

**COMPARISON OF SUPERCONDUCTING MAGNETS WITH MECHANICAL SUPPORT ELEMENTS PLACED INSIDE TORUS****Yu.M.Vasetsky, I.L.Mazurenko, A.V.Pavlyuk****Institute of Electrodynamics National Academy of Science of Ukraine,****Peremogy Av., 56, Kyiv-57, 03680, Ukraine,****e-mail: [Yuriv.Vasetsky@gmail.com](mailto:Yuriv.Vasetsky@gmail.com)**

*The open space inside torus of superconducting magnetic devices, as against a fusion device, may be used for placing elements of mechanical support structure. The toroidal systems with O-shaped and racetrack-shaped coils and support elements inside torus are investigated in this paper. Two problems are solved proposed magnets configurations: elimination of bending moments in the support system and ensuring uniform mechanical stress in all support elements. Support system requirements as well as the volume of a superconducting winding and sizes of toroidal system are analyzed. The racetrack-shaped torus support system requirements are approximately the same as corresponding values for O-shaped torus with spokes and for traditional systems with geometrically more complicated banded D-shaped coils. The volume of superconducting winding of toroidal system with racetrack coils is considerably lesser in comparison with O-shaped coils and approaches to theoretical minimum for D-shaped torus. Also, the toroidal magnet with racetrack coils has minimal radial sizes as compared with other researched configurations. References 25, tables 2, figures 8.*

**Key words:** toroidal superconducting magnetic energy storage, support system, spokes inside torus, size and mass parameters.

**Introduction.** Recent studies have explored some of the important benefits that large-scale SMES technology could provide to both the electric power industry [1,2,12] including applications in power systems with renewable energy sources [3, 4] and industrial end-users of electric power, particularly those that critically depend on a supply of reliable, high quality electricity, such as computer chip and pharmaceutical manufacturers, and electric arc steel mills.

Nowadays it is designed SMES with different value of stored energy. The energy of magnets for electric power industry has values from 1-10 MJ [11,16,19] up to  $10^3$ - $10^5$  MJ for large-scale devices [8,15,17,24]. The mass of the support structure increases proportionally to the magnetic energy [13] and for magnets with large accumulated energy its mass constitutes a major fraction of the mass of all structural elements. One of the primary challenges in designing suitable SMES devices is to accommodate as efficiently and economically as possible the large mechanical stresses arising from the electromagnetic forces.

Two ways are known to reduce the mass of the magnet support system in toroidal energy storage devices: application of a helical winding configuration resulting in reduced forces [5,21,22,25] and application of constant tension D-shaped toroidal coils [9], [20]. In both cases, the winding geometry and overall design of the magnet systems become complicated in comparison with storage using simpler and cheaper circular, O-shaped coils.

Support systems usually contain clamping belts around coil windings to resist the large forces that are developed when the coils are energized. Because tension is not constant and bending moments are not zero in toroidal systems with O-shaped coils, the mass of the support structure becomes much greater than the idealized value calculated based on average tension and zero bending moments [7]. As developed in magnetic fusion research, toroidal systems with D-shaped coils provide a more optimal distribution of mechanical stresses than those with O-shaped coils, eliminating bending moments and producing constant tensile stress along the length of the coil case. However, these devices are more complicated and expensive to fabricate, resulting in preference often given to SMES devices with O-shaped coils.

Many studies have sought to define and develop the most advantageous method of supporting the coils of toroidal fusion devices, however, none of these studies has considered internal spoke supports since the center of the torus of a fusion device contains plasma inside a vacuum vessel. SMES devices, however, do not have such restrictions, and may take advantage of the open space inside the individual toroidal coils. For this reason, the conceptual design of SMES with spokes placed inside of torus has been proposed in [10, 14, 23]. The main aim of this paper is to investigate toroidal systems with different configurations of coils to reach the minimization of both the volume of structural materials and the mass of a superconductor. For this purpose the toroidal magnets composed of circular-shaped coils with various support systems and the

magnets consisting of racetrack-shaped coils with spokes are considered. In this type of spoke design all components of the support system have no bending moments. There are only either tension (spokes) or compression (support rings). The main features of proposed SMES spoke support systems and its parameters as well as comparisons with more conventional SMES designs are presented below.

**Mathematical model.** An ideal model of toroidal system that has a sufficient number of coils is

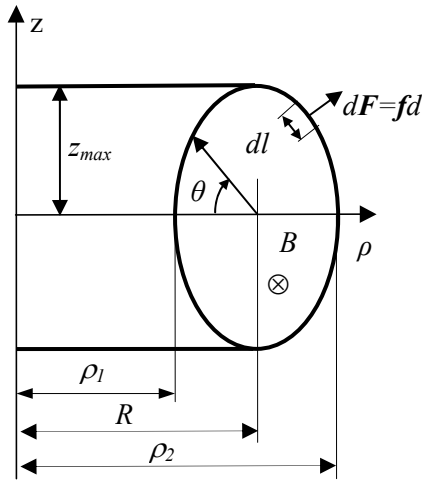


Fig. 1

considered. The configuration of each filament is the same as the configuration of the curvilinear axis of a coil. The following parameters define the geometrical sizes and the configuration of toroidal system: a large radius  $R$ , a relative radial size of torus cross-section (aspect ratio)  $\varepsilon$  and ratio of the vertical size to the radial size of torus cross-section  $\lambda$  (Fig. 1):

$$R = (\rho_1 + \rho_2) / 2, \quad \varepsilon = (\rho_2 - \rho_1) / (\rho_2 + \rho_1), \quad \lambda = 2z_{\max} (\rho_2 - \rho_1)^{-1}. \quad (1)$$

The magnetic field confined within the torus and the induction  $B$  has only an azimuthal component, which is inversely proportional to the distance from the  $z$ -axis

$$B = \mu_0 I N (2\pi\rho)^{-1}, \quad (2)$$

where  $N$  is the total number of coils in the torus,  $I$  is the current per coil ( $NI$  is the total of ampere turns in the current sheet or the total poloidal current of the torus).

Equation for energy stored in magnetic field is determined by integration with respect to the volume of torus  $V$  and can be written as

$$W = \int_V \frac{B^2}{2\mu_0} dV = \frac{\mu_0 I^2 N^2 R}{4} k_W, \quad (3)$$

where  $k_W$  is a dimensionless parameter.

Each incremental length  $dl$  of the coil experiences an electromagnetic force  $dF = f dl$  which is perpendicular to the surface. The distribution of linear force density,  $f$  along the perimeter of each coil, is given by

$$f = IB^+ / 2, \quad (4)$$

where  $B^+$  is magnetic fields inside the torus near its surface.

The total electromagnetic force  $F_R$  that acts on each coil is non-zero and it is directed to vertical  $z$ -axis

$$F_R = \int_l f(l) \cos\theta dl. \quad (5)$$

**Superconducting magnets with support structure inside torus.** The toroidal magnets with circular and racetrack shaped coils and support structure inside of torus are investigated below. These toroidal configurations have some similar elements and its main parameters are the same while as relative vertical size of torus is equal to  $\lambda=1$ . At first more simple configuration with support structure placed inside circular coils (Fig. 2) is examined. Then it is examined the magnets with racetrack shaped coils ( $\lambda > 1$ ) that have some additional elements (Fig. 3).

**1. Toroidal magnets with circular coils and support rings outside of torus.** Toroidal system with circular coils of radius  $a$  has geometrical parameters  $\varepsilon = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1} = \frac{a}{R}$  and  $\lambda = 1$ . The energy stored in the magnetic field of torus is given by [20]

$$W = 0,25\mu_0 I^2 N^2 R k_W, \quad k_W = 2 \left( 1 - \sqrt{1 - \varepsilon^2} \right) \quad (6)$$

and the linear density force is

$$f = \frac{1}{2} IB = \frac{\mu_0 I^2 N}{4\pi(R - a \cos\theta)}. \quad (7)$$

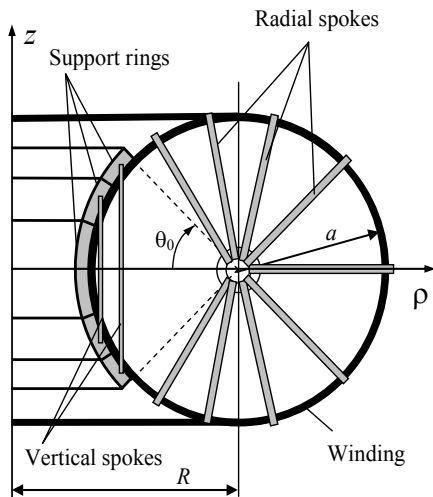


Fig. 2

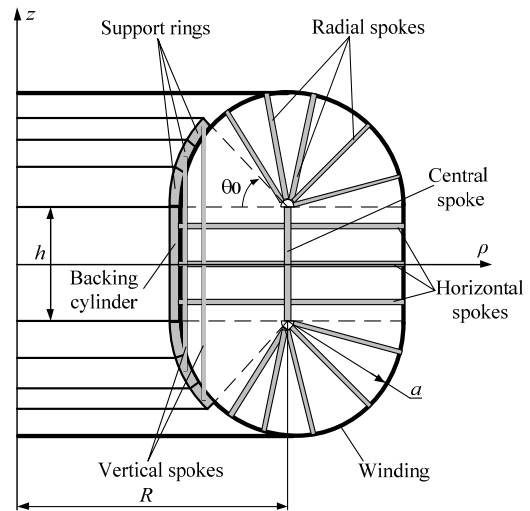


Fig. 3

Instead of clamping belts, each circular coil is supported by a number of internal spokes. The spoke system aims to solve two mechanical problems: to eliminate the bending moments in the support system and to ensure a uniform mechanical stress in all spokes.

The main components of toroidal SMES and support rings arranged outside of torus are shown in Fig. 2. All radial spokes are connected at one central unit. As the central unit has no additional support, the sum of forces in the spokes must be equal to zero. For this reason the radial spokes are absent in the sector with an angular size  $2\theta_0$  as it is seen in Fig. 2.

The value of the angle  $\theta_0$  is determined by 
$$\int_{\theta_0}^{\pi} f(\theta) \cos \theta d\theta = 0. \quad (8)$$

Combining (7) and (8) the following equation to determine the angle  $\theta_0$  is given as

$$\pi - \theta_0 - \frac{2}{\sqrt{1 - \varepsilon^2}} \tan^{-1} \left[ \frac{\sqrt{1 - \varepsilon}}{\sqrt{1 + \varepsilon}} \tan^{-1} \left( \frac{\theta_0}{2} \right) \right] = 0. \quad (9)$$

The electromagnetic forces distributed along the arc  $-\theta_0 \leq \theta \leq \theta_0$  must be resisted by additional components, support rings, of the support system. The forces arising in the support rings compensate for the radial component of magnetic forces and, therefore, the support rings are in compression. The z-directed forces that act on the support rings are equilibrated by vertical spokes, which are in tension.

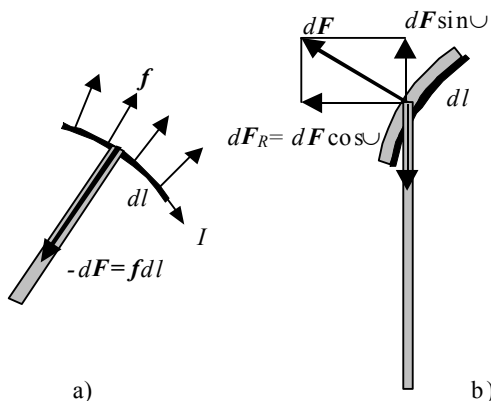


Fig. 4

In this proposed support structure design, all components of the support system have no bending moments. They are either in tension (radial and vertical spokes) or compression (support rings).

To determine the required volumes of the structural materials, radial and vertical spokes as well as support rings, it is assumed that all spokes have equal stresses  $\sigma_t$  in tension and all support rings have equal stresses  $\sigma_c$  in compression.

All spokes are assumed to have equal stresses in tension  $\sigma_t$ . The support ring has stress in compression  $\sigma_c$ .

If the radial spokes are located densely enough, the mechanical stress in each spoke can be approximated as

$\sigma_t = dF / dS = f(l) dl / dS$ , where  $dS$  is the area of the radial spoke cross section (Fig. 4, a).

The volume element of the spoke is given by  $dV_t = adS = (a / \sigma_t) f(l) dl$ . For circular coils the elemental length of perimeter is  $dl = a d\theta$ . The total volume of the spokes is determined by the integration for  $\theta$  gaping from  $\theta_0$  to  $2\pi - \theta_0$ . The volume of the radial spokes of all  $N$  coils is given as

$$V_{t1} = \frac{2a^2 N}{\sigma_t} \int_{\theta_0}^{\pi} f(\theta) d\theta = \frac{\mu_0 I^2 N^2 R}{2\pi\sigma_t} \varepsilon^2 (\pi - \theta_0). \quad (10)$$

The volume of the vertical spokes is determined in the condition of equilibrium in the z-direction. In this case, as shown in Fig. 4b,  $\sigma_t = dF_z / dS_2 = f(\theta) \sin \theta a d\theta / dS_2$ , where  $dS_2$  is the area of the vertical spoke cross section. The length of the spokes is not constant,  $l_2(\theta) = 2a \sin \theta$ . After integration, the volume  $V_{t2}$  of vertical spokes of all coils is given as

$$V_{t2} = N \int_{-\theta_0}^{\theta_0} l_2(\theta) dS_2 = \frac{\mu_0 I^2 N^2 R}{2\pi\sigma_t} \varepsilon^2 \left[ \frac{\pi}{1 + \sqrt{1 - \varepsilon^2}} + \frac{\sin \theta_0}{\varepsilon} - \pi + \theta_0 \right]. \quad (11)$$

Note that the volume of the vertical spokes is very negligible and these spokes may have a rather theoretical interest as, in practice, they may be replaced by an unbroken non-straight cylinder as the compression structure [10].

The total volume of the structural materials that are in tension will be the sum of the radial and vertical spoke volumes,  $V_t = V_{t1} + V_{t2}$ . Utilizing a  $\mu_0 I^2 N^2 R$  factor appearing in the equation for stored energy  $W$  in (6), the volume of spokes is as follows:

$$V_t = W \sigma_t^{-1} Q_t, \quad \text{where} \quad Q_t = \pi^{-1} \left( 1 + \varepsilon (1 - \sqrt{1 - \varepsilon^2})^{-1} \sin \theta_0 \right). \quad (12)$$

The volume of the support rings that are in compression can be easily defined taking into consideration the fact that each support ring is under the action of the uniformly distributed forces, which are directed to the center of the ring. Each of the  $N$  coils presses down with force  $dF_R = dF \cos \theta$  on a ring element with length  $dl$  (Fig. 4, b). In this case, the ring is in compression with stresses  $\sigma_c = dF_R N / 2\pi dS_c$ , where  $dS_c$  is the area of a ring cross section element. The volume of each ring element will be  $dV_c = 2\pi \rho dS_c = \sigma_c^{-1} f(\theta) N a (R - a \cos \theta) \cos \theta d\theta$  and the volume of all rings is given by

$$V_c = \frac{aN}{\sigma_c} \int_{-\theta_0}^{\theta_0} f(\theta) \rho \cos \theta d\theta = \frac{\mu_0 I^2 N^2 R}{2\pi\sigma_c} \varepsilon \sin \theta_0. \quad (13)$$

Using (6), the volume of the structural materials in compression is

$$V_c = W \sigma_c^{-1} Q_c, \quad \text{where} \quad Q_c = \pi^{-1} \varepsilon (1 - \sqrt{1 - \varepsilon^2})^{-1} \sin \theta_0. \quad (14)$$

The comparison of (14) and (12) gives

$$Q_t = Q_c + 1. \quad (15)$$

A similar equation is valid for toroidal solenoids with momentless constant tension coils. The equation is also valid for magnets with O-shaped coils and support belts around the coils, but only if  $\sigma_t$  is the average stress value and the bending moments are not taken into consideration [6]. The research has shown that equation (15) has a more general application; it is appropriate for toroidal magnet systems with spokes as the tension structure.

**2. Toroidal magnets with racetrack shaped coils.** The toroidal systems with the racetrack coil configuration also allow the usage of spokes as a support structure, and systems with O-shaped coils make a particular case of this configuration. Each coil has upper and lower semicircular parts with radius  $a$  and a central straight part with length  $h$  (Fig. 3).

The geometrical parameters which characterize the toroidal magnet configuration with a racetrack-shaped system include a aspect ratio of torus  $\varepsilon = a/R$  and a ratio of the vertical to the radial sizes of the torus cross-section  $\lambda = (2a + h)/2a$ . In comparison with an O-shaped torus the racetrack-shaped coil system has additional elements, namely, a central spoke, horizontal spokes, a backing cylinder. As well as for O-shape torus, all construction elements have no bending moments, additional spokes have constant stresses in tension and the backing cylinder has constant stresses in compression.

Equation for dimensionless parameter  $k_W$  of the magnetic field energy  $W = \mu_0 I^2 N^2 R k_W(\varepsilon, \lambda) / 4$  is

$$k_W(\varepsilon, \lambda) = 2 \left( 1 - \sqrt{1 - \varepsilon^2} + \frac{\varepsilon(\lambda - 1)}{\pi} \ln \frac{1 + \varepsilon}{1 - \varepsilon} \right). \quad (16)$$

The sum of forces in the radial spokes must be equal to zero. So, the radial spokes are absent within the angles  $0 \leq \theta \leq \theta_0$ , where  $\theta_0$  is the same as for O-shaped coils (14). The electromagnetic forces here, as well as at the straight part, are balanced by the reaction compression forces in the support rings and the backing cylinder.

The choice of the permissible mechanical stress in vertical spokes allows to determine their cross section  $dS_2 = \sigma_t^{-1} f(\theta) a \sin \theta d\theta$ . Taking into account the fact that the lengths of vertical spokes are  $l_2(\theta) = 2(a \sin \theta + h/2)$ , after the integration for  $\theta$  from 0 to  $\theta_0$  the volume of all vertical spokes will be the following:

$$V_{i2} = \frac{\mu_0 I^2 R}{2\sigma_t} k_{i2}, \text{ where } k_{i2} = \frac{\varepsilon^2}{\pi} \left( \frac{\pi}{1 + \sqrt{1 - \varepsilon^2}} + \frac{\sin \theta_0}{\varepsilon} + \theta_0 - \pi + \frac{(\lambda - 1)}{\varepsilon} \ln \frac{1 - \varepsilon \cos \theta_0}{1 - \varepsilon} \right). \quad (17)$$

The horizontal spokes compensate the electromagnetic forces at the straight part of the winding on the external side of torus as shown in Fig. 5 a. If the spokes are located densely enough, their volume is

$$V_{i3} = \frac{2aN}{\sigma_t} \int f dl = \frac{\mu_0 I^2 N^2 R}{2\sigma_t} k_{i3}, \text{ where } k_{i3} = \frac{2\varepsilon^2(\lambda - 1)}{\pi(1 + \varepsilon)}. \quad (18)$$

The volume of the central spokes  $V_{i4} = hS_4$  is determined in the condition of equilibrium in the z-direction. The cross-section of all central spokes  $S_4 = F_z / \sigma_t$  is defined as the total force  $F_z$  that acts on the upper-half torus in the sector with the angular size  $\theta_0 < \theta < \pi$  (Fig. 5b). Therefore, this volume is given by

$$V_{i4} = \frac{hN}{\sigma_t} \int_{\theta_0}^{\pi} f \sin \theta dl = \frac{\mu_0 I^2 N^2 R}{2\sigma_t} k_{i4}, \text{ where } k_{i4} = \frac{\varepsilon(\lambda - 1)}{\pi} \ln \frac{1 + \varepsilon}{1 - \varepsilon \cos \theta_0}. \quad (19)$$

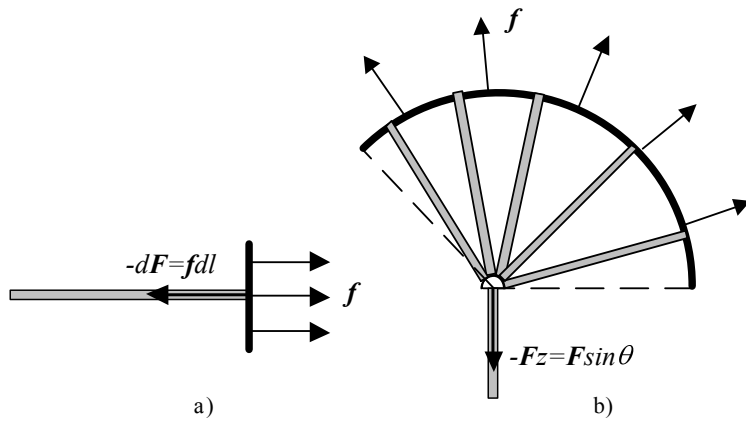


Fig. 5

The volume of the support rings is the same as for circular coils (13) and can be written as follows:

$$V_{c1} = (2\sigma_t)^{-1} \mu_0 I^2 N^2 R k_{c1}, \text{ where } k_{c1} = \varepsilon \pi^{-1} \sin \theta_0. \quad (21)$$

The backing cylinder is under the action of the uniformly distributed forces, which are directed to the center of the cylinder. This support element compensates the difference between the forces  $F_R = F_1 - F_2$ , acting on the straight part of the winding:  $F_1$  at  $\rho = \rho_1$  and  $F_2$  at  $\rho = \rho_2$  of the torus. The backing cylinder is in compression with stresses  $\sigma_c = F_R N (2\pi S_c)^{-1}$ , where  $S_c$  is the cross-section area of the backing cylinder wall. From this equation the volume of the backing cylinder is given by

$$V_{c2} = 2\pi \rho_1 S_c = \frac{\mu_0 I^2 N^2 R}{2\sigma_c} k_{c2}, \text{ where } k_{c2} = \frac{2\varepsilon^2(\lambda - 1)}{\pi(1 + \varepsilon)}. \quad (22)$$

The total volume of the structural materials being in compression is the sum of the support rings and the backing cylinder volumes  $V_c = V_{c1} + V_{c2}$ . Using (16), the volume of the structural materials in compression is

$$V_c = W \sigma_c^{-1} Q_c, \text{ where } Q_c = (k_{c1} + k_{c2}) k_W^{-1}. \quad (23)$$

The total requirements of the structural materials that are in tension is the sum of the radial, vertical and central spoke volumes  $V_t = V_{i1} + V_{i2} + V_{i3} + V_{i4}$ . Utilizing the factor  $\mu_0 I^2 N^2 R$  appearing in the equation for the stored energy  $W$  in (16), the volume of spokes is as follows:

$$V_t = \frac{W}{\sigma_t} Q_t, \text{ where } Q_t = \frac{k_{t1} + k_{t2} + k_{t3} + k_{t4}}{k_W}. \quad (20)$$

The volume of the support rings and the backing cylinder comprises the volume of the structural materials in compression.

The comparison of (20) and (23) gives the same condition  $Q_t = Q_c + 1$  as for O-shaped coils and in this case for racetrack-shaped coils it may be written as

$$\sum k_t = \sum k_c + k_w. \quad (24)$$

**3. Comparison of Support System Requirements.** If the permissible mechanical stresses in tension and compression are equal, a general dimensionless parameter characterizing the requirements of the structural materials can be applied. As the volume of the structural materials is determined by  $V = V_t + V_c = W \sigma^{-1} (Q_t + Q_c)$ , general parameter is the sum

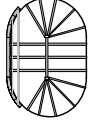
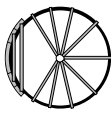
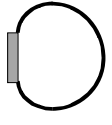
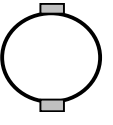
$$Q = Q_t + Q_c. \quad (25)$$

The values of the  $Q$  parameter for toroidal magnet systems with spokes at different aspect ratios of torus  $\varepsilon$  are presented in Table 1. The factor  $Q$  is compared for torus with spokes inside O-shaped coils and racetrack-shaped coils at  $\lambda = 1.89$ , and traditional configuration systems with support belts for O-shaped coils and D-shaped coils.

The results (Table 1) show that for the racetrack-shaped coils the  $Q$  parameter practically does not depend on  $\lambda$  and the volume of the support system requirements does not exceed the values for D-shaped toroidal system at the same  $\varepsilon$  as well as for O-shaped coils with spokes and support rings outside of torus.

**Size and volume of superconducting materials.** The total cost of superconducting magnets depends on the geometrical configuration which determines the size and the mass of storage. The superconducting winding consists of expensive materials so finding an optimal configuration is important.

**Table 1**

$\varepsilon$	*1	2	3	4
				
0.1	2.812	2.81	2.813	3.01
0.2	2.643	2.638	2.649	3.041
0.3	2.488	2.481	2.5	3.097
0.4	2.345	2.338	2.362	3.182
0.5	2.21	2.204	2.232	3.309
0.6	2.081	2.079	2.104	3.5
0.7	1.953	1.958	1.976	3.801
0.8	1.823	1.836	1.838	4.333

\*1 – Racetrack Coils, 2 – O-coils with spokes and support rings outside of torus, 3 – D-coils, 4 – O-coils with support belts

The maximal induction of the magnetic field  $B_m$  on the superconducting winding takes place on the internal side of torus at  $\rho = \rho_1$

$$B_m = \mu_0 IN (2\pi R)^{-1} k_B(\varepsilon), \text{ where } k_B(\varepsilon) = (1 - \varepsilon)^{-1} \quad (26)$$

Expressions (6) and (26) give a system of equations for large radius  $R$  and the total ampere turns  $IN$  of coils as functions of initial SMES parameters: energy  $W$  and the maximal value of the magnetic field  $B_m$ . Using (6) and (26) the required parameters are the following:

$$R = \frac{W^{1/3} \mu_0^{1/3}}{B_m^{2/3}} k_R(\varepsilon), \text{ where } IN = \frac{W^{1/3} B_m^{1/3}}{\mu_0^{2/3}} k_I(\varepsilon). \quad (27)$$

Here dimensionless parameters are

$$\text{where } k_I = 2\pi^{1/3} (k_w k_B)^{-1/3}. \quad (28)$$

Now let us determine the equations to define the volume of the superconducting winding of all coils with constant cross-section

$$V_{sc} = NSl. \quad (29)$$

The area of coil cross-section is  $S = I/j_m$ , where  $j_m$  is a permissible maximal density current of the superconducting winding. The length of the coil curvilinear axis can be written as  $l = k_l R$ , where a dimensionless parameter  $k_l$  connects the length of a coil with a large radius of the torus,  $R$ .

The combination of (28) and (29) yields

$$V_{sc} = \frac{IR}{j_m} k_l = \frac{W^{2/3}}{j_m B_m^{1/3} \mu_0^{1/3}} k_{V_{sc}}, \quad (30)$$

where the dimensionless parameter is  $k_{V_{sc}} = 2k_l k_B^{1/3} k_w^{-2/3}$ .

In the general case parameter  $k_l$  is  $k_l(\varepsilon, \lambda) = 2\varepsilon[\pi + 2(\lambda - 1)]$  and other dimensionless parameters for the racetrack-shaped coil system are determined according to the equations (16) and (26) above.

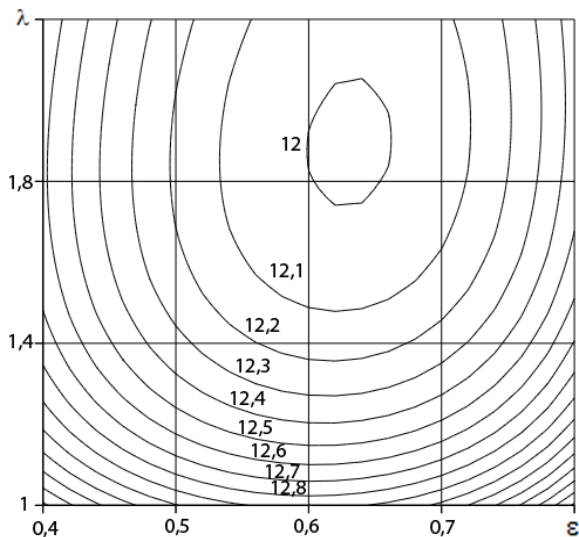


Fig. 6

$$k_{VscD}(\epsilon_{\min D}, \lambda_{\min D}) = 11.70 \text{ at } \epsilon_{\min D} = 0.68, \lambda_{\min D} = 1.75$$

and corresponds to the theoretical minimum for toroidal system [18].

The calculated results in Fig. 7 indicate that the minimal volume of superconducting materials for system with racetrack coils is practically the same as for D-shaped coil torus at aspect ratio  $\epsilon \approx 0.6$  for both configurations. On the contrary, the racetrack-shaped coil system ( $\lambda = \lambda_{\min}$ ) has a considerably lesser volume of superconducting winding in comparison with the usual O-shaped coils ( $\lambda = 1$ ).

Fig. 8 illustrates the influence of the relative size of the torus cross-section on the value of the large radius. As indicated in the figure, the lower values of the dimensionless parameter  $k_R$  have magnets with the racetrack-shaped coils. The parameter  $k_R$  has a minimum value at  $\epsilon = 0.52$  for O-torus, at  $\epsilon = 0.53$  for racetrack torus, and at  $\epsilon = 0.6$  for D-torus.

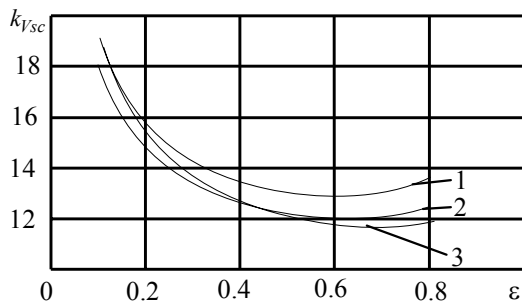


Fig. 7

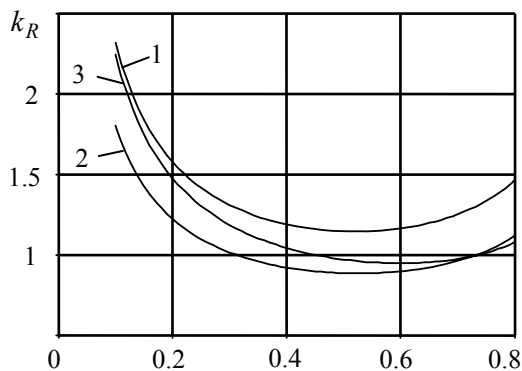


Fig. 8

This allows to give the following expressions:

$$k_R(\epsilon, \lambda) = \left[ 2\pi^2(1-\epsilon)^2 \left( 1 - \sqrt{1-\epsilon^2} + \frac{\epsilon(\lambda-1)}{\pi} \ln \frac{1+\epsilon}{1-\epsilon} \right) \right]^{-1/3} \quad (31)$$

$$k_{Vsc}(\epsilon, \lambda) = 4\epsilon(\pi + 2(\lambda-1)) \times \left[ 4\pi(1-\epsilon) \left( 1 - \sqrt{1-\epsilon^2} + \frac{\epsilon(\lambda-1)}{\pi} \ln \frac{1+\epsilon}{1-\epsilon} \right)^2 \right]^{-1/3} \quad (32)$$

The results of the  $k_{Vsc}$  parameter calculation is plotted in Fig. 6 as a function of two geometrical parameters  $\epsilon$  and  $\lambda$ . The dimensionless parameter  $k_{Vsc}$  has a minimum value  $k_{Vsc}(\epsilon_{\min}, \lambda_{\min}) = 11.99$  at  $\epsilon_{\min} = 0.63$ ,  $\lambda_{\min} = 1.89$ . The defined  $k_{Vsc}(\epsilon_{\min}, \lambda_{\min})$  value is approaching to the minimum of analogous parameter for D-torus that equals

**Example of SMES.** The calculated size and volume parameters of SMES are given in Table 2 as an example for the following initial parameters:  $\epsilon = 0.4$ ,  $\lambda = 2.2$ ,  $\sigma_t = \sigma_c = 350$  MPa,  $W = 450$  MJ,  $B_m = 5$  T,  $j_m = 4 \cdot 10^7$  A/m<sup>2</sup>.

In Table 2  $\rho_2$  is a maximal radial size of torus  $\rho_2 = R(1+\epsilon)$  (Fig. 1);  $V_\Sigma$  being the total volume of the support system and  $V_1$  being the volume of the spokes or support belts. For momentless constant tension systems  $V_1$  is equal to  $V_t$  (D-shaped torus and coils with spokes). For O-shaped coils with the support belts the volumes  $V_1$  (upper values) are calculated with regard to the bending moments and the non-uniform tension along the coil perimeter. In this case, the belt of the constant cross section calculated at the maximum mechanical stress has been chosen, and position of the support rings corresponds to the minimal value  $V_\Sigma$  for such designs. The data under the line for volumes  $V_1$  and  $V_\Sigma$  are calculated at average values of stresses and without the bending moments. These data are, probably, a theoretical limit for the volume of this support structure.

**Table 2**

	*1	2	3	4
$V_c, m^3$	0.864	0.860	0.87	1.401
$V_1, m^3$	2.149	2.146	2.16	$\frac{6.870}{2.691}$
$V_\Sigma, m^3$	3.013	3.006	3.03	$\frac{8.270}{4.091}$
$R, m$	2.645	3.366	2.954	3.466
$\rho_2, m$	3.703	4.712	4.136	4.712
$z_{max}, m$	1.926	1.346	1.607	1.346
$V_{sc}, m^3$	10.040	10.677	10.075	10.677
*1 – Racetrack Coils, 2 – O-coils with spokes and support rings outside of torus, 3 – D-coils, 4 – O-coils with support belts				

minimum which is realised for toroidal magnets with D-shaped coils. The volume of the support system requirements of racetrack-shaped torus is approximately the same as the corresponding values for O-shaped torus with spokes and for geometrically more complicated traditional D-shaped toroidal system. Thus, the toroidal SMES with spokes and racetrack-shaped coils could provide attractive alternatives to conventional coil systems in the development of SMES devices.

**Conclusion.** The internal volume of toroidal coils of SMES devices can be used for the placement of the support system components, to minimize the volume and the cost of the structure material. The proposed systems, with spokes placed inside each simple O-shaped and racetrack-shaped toroidal magnet coil, solve two mechanical problems: the elimination of the bending moments in the support system and ensuring a uniform mechanical stress in all the spokes and the supporting structure.

The toroidal magnetic system composed of racetrack-shaped coils with spokes inside torus has a considerably lesser volume of a superconducting winding in comparison with O-shaped coils and is close to the theoretical

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**ПОРІВНЯННЯ НАДПРОВІДНИХ ІНДУКТИВНИХ НАКОПИЧУВАЧІВ ЕНЕРГІЇ (НПІН) З КОТУШКАМИ РІЗНОЇ КОНФІГУРАЦІЇ ТА МЕХАНІЧНИМИ УТРИМУЮЧИМИ ЕЛЕМЕНТАМИ ВСЕРЕДИНИ ТОРОЇДАЛЬНОГО ОБ'ЄМУ**

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*Розглянуто тороїдальні магнітні системи, внутрішній об'єм яких використовується для розміщення там елементів механічної утримуючої системи. Досліджено системи з котушками круглої і рейстреккової форм. Запропоновано утримуючу систему, у якій відсутні згинаючі моменти і забезпечено однакові значення механічних напруг в елементах конструкції. Проведено аналіз розмірів тороїдальних соленоїдів і об'ємів надпровідної обмотки та матеріалу механічної утримуючої системи. Показано, що об'єм утримуючої системи практично однаковий для соленоїдів із стяжками для котушок як рейстреккової, так і круглої форм, а також для більш складних за геометрією D-подібних котушок, для яких необхідним є механічний бандаж навколо периметру котушок. Об'єм надпровідної обмотки для накопичувачів з рейстрекковими котушками є значно меншим, ніж з круглими і наближається до теоретичного мінімуму, який мають тороїдальні соленоїди з D-подібними котушками. Показано, що тороїдальні магнітні системи з рейстрекковими котушками мають найменші радіальні розміри порівняно з іншими досліджуваними конфігураціями. Бібл. 25, табл. 2, рис. 8.*

**Ключові слова:** тороїдальний надпровідний індуктивний накопичувач енергії, механічна утримуюча система, масогабаритні параметри.

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**СРАВНЕНИЕ СВЕРХПРОВОДЯЩИХ ИНДУКТИВНЫХ НАКОПИТЕЛЕЙ ЭНЕРГИИ (СПИН) С КАТУШКАМИ РАЗНОЙ КОНФИГУРАЦИИ И МЕХАНИЧЕСКИМИ УДЕРЖИВАЮЩИМИ ЭЛЕМЕНТАМИ ВНУТРИ ТОРОИДАЛЬНОГО ОБЪЕМА**

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*Рассмотрены тороидальные магнитные системы, внутренний объем которых используется для размещения там элементов механической удерживающей системы. Исследованы системы с катушками круглой и рейстрекковой форм. Предложена конфигурация удерживающей системы, у которой отсутствуют изгибающие моменты и обеспечены одинаковые механические напряжения в элементах конструкции. Проведен анализ размеров тороидальных соленоидов и объемов сверхпроводящей обмотки и материала механической удерживающей системы. Показано, что объем удерживающей системы практически одинаков для соленоидов со стяжками для катушек как рейстрекковой, так и круглой форм, а также для более сложных по геометрии D-образных катушек, для которых необходимо также механический бандаж вокруг периметра катушек. Объем сверхпроводящей обмотки для накопителей с рейстрекковыми катушками является значительно меньшим, чем с круглыми и приближается к теоретическому минимуму, который реализуется в тороидальных соленоидах с D-образными катушками. Показано, что тороидальные магнитные системы с рейстрекковыми катушками имеют наименьшие радиальные размеры по сравнению с другими исследованными конфигурациями. Библ. 25, табл. 2, рис. 8.*

**Ключевые слова:** тороидальный сверхпроводящий индуктивный накопитель энергии, механическая удерживающая система, массогабаритные параметры.

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