LAMINA ELASTIC CONSTANTS IDENTIFICATION BASED ON DYNAMIC TESTING AND REFINED THEORY

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Abstract *—Elastic constants of laminates have been determined by using a multi-level modelling and combined identification procedure. The necessary conditions for the existence and uniqueness of identification procedure based on eigen-frequencies calculation and combined identification procedure have been established. Some practical algorithms for modules identification are proposed and proofed.*

Keywords*: Laminated structure, beam and plate vibration, eigen-frequencies, elastic constants, combined identification procedure*

Introduction

Noise and vibration are of concern with many mechanical systems including industrial machines, home appliances, surface vehicle transportation systems, aerospace systems, and building structures. Many such mechanical structural system components are comprised of beam and plate like elements. Such structure is typical for aerospace design. The vibration of beam and plate structural systems can be reduced by the use of passive damping, once the system parameters, such as dynamic stiffness of the plate or beam, have been identified [1,2]. It is important to identify the plate or beam structural system dynamic parameters no matter what methods of noise and vibration control are eventually chosen.

The rapid increase in the industrial use of these structures has necessitated the development of new analytical and numerical tools that are suitable for the analysis and study of the mechanical behaviour of such structures. The determination of stiffness parameters for complex materials such as fibre-reinforced composites ore multilayer design is much more complicated than for isotropic materials, since its are anisotropic and non-homogeneous. Many different approaches have now been produced for the identification of the physical parameters which directly characterise structural behaviour.

Since the late 1950s, many papers have been published on the vibration of sandwich structures [3-7]. All of the models discussed so far are based on the following assumptions: a) the viscoelastic layer undergoes only shear deformation and hence the extensional energy of the core is neglected; b) the face sheets are elastic and isotropic and their contribution to the shear energy is neglected, and c) in the face-sheets, plane sections remain plane and normal to the deformed centrelines of the face-sheets. However, as the frequency increases, the results calculated from these models disagree strongly with measurements.

For modelling laminated composite plates, it is important to have an effective general theory for accurately evaluating the effects of transverse shear stresses on the plate performance. It has long been recognized that higher- order laminated plate theories may provide an effective solution tool for accurately predicting the deformation behaviour of composite laminates subjected to bending loads [8-12].

This study aims to predict the elastic properties of composite laminated plates. The present method for the modelling of laminated composite plates does not rely on strong assumptions about the model of the plate. In this paper, numerical evaluations obtained for vibrations in isotropic, orthotropic and composite laminated plates have been used to determine the displacement field for the efficient analysis of vibrations in laminated composite plates. The numerical method developed follows a semi-analytical approach with an analytical field applied in the longitudinal direction and a layer-wise displacement field employed in the transverse direction.

Adaptive plate cylindrical bending equations

Let us consider now a symmetrical three-layered plate of thickness *$2H_p$* and core of thickness *$2H$* (Fig.1.):

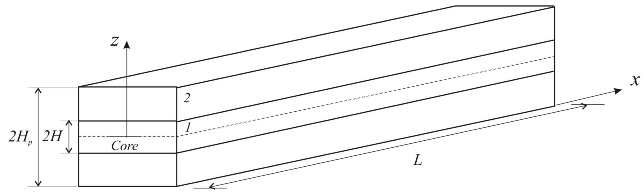


Fig. 1. Sandwich plate scheme

Let us consider cylindrical bending of plate by such kinematic assumptions ([8-12])

$$U_e - \begin{cases} u = \sum_{i,k} u_{ik}{}^e z^{2i-1} \varphi_k(x), & 0 < z < H \\ w = \sum_{i,k} w_{ik}{}^e z^{2i-2} \gamma_k(x), & 0 < x < L \end{cases} ; \quad U_d - \begin{cases} u = \sum_{i,k} u_{ik}{}^d (z-H)^i \varphi_k(x), & H < z < H_p, \\ w = \sum_{i,k} w_{ik}{}^d (z-H)^i \gamma_k(x) & 0 < x < L. \end{cases} \quad (1)$$

Here $\varphi_k(x)$, $\gamma_k(x)$ – are a priory known coordinate functions (for every beam clamp conditions), $u_{ik}{}^e$, $w_{ik}{}^e$, $u_{ik}{}^d$, $w_{ik}{}^d$ – unknown set of parameters. The solutions which express Hooke's law with respect to the stress components have the form

$$\sigma_{xx} = C_{xx}\varepsilon_{xx} + C_{xz}\varepsilon_{zz}, \quad \sigma_{zz} = C_{zx}\varepsilon_{xx} + C_{zz}\varepsilon_{zz}, \qquad \tau_{xz} = G\gamma_{xz}. \quad (2)$$

By substituting Eqs. (1,2) into the following Hamilton-Ostrogradsky variation equation

$$\int_{t_1}^{t_2}\left(\int_V (\sigma_{xx}\delta\varepsilon_{xx} + \sigma_{zz}\delta\varepsilon_{zz} + \tau_{xz}\delta\varepsilon_{xz} - \rho\frac{\partial u}{\partial t}\delta\frac{\partial u}{\partial t} - \rho\frac{\partial w}{\partial t}\delta\frac{\partial w}{\partial t})dV\right)dt + \int_{t_1}^{t_2}\left(\int_V (\int_{S_K} KU\delta U dS + \int_{S_P} P\delta U dS\right)dt = 0. \quad (3)$$

(V- beam volume, S_K – clamp contact surface, S_P – boundary forces surface t_i – arbitrary time moment) for Winkler foundation clamp conditions with the rigidity coefficient K and also assuming single frequency vibration ($u_{ik}^e = \overline{u}_{ik}^e e^{i\omega t}$, $w_{ik}^e = \overline{w}_{ik}^e e^{i\omega t}$, $u_{ik}^d = \overline{u}_{ik}^d e^{i\omega t}$, $w_{ik}^d = \overline{w}_{ik}^d e^{i\omega t}$) we obtain the set of linear algebraic equations for the amplitudes.

$$[A]\overline{U} = \begin{bmatrix} A_1 & A_d \\ A_d^T & A_2 \end{bmatrix}\begin{bmatrix} \overline{U}_e \\ \overline{U}_d \end{bmatrix} = f. \quad (4)$$

For a greater number of lamina equations (1) has the following form for each additional layer

$$U_d^n - \begin{cases} u = \sum_{i,k} u_{ik}\left(z - H^{(n)}\right)^i \varphi_k(x), & H^{(n)} < z < H^{(n+1)}, \\ & 0 < x < L, \\ w = \sum_{i,k} w_{ik}{}^n \left(z - H^{(n)}\right)^i \gamma_k(x), & n = 1, \ldots, N, \end{cases} \quad (5)$$

Here $H_p^{(n+1)} - H_p^{(n)} = H_n$, $H_p^{(1)} = H$; H is one-half of the thickness of the core and $H^{n+1} - H^n$ are thicknesses of the n*-th* layer, respectively. Matrix $[A]$ is found by double integration through the thickness and along the length of the beam, and can can be represented as follows.

$$[A] = \begin{bmatrix} A_1 & A_{1d}^1 & A_{2d}^1 & \ldots & A_{N-1,d}^1 & A_{Nd}^1 \\ A_{1d}^{1,T} & A_2 & A_{1d}^2 & \ldots & A_{N-2,d}^2 & A_{N-1,d}^2 \\ A_{2d}^{1,T} & A_{1d}^{2,T} & A_3 & \ldots & & \\ & & & \ldots & & \\ & \ldots & & & A_{N-1} & A_{1d}^{N-1} \\ & \ldots & & & A_{1d}^{N-1,T} & A_n \end{bmatrix} \quad (6)$$

Note that, $N = 1$ and $N = 2$ represent the cases of symmetrical three- and five-layered plates, respectively.

Experimental setup

Dynamic bending of beams from a foam plastic 3715 and 6718 (General Plastic, USA specification) and three-layered (sandwich) beam with face layers from carbon fibre composite material and a core from a foam plastic 3715 are tested. Length of all beams – *0,611* m, width and height of section accordingly *b = 0,028* m; *h= 0,0264* m, thickness of face layers – *t=0,0005* m. Material density of foam plastic 3715 is *240* kg/m^3 and 6718 – *288* kg/m^3 accordingly, and linear sandwich weight is *0,207* kg/m. In the experiment the beam was excited with white noise by a shaker mounted at the middle. The experimental Bruel&Kjaer PULSE system analyzes the stochastic signals. Measurings of eigen- frequencies (Tabl. 1) were conducted. Beams (material *6718*, section b × h = *12,7 × 25,4* mm^2 and length *0.6*m) experimental eigen-frequencies are*Table 1*

Mode	frequencies $f_i^{\exp}$, Hz	Mode	frequencies $f_i^{\exp}$, Hz
1	60	5	1154
2	144	6	1666
3	376	7	2279
4	713	8	2953

Coordinate functions in (1) are taken as trigonometric functions:

$$\varphi_k(x) = \sin\left({(2k-1)\pi x}/{2L} \right),$$

$$\gamma_k(x) = \cos\left({(2k-2)\pi x}/{2L} \right).$$

Identification

A detailed sensitivity study has shown that the solution of the one level optimization problem is very sensitive to the variations of one group of modules match less sensitive to another module. Since the existence of noise in measurement data is inevitable, the above formulation of the material constants identification problem then becomes inappropriate if the determination of accurate elastic constants is desired. This may produce erroneous results even when the variations of the measured frequencies or other characteristics are relatively small.

The general form of error function is as follows [2]

$$F_C = \sum_i^{N_f} \alpha_i \frac{\left| f_i^{\exp} - f_i^c \right|^2}{{f_i^{\exp}}^2}. \quad (7)$$

The numeral calculation of vibrations is conducted on the basis of (1) –(6) taking into account elastic and inertias properties of beam and elements of vibrator. A quadratic rejection as a function of error was examined between the experimentally measured

f_i^{exp} and calculated values f_i^c of eigen-frequencies.

In a Fig. 2 the processes of identification of the modules of homogeneous beam by genetic algorithms (GA) is shown. Here *N* is the number of steps in GA.

Combined identification schemes

From a [8-12] is evidently, that it is possible to use additional information for one-digit determination of the transversal modules E_2, ν, for an internal layer and simultaneously modules of external layer. For this purpose sufficiently, that hyper surfaces of their values of the same levels of function of error were concave and tightened pithily at diminishing of level. If on some criteria maps are bars it is necessary to search variants, where at other schemes of experiment they become concave.

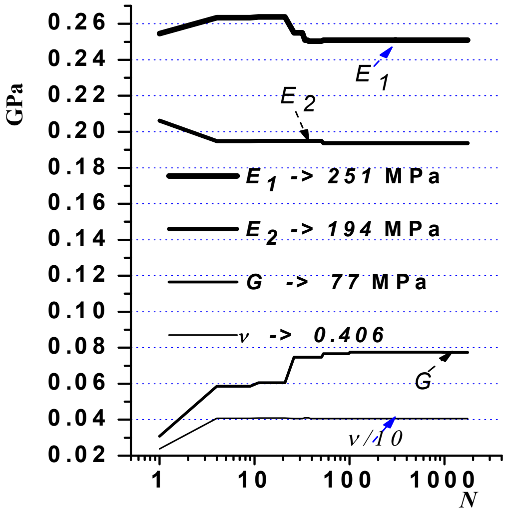


Fig. 2. Incremental identification of the modules E_1, E_2 *,* ν*, G of homogeneous beam: material 6718*

Here *α, β*, are weight coefficients. They were determined on the basis of numeral experiments in the process of minimization and approximately corresponded to the equal deposit of errors.

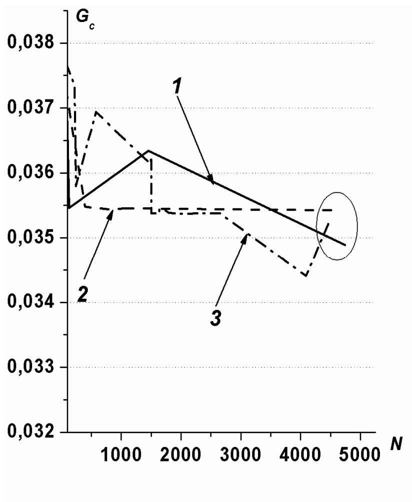
We will consider identification of the modules on the basis of the simultaneous use of information on a homogeneous beam and beam with an internal layer, to identical mechanical properties to the homogeneous beam. The error function we will take in a kind (9). For parameters will take the modules E_f, G_c, E_{1c}, E_{2c}, ν. the other modules of sandwich were set approximately, so their small influence is shown on dynamic and static properties. On a Fig. 3 the processes of identification by means of GA are resulted.

In spite of single identification method (see Fig. 2) now a process is acquired by stable character.

Values of the shear modules *G* for a beam from material 3715 are within the limits of 33-35MPa. Longitudinal Young module for this purpose material lies within the limits of 173-179 MPa. Longitudinal module Young module for material 6718 lies in an interval 240-260 Mpa, value of the shear module within the limits of 51-55 MPa. The got results are near to information of company General Plastics (THE USA).

Conclusion

The module identification by dynamic tests of FRF functions is not a trivial tusk. This problem needs both the various experimental schemes and a various refined difference functions. The direct application of any search method ore nonlinear programming or genetic algorithms or neural nets in many cases brings doubtful results. The strategy of an anisotropic beam module identification seem to be such: The raw of models can be applied at different vibration conditions of the beam by a suitable analytical ore approximation method, research of sensitiveness in relation to the parameters of fixing and material anisotropy, numerical experiments on identification of elastic modules, practical module identification by exploring different schemes of experimental setup and, finally, posterior analysis of identification quality. The combined method of identification was proposed on the basis of the simultaneous use of information on a homogeneous beam and beam with an internal layer, to identical mechanical properties to the homogeneous beam. The one-digit values are established not only for longitudinal Young module and shear module, but also for transversal elastic modules.



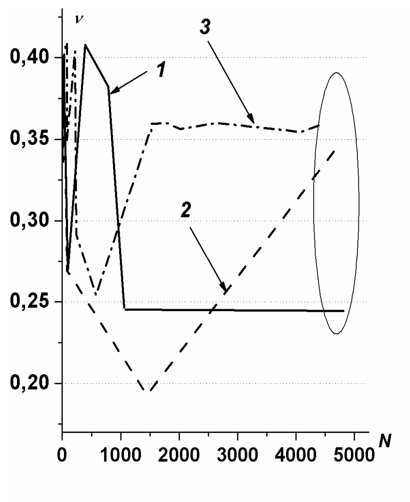


Fig. 3. Incremental identification of the shear modules in the combined chart after parameters G_c, *ν (1 – uniform beam; 2 – sandwich; 3 – combined scheme*

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