $n = 2$ equations of (4) give the optimal distribution for the objective functional (3).

If each of the induced local fields v_1 and v_2 is compared with the threshold activation potential v_0, the objective functional likewise (3) optimization leads to the solution in the view of canonical expressions equivalent with (4), however separately for the pairs of v_0, v_1 and v_0, v_2 correspondingly.

Making simplest transformations, it is possible to get the following equation in general case, for any separate pair of v_0 and v_i

$$h_{0/i} = \frac{1}{1 + \exp\left[-\frac{v_i - v_0}{T}\right]}. \tag{5}$$

Comparing equations (5) and (1) one can notice that

$$P(v) = h_{0/i}, \qquad v = v_i - v_0. \tag{6}$$

Model (1-6), at $T = 2$; $v_0 = 0$; $v_1 = -10\ldots10$; $v_2 = 2$; gives result shown in fig. $f_i(0, v_1, 2)$ – solid red, dash blue, and green dot lines, are (4) at $i = \overline{0,2}$ and $n = 3$.

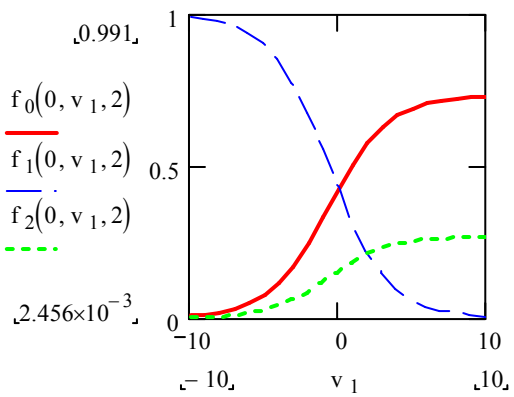

Fig. Hybrid multi-optional function

Conclusions. It is revealed a possibility of the hybrid combined relative pseudo-entropy application for the neuron stochastic model sigmoid firing function at the consideration of the probability of a neuron firing function as a logistic one. Parameters of the problem need further investigation.

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МОДЕЛЬ АКТИВАЦІЇ НЕЙРОНУ ЗІ СТОХАСТИЧНОЮ ФУНКЦІЄЮ АКТИВАЦІЇ ПОБУДОВАНА НА ДОКТРИНІ УМОВНОЇ ОПТИМАЛЬНОСТІ ЕНТРОПІЇ БАГАТООПЦІЙНИХ ФУНКЦІЙ

У наведеній роботі здійснено спробу запропонувати певні моделі, що підходять, процесів тертя та зношування, котрі могли би відбуватися в конструктивних елементах технічних виробів, які беруться до розгляду. Міра невизначеності у даному розгляді це є ентропія спеціальних гібридно-опційних функцій ефективності. Ентропія такого виду походить від принципу Джейнса, будучи адаптованою до принципу максимуму суб'єктивної ентропії, із імплементаційними можливостями до авіаційних галузей застосування, як це походить зі списку наведених посилань. Стаття розглядає теоретично можливість застосування доктрини умовної оптимізації ентропії багатоопційних гібридних функцій з метою відкриття обґрунтованої причини існування спеціальних гібридно-опційних функцій ефективності, а також причин оптимальності формули, що наводиться. Така формула, що вжита у якості моделі функції активації нейрону, або так звана «*squashing function*», типа сигмоїдальної функції, подібної до логістичної функції, отримується з урахуванням ступеня невизначеності певного типу гібридно-опційних функцій ефективності. За допомогою даного варіаційного принципу було показано оптимальність такого виду формули. Еволюція запропонованого у цій статті підходу, який є розвитком від суб'єктивного аналізу до оптимізаційної доктрини гібридних багатоопційних функцій, передбачає використання вказаних гібридних багатоопційних функцій, у якості об'єктивно існуючої характеристики певного явища, замість описаних суб'єктивно переважних, людиною, функцій, оскільки ніхто не обирає об'єктивно існуючої реальності. Даний підхід має значущість правдоподібного пояснення для таких явищ, обумовлених багатоопційністю. Теоретичні міркування проілюстровано побудованою діаграмою, яка представляє проведене числове моделювання.

***Ключові слова*:** трибологія; тертя; зношування; оптимізація; ентропійна доктрина; багатоопційність; гібридна опційна функція; активація нейрону; сигмоїдальна функція.

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