# ДОСЛІДЖЕННЯ ІНТЕРВАЛІВ ПРИБУТТЯ ABTOBYСIB TA ЧАСУ ОЧІКУВАННЯ ПАСАЖИРІВ МІСЬКИХ МАРШРУТІВ 

Наведені результати емпіричного дослідження статистичних розподілів часових інтервалів між прибуттям автобусів та тривалістю очікування пасажирів на зупиночних пунктах міського громадського транспорту, які розташовані на спільній діляниі декількох автобусних маршрутів. Встановлено, що неузгодженість розкладів руху автобусів різних марирутів збільшує тривалість очікування пасажирів на зупиниі в середнвому на 15-25 \%.

Ключові слова: міський громадський транспорт, інтервал руху, нерегулярність, тривалість очікування, закон розподілу.

## 1. Introduction

The problem of determining the passenger waiting time at bus stops appears when solving a number of practical problems associated with the design and operation of urban public transport (UPT) [1]. The task of establishing the level of passenger satisfaction by transport services provided by UPT is related to these problems.

The proper level of UPT services makes it more attractive for city residents and attracts a certain share of the last to use it in comparison with a variant of transportation in own car. This makes it possible to significantly reduce the severity of the problem of congestion of modern streets and roads of major cities by road traffic or completely remove it [2].

In most of the proposed approaches to assessing the quality of UPT services directly or indirectly take into account the duration of the transportation of passengers, one component of which is the passenger waiting time for boarding the bus at the start stop of the trip or transfer stop in case of need [3].

Given that passenger waiting time at the UPT stop is psychologically perceived as the most desirable part of the transportation process [4], the problem of reliable assessment of its duration is relevant.

## 2. The ohject of research and its technological audit

The object of research is the process of passenger waiting time for boarding the bus at the stop of public transport.

With the use of the most common routing technology passenger transport services in urban areas the average waiting time for boarding the bus at the UPT stop is generally defined as the sum of three components [5]:

$$
\begin{equation*}
T_{w}=t_{1}+t_{2}+t_{3}, \tag{1}
\end{equation*}
$$

where $t_{1}$ - waiting time depending on entirely on the planned UPT headway at the stop; $t_{2}$ - additional waiting time, depending on the accuracy of schedule compliance by drivers of buses and unpredictable breakdown of buses (transportation regularity); $t_{3}$ - additional waiting time, which
taking into account the passengers refusal for boarding the bus due to lack of the free seats or passenger transportation in unsatisfactory conditions. Ignore these terms and will assume that $t_{3}=0$.

In conditions of relatively high bus headway on city routes, value of the first component is usually determined by the formula [6, 7]:

$$
t_{1}=\left\{\begin{array}{l}
\frac{H}{2}, \text { at } H \leq 10 \mathrm{~min}  \tag{2}\\
\sqrt{H}, \text { at } H>10 \mathrm{~min}
\end{array}\right.
$$

where $H$ - planned headway, min.
The first component of the formula (2) is based on theoretical assumptions and the results of field observations indicating that in the case of relatively small bus headway, which do not exceed 10 minutes, passengers no need for information on the schedule and they arrive at stop at random times. By increasing the bus headway more than 10 minutes, passengers adapt to the schedule of buses and arrive at a stop in accordance with this schedule, and the waiting time is relatively smaller.

The presence of the second component in the formula (1) is due to the fact that in the real time buses arrive to a stop with some deviation from the schedule in greater or smaller side. As shown in [8], the value of this component is determined by:

$$
\begin{equation*}
t_{2}=\frac{\sigma_{H}^{2}}{2 H}, \tag{3}
\end{equation*}
$$

where $\sigma_{H}$ - the standard deviation of the bus headway at stop from planned, defined by the formula:

$$
\begin{equation*}
\sigma_{H}=\sqrt{\frac{\sum_{i=1}^{n}\left(h_{i}-H\right)^{2}}{n}}, \tag{4}
\end{equation*}
$$

where $h_{i}$ - the headway time interval between the bus arrival at stop registered in the $i$-th observation in a series of $n$ successive intervals.

Thus, the average passenger waiting time at bus stop in the first approximation is determined by vehicles interval and accuracy of schedule compliance by drivers.

In determining the average waiting time at bus stop by the formula (1) is usually considered a case where the passengers are waiting a certain route. However, in terms of route overlapping, a situation arises when several routes arrive at stop and passenger can use one or more of them.

Joint (network) headway at the $\bar{H}$ stop in this case is determined by:

$$
\begin{equation*}
\frac{1}{\bar{H}}=\sum_{i=1}^{m} \frac{1}{H_{i}} \tag{5}
\end{equation*}
$$

where $m$ - the number of routes at the $\bar{H}$ stop; $H_{i}-$ planned headway on each route and $i$-th, min.

Under such conditions, outlined above approach to determine the average waiting time at the bus stop by the formula (1) is true. However, it should pay attention to some features of its application.

First, the route overlapping almost inevitably lead to additional waiting time. Indeed, even if each of the routes at the stop, schedule is done exactly, the movement of vehicles at the bus stop will be regular $\left(t_{2}=0\right)$ only when the headways are identical in size and schedules of various routes are well coordinated.

Secondly, in most cases it is unknown which of the available routes passenger choose. It could be any one of the routes or some combination of them. It is clear that in each of those cases, the range of network headway and its standard deviation in terms of passenger waiting time will be different.

These differences need to study and refine models for determining the average passengers waiting time at the stops in terms of route overlapping and uncertainties of passenger choosing certain route from several alternatives.

## 3. The aim and objectives of research

Therefore, this article deals with the bus headway at UPT stops in terms of route overlapping and estimates the average passenger waiting time at bus stop in such circumstances.

To achieve this aim the following tasks are solved:

- The collection of field data on the bus headway on different routes at the joint stop and their processing by the methods of mathematical statistics.
- Calculation and determination of form of the link between statistical characteristics of bus headway of different groups, each of which represents passenger choice variant of future travel.
- The establishment of the headway distribution for different groups and assess the average passenger waiting time length at the bus stop according to this distribution.


## 4. Literature review

A number of works, both domestic and foreign scientists, are devoted to study to evaluate the passenger waiting time on UPT routes. The importance of this research proved that the value of this duration directly determines the passenger service quality and, consequently,
the degree of UPT satisfaction and attractiveness for city residents.

Most of theoretical studies based on certain assumptions about the passenger arrivals at stops and transport organization for them.

One of the key provisions, in which dependence (1) is correct, there is an accident of the passenger arrivals at stops. In [9] on the basis of the research in Austin (USA) found that for the bus headway over 10 minutes, passenger waiting time evaluation based on this interval is unreliable and requires the involvement of additional factors. Similarly, the results obtained in the studies [6] at UPT stops in Zurich (Switzerland) showed that even for bus headway, equal to about 5 minutes, a certain percentage of passengers adapt to the schedule of buses and arrive at stop coincidental way. The process of passenger arrival at at the stops can be described by a combination of laws of uniform distribution and shifted Johnson distribution.

Assuming a uniform flow of passenger arrival at stops, dependence (1) is free from the law of distribution of intervals for bus headway at stops, so previous studies have focused on the establishment of laws of dependency of the standard deviation of bus headway on its expectation. Thus, in [10] based on observations of the movement of buses on three routes of five stops in Columbus (USA) obtained empirical dependence of variation coefficient of intervals for buses $v_{H}$ on scheduled interval on the route:

$$
\begin{equation*}
v_{H}=\frac{\sigma_{H}}{H}=\frac{1,296}{\sqrt{H}} . \tag{6}
\end{equation*}
$$

The obtained dependence suggests that headway coefficient for buses is nonlinearly reduced with headway increase, which reduces the proportion of time $t_{2}$ associated with irregular transportation in the overall average passenger waiting time.

Not only knowledge of schedule by the passenger, but uniform bus headway influence on passenger waiting time at a stop in the case of the high frequency of buses that are inherent for peak hours. Under these conditions, the distribution of bus headway cease to be normal [11] and its mode is shifted to the left of expectation [12]. This gave grounds for scientists to propose and empirically test hypotheses about the bus headway distribution under the laws of the gamma distribution [13], Weibull distribution, log-normal and logistics law [14]. Accordingly, similar distribution laws of random variable of average passenger waiting time at a bus stop and connection of their parameters with expectation of headway were studied [15].

However, based on our literature review, we can conclude that the objects of study are isolated certain UPT bus routes and distribution of its headway, so the average waiting time and its distribution to include passengers awaiting the arrival of a vehicle a particular route. In most practical situations UPT stop is served by several bus routes (or other UPT types). As shown by studies conducted at UPT stops in Portland (USA) [16], in terms of route overlapping random route of bus arrival at stops losing ordinariness property, thus it is significantly evident mutual influence of headway for some routes with a common area. Thus, determining patterns of headway distribution of buses and average passenger waiting time in terms of route overlapping require additional research.

## 5. Methods and materials of research

Research of bus headway was held at four UPT stops in Zaporozhye (Ukraine) with different number of routes within an hour of the morning «peak», which is characterized by high motion frequency. General characteristics of the investigated stops and bus headway listed in the Table 1.

Table 1
General characteristics of stops

| Number <br> of stop | Number <br> of route | Route frequency, <br> item/hour | Headway, min |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Network |  |  |
| 1 | 4 | 50 | 3,1 | 7,0 | 1,22 |
| 2 | 5 | 37 | 4,1 | 15,0 | 1,62 |
| 3 | 6 | 44 | 4,7 | 16,0 | 1,33 |
| 4 | 7 | 51 | 5,9 | 12,0 | 1,16 |

Investigated stops on the route UPT network were selected so that all routes that pass through it, have at least one joint stop located on the further way of buses. However, all stops are not primary for any of the routes. This choice enables the assumption of the existence of alternatives for some passengers when choosing a route. During the observation the bus headway at the stop for all routes was recorded.

Let the stop is served by $m>1$ bus routes. Then the number of combinations of them in two or more that are different from each other by at least one route is $k=2^{m}-m-1$. Accordingly, a plurality of passengers arrived at stop can be divided into $k$ groups that can be used to travel the routes that are part of each of these groups. Based on the principle of choice stops assume that passengers are equally divided between groups of routes.

Groups of routes that consist of only one element are excluded. Indeed, this case is similar to the previously investigated case of passenger bus waiting for one particular route. In addition, the relative proportion of these groups in their total number decreases rapidly as the increase of the number of routes (at investigated stops $26,7 \%$ for $m=4 ; 16,1 \%$ for $m=5 ; 9,5 \%$ for $m=6$ and 5,5 \% for $m=7$ ).

Network headway $\bar{H}_{k}$ (5), its standard deviation $\sigma_{H(k)}$ (4) and average passenger waiting time $T_{w(k)}$ using the (1) for each $k$-th combination of routes are calculated for each of the investigated stops on the basis of observation. Later obtained statistical sets were elaborated by the methods of mathematical statistics to study the parameters of bus headway at stops and average passenger waiting time.

## 6. Research results

Analysis of the standard deviation dependency of the network headway calculated for each group of routes $\sigma_{H(k)}$ on its expectation $\bar{H}_{k}$ showed that between them there is a significant linear correlation (Fig. 1):

$$
\begin{equation*}
\sigma_{H(k)}=a_{0}+a_{1} \cdot \bar{H}_{k}, \tag{7}
\end{equation*}
$$

where $a_{0}, a_{1}-$ constants determined by least squares method.


Fig. 1. Correlation field and regression equation graphs $\sigma_{H(k)}=f\left(\bar{H}_{k}\right)$ : $a-\operatorname{stap}$ № 1 ; $b$ - stap №2; $c$ - stap № 3; $d$ - stop № 4

As shown in the graphs in Fig. 1, while increasing network headway dispersion of random errors of linear regression models increases, in other words, there is heteroscedasticity phenomenon. However, given a sufficient number of observations in developing pair regression models, least squares method makes it possible to get unbiased able estimation of parameters of these models.

Value of regression coefficients and indicators for estimation of the regression equations quality derived from analysis using the statistical package STATISTICA and presented in Table 2.

Table 2

| Hesults of paired linear regression analysis |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number <br> of stap | Regression <br> coefficients | Determina- <br> tion coef- <br> ficient $R^{2}$ | Standard <br> approxima- <br> tion error $S$ | Value of $F$-Fisher <br> criterion ( $\alpha=0,05)$ |  |  |  |
|  | $a_{0}$ | $a_{1}$ | Calculated | Critical |  |  |  |
| 1 | 0,082 | 0,614 | 0,516 | 0,393 | 9,59 | 5,12 |  |
| 2 | 0,044 | 0,712 | 0,892 | 0,341 | 198,59 | 4,26 |  |
| 3 | 0,298 | 0,620 | 0,735 | 0,426 | 152,32 | 4,02 |  |
| 4 | 0,438 | 0,573 | 0,763 | 0,283 | 379,57 | 3,95 |  |

The results in Table 2 show that the linear regression model is adequate to output data and statistically significant.

Linearity of statistical relationship between network headway expectation and its standard deviation gives reason to adopt advanced earlier $[13,14]$ assumptions about the distribution of the random bus headway at stop according to the gamma distribution law with probability density:

$$
\begin{equation*}
f(H)=\frac{H^{k-1}}{\theta^{k} \cdot G(k)} \cdot e^{-\frac{H}{\theta}} \tag{8}
\end{equation*}
$$

where $k, \theta$ - parameters of shape and size distribution, respectively; $G(k)$ - Euler gamma function.

For a random variable, distributed by (8) it is true that:

$$
\left\{\begin{array}{l}
M[H]=k \cdot \theta ;  \tag{9}\\
\sigma_{H}^{2}=k \cdot \theta^{2} .
\end{array} \Rightarrow \sigma_{H}=\frac{1}{\sqrt{k}} M[H],\right.
$$

where $M[H], \sigma_{H}$ - respectively the expectation and standard deviation of a random variable.

Given the ratio (9) it can be written expression to calculate the average passenger waiting time at bus stop (1) as:

$$
\begin{equation*}
T_{w}=\frac{M[H]}{2}\left(1+\frac{1}{k}\right) . \tag{10}
\end{equation*}
$$

According to (10), the average passenger waiting time at stop depends on the shape parameter of gamma distribution law and does not depend on the parameters of its scale. On the other hand, the formula (1) with accounting (8) becomes:

$$
\begin{align*}
& T_{w}=\frac{M[H]}{2}+\frac{\left(a_{0}+a_{1} M[H]\right)^{2}}{2 M[H]}= \\
& =\frac{M[H]}{2}\left(1+a_{1}^{2}\right)+\frac{a_{0}^{2}}{2 M[H]}+2 a_{0} a_{1} . \tag{11}
\end{align*}
$$

Taking into account that on the basis of observations and obtained values of regression coefficients $\left|a_{0}\right|<1,\left|a_{1}\right|<1$, $M[H]>1$, the last two summands in (11) can be neglected and evaluate $k$ parameter of gamma distribution by value regression coefficient $a_{1}$ :

$$
\begin{equation*}
k \approx \frac{1}{a_{1}^{2}} . \tag{12}
\end{equation*}
$$

Therefore, according to the results of regression analysis (Table 2), the value of the gamma distribution parameter describing the bus headway at stops is within $k=1,97 \ldots 3,04$. According to this, the average passenger waiting time at joint stop of several routes with high traffic frequency and inconsistent schedules according to (10) in practical calculations can be taken as $T_{w}=(0,65 \ldots 0,75) \bar{H}$.

## 7. SWOT-analysis of research results

Obtained recommendations to determine the average passenger waiting time at joint stop of several UPT routes enable sufficiently for practical calculation accuracy justify design decisions to optimize the route network, technological transportation support and evaluation of the passenger service quality without the need for field observations directly at bus stops.

However, the applicability of the results is somewhat limited, because it much depends on the actions of controlled and uncontrolled factors that are internal and external to the research facility. Among the first there are feasibility of providing passengers about the bus schedule on certain routes, coordination of schedules of buses of different routes on their joint areas and change the route and, accordingly, the network bus headway according to the passenger traffic flow and through the use of buses with different passenger capacity. Among the second there are personal preferences of passengers in the further transportation and general uncertainty of collocation of bus route area that follow under considered.

The presence of indirect or combined effect of these factors gives reason to believe the obtained expressions as an upper bound for the size of the average passenger waiting time at bus stop.

## 8. Conclusions

As a result of research:

1. Field observations of bus headway at stops, located at the joint area of several routes in the city Zaporozhye (Ukraine) during the period of morning peak hours are developed.
2. The presence of significant statistical linear relation between expectation of network headway and its standard deviation when considering all possible combinations of routes is defined by the method of correlation and regression
analysis. Each of them is a variant of passenger choosing to further transportation.
3. It is established that the headway at bus stop for various groups of routes having inconsistent schedules with sufficient accuracy for practical calculations can be described by the law of the gamma distribution with shape parameter $k \approx 2 \ldots 3$ that, subject to the random passenger arrival at stop and equiprobable passenger distribution for groups of routes increases the average passenger waiting time at bus stop by $15-25 \%$ in comparison with the case of regular bus traffic.

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## ИССЛЕДОВАНИЕ ИНТЕРВАЛОВ ПРИБЫТИЯ АВТОБУСОВ И ВРЕМЕНИ ОЖИДАНИЯ ПАССАЖИРОВ ГОРОДСКИХ МАРШРУТОВ

Приведены результаты эмпирического исследования статистических распределений временных интервалов между прибытиями автобусов и временем ожидания пассажиров на остановочных пунктах городского общественного транспорта, расположенных на общем участке нескольких автобусных маршрутов. Установлено, что несогласованность расписаний движения автобусов различных маршрутов увеличивает время ожидания пассажиров на остановке в среднем на 15-25 \%.

Ключевые слова: городской общественный транспорт, интервал движения, нерегулярность, время ожидания, закон распределения.

Кузъкін Олексій Феліксович, кандидат технічних наук, доцент, кафедра транспортних технологій, Запорізький національний технічний університет, Україна, e-mail: kuzkin@zntu.edu.ua.

Кузъкин Алексей Феликсович, кандидат технических наук, доцент, кафедра транспортных технологий, Запорожский национальный технический университет, Украина.

Kuzkin Olexiy, Zaporizhzhya National Technical University, Ukraine, e-mail: kuzkin@zntu.edu.ua

