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# PROGRAMMING OF THE SEARCH ALGORITHM FOR POINT BELONGING TO THE POLYGON AND THE MUTUAL NON-INTERSECTION OF THE FIGURES 

Показано, що щільність укладання деталей в схемі розкрою прямо пропорційно впливає на відсоток використання матеріалу. Зважаючи на технологічні вимоги до використання листового матеріалу, розроблені алгоритми додавання та вилучення деталей із схеми. По даним алгоритмам написані та протестовані програмні модулі в інтегрованому об'єктно-орієнтованому середовищі Delphi.

Ключові слова: метод трасування променя, метод Грєхєма, розкрійна схема, додавання та вилучення деталей із схеми.

## 1. Introduction

At present, the need for high-quality and comfortable footwear has become one with other physiological needs of man. And along with quality and comfort, the need for the quantity of this footwear has significantly increased. According to statistics and research, the need for quantity of shoes in recent years has grown to more than 13 billion pairs per year.

To meet this growing demand, the footwear industry of light industry should improve the methods of rationalizing nesting patterns. When designing rational nesting patterns, the cutter must adhere to the rule of tight placement of parts, while ensuring that they are not mutually non-negotiable. To solve this problem, it is necessary to construct around the details the hodographs of vector-functions of the dense distribution of the figures of these parts.

## 2. The object of research and its technological audit

The object of research is shoe parts of various configurations that are manufactured at the enterprises of the footwear industry of light industry. The study of various variants of cutting is conducted, the main feature of which is that around the details are built hodograph curves.

In fact, the hodograph is the trajectory of the movement of the pole of one figure around the other, preserving the conditions for their mutual contact [1-3]. Let's illustrate the fulfillment of this condition by the example of two polygons (Fig. 1).

For interactive extraction of the parts from a nesting pattern, it is necessary to solve the problem of belonging to a polygon point.

Let's suppose that there is an abstract polygon A given by arrays of coordinates of the points $d[i] \cdot x, d[i] \cdot y$, where $i=0 \ldots n-1$, where n is the number of points, and some point $C$ with coordinates ( $x_{0}, Y_{0}$ ) (Fig. 2). It is necessary to prove that the point belongs to the figure.

In Fig. 1, 2, a convex-concave polygon is represented, however, the essential drawback of the hodograph curves
is precisely that they are correctly constructed only for convex-concave figures.


Fig. 1. Construction of a hodograph around the figure: $A, B$ - figures; $D_{1}, D_{2}$ - poles of figures


Fig. 2. The problem of point belonging to a polygon: A - polygon; $[$ - point

## 3. The aim and ohjectives of research

The aim of research is development of software module for extracting parts from the nesting pattern, which are manufactured at the enterprises of the shoe industry of light industry. This software module, after development, will become part of the developed software.

To achieve this aim, it is necessary to perform the following tasks:

1. To analyze algorithms for finding the point belonging to the polygon.
2. To develop an algorithm for extracting parts.

## 4. Research of existing solutions of the problem

The problem of localizing a point has many subspecies and they are solved by different methods. Among the existing solutions to the problem, it should be noted:

- ray tracing method [4, 5];
- summing angles method [6, 7];
- Graham circumventing method [8, 9].

In particular, the paper [4] is completely devoted to the problem of the ray tracing method, its advantages and disadvantages, analysis of its complexity and execution time.

As the author notes in [6], despite the complexity of the method of summing angles, its simplified form gives good prospects for its wide application.

The author of [10] states that when constructing a convex simple polygon, the execution time of algorithms can be reduced to $O(c n)$, where $c$ is a constant.

However, this article explores the design of nesting patterns, which means that the focus of the problem is somewhat different.

In the opinion of the authors [1-3], for construction of rational nesting patterns, a dense placement of the details is necessary, that is, a hodograph must be created. At the same time, the authors of [1] note that an essential disadvantage of hodographs is that they are constructed only for the same type of figures.

## 5. Methods of research

To solve the problem, there are many algorithms. Let's analyze the main:

The essence of the algorithm for finding point belonging to the polygon using the Graham circumventing method [4] is quite simple. A vector $\vec{a}$ is drawn from one vertex to another, and a vector $\vec{b}$ is drawn to the third. The direction of traversal from vector $\vec{a}$ to vector $\vec{b}$ will show whether to remove the item or not. If we go around clockwise, then the point belongs to a polygon, and if it is against, it does not belong. However, this method does not provide a situation where the point lies counterclockwise from the vector $\vec{a}$, but is within the polygon.

The essence of the method of angles lies in the search for the sign of the sum of the angles [7]. From the point, the belonging of which must be proved, rays are taken to other vertices of the polygon. Angles between the rays are attached. The sign of this sum determines the point belonging to the polygon. The disadvantages of the algorithm are the complexity of calculating the angles and the uncertain position of the reference point.

Ray tracing algorithm method looks for the point belonging to the polygon by the number of intersections with the ray of the sides of the polygon [6]. From some remote point to the point, belonging of which must be proved, a ray is drawn, and the number of its connections of polygon boundaries determines the point belonging to the polygon. In Fig. 3 shows three advantages of the ray
method. First, any ray that passes through a given point is suitable for solving the problem. Secondly, the order of intersection of the sides of a polygon is important, only the pairing of their total is important. And, thirdly, most importantly, it will be much easier to implement programmatically.


Fig. 3. Features of the ray method: $S$ - figure; $A, B$ - points; гa, $\Gamma b$ - rays
5.1. Ray construction. It is necessary to find the number of intersections of the ray with the edges of the polygon. Physically, in order to construct a ray (in fact, a ray is a straight line segment), it is necessary two points. The first is the delete point. The second point is selected within the part, and for simplicity, the pole of the part. Then the ray equation can be written in the form:

$$
\begin{align*}
& \frac{x-x_{0}}{x p-x_{0}}=\frac{y-y_{0}}{y p-y_{0}}=0 \Rightarrow \\
& \Rightarrow\left(y p-y_{0}\right) x+\left(x_{0}-x p\right) y+y_{0} x p-y p x_{0}=0, \tag{1}
\end{align*}
$$

that is $A_{1} x+B_{1} y+C_{1}$.
And the equation of the polygon edge is in the form:

$$
\begin{align*}
& \frac{x-d[i] \cdot x}{d[i+1] \cdot x-d[i] \cdot x}=\frac{y-d[i] \cdot y}{d[i+1] \cdot y-d[i] \cdot y}=0 \Rightarrow \\
& \Rightarrow(d[i+1] \cdot y-d[i] \cdot y) x+(d[i] \cdot x-d[i+1] \cdot x) y+ \\
& +d[i] \cdot y d[i+1] \cdot x-d[i+1] \cdot y d[i] \cdot x=0, \tag{2}
\end{align*}
$$

that is $A_{2} x+B_{2} y+C_{2}$.
Thus, the coordinates of the intersection point $D$ will be the solution of the system of equations (1) and (2):

$$
\begin{equation*}
x=\frac{B_{1} C_{2}-C_{1} B_{2}}{A_{1} B_{2}-B_{2} A_{1}}, y=\frac{A_{2} C_{1}-C_{2} A_{1}}{A_{1} B_{2}-B_{2} A_{1}} . \tag{3}
\end{equation*}
$$

So, if the coordinates of the point $D$ satisfy equation (3), the intersection is, but it is not known whether the point belongs to the polygon or not. To do this, let's find a point $D$ for each edge of the polygon, and, in each time, increments the number of intersections. By the total number of intersections, it is possible to see whether the polygon point belongs or not. If the number of intersections
is odd, then, as can be seen from Fig. 2, the point $C$ belongs to the polygon.

Mutual non-negotiable details are possible provided the non-repainted circles described around them, that is:

$$
\begin{equation*}
R_{1}+R_{2}>=D \tag{4}
\end{equation*}
$$

where $R_{1}$ and $R_{2}$ - the radii of the circles described around the selected parts, and $D$ - the distance between their real poles.

Let's illustrate the fulfillment of this condition (Fig. 4).


Fig. 4. The fulfillment of the non-intersection condition: $a$ - dense placement; $b$ - simple non-intersection; $c$ - cross section

An ideal variant is shown in Fig. 4, $a$, when the parts only touch. The variant when the parts are placed too far apart is shown in Fig. 4, b. However, condition (4) is satisfied and, therefore, this arrangement of details satisfies the non-intersection condition. The variant when the parts are placed too close and intersect is shown in Fig 4, c.

Fig. 4 more clearly illustrates the criterion by which it becomes clear that the parts should be excluded from the pattern.
5.2. Procedure algorithm. As already noted, the standard algorithm for the ray tracing method is modified according to the conditions of the problem. Let's emphasize that the edge in this algorithm acts as a straight line, and the points on the ray will be on one or the other side of the edge. Thus, to solve the problem of excluded parts, let's already apply formulas:

$$
\begin{equation*}
D_{1}=A_{1} x_{d i}+B_{1} y_{d i}+C_{1}, \tag{5}
\end{equation*}
$$

$$
\begin{align*}
& D_{2}=A_{1} x_{d i+1}+B_{1} y_{d i+1}+C_{1}  \tag{6}\\
& D_{3}=A_{2} x_{p}+B_{2} y_{p}+C_{2}  \tag{7}\\
& D_{4}=A_{2} x_{e x}+B_{2} y_{e x}+C_{2} \tag{8}
\end{align*}
$$

where $x_{d i}, y_{d i}, x_{d i+1}, y_{d i+1}-$ coordinates of the points of the beginning and the end of the edge; $x_{p}, y_{p}$ - part pole coordinates; $x_{e x}, y_{e x}$ - coordinates of the exclusion point.

So, the algorithm looks like this:

1) zero the contents of the cross section counter;
2) draw a straight line $A_{1} x+B_{1} y+C_{1}$ between the exclusion point and the part pole (1);
3) a straight line $A_{2} x+B_{2} y+C_{2}$ is on the edge of the polygon (2);
4) find the value of $D_{1}, D_{2}, D_{3}, D_{4}$ according to formulas (5)-(8);
5) if the products $D_{1}{ }^{*} D_{2}<0$ and $D_{3}{ }^{*} D_{4}<0$, then the intersection is and add to the contents of the connection counter unit and go to step 2, otherwise - skip the step;
6) repeat the steps from the second to the fifth for each edge of the polygon;
7) analyze the contents of the connection counter and exclude or not exclude this part.
5.3. Procedure listing. This procedure describes the steps from the second to the fifth of above described algorithm.
function peretyn( $\mathrm{x} 1, \mathrm{y} 1, \mathrm{x} 2, \mathrm{y} 2, \mathrm{x} 3, \mathrm{y} 3, \mathrm{x} 4, \mathrm{y} 4$ :integer): boolean; var A1,B1,C1,A2,B2,C2,D1,D2,D3,D4:integer;
begin
peretyn:=false;
A1:=y2-y1;
B1:=x1-x2;
C1:=x2*y1-x1*y2;
A2:=y4-y3;
B2:=x3-x4;
C2:=x4*y3-x3*y4;
if ( $\mathrm{A} 1 * \mathrm{~B} 2-\mathrm{A} 2 * \mathrm{~B} 1<>0$ ) then begin
D1:=A1*x3+B1*y3+C1;
D2: $=\mathrm{A} 1 * \mathrm{x} 4+\mathrm{B} 1 * \mathrm{y} 4+\mathrm{C} 1$;
D3: $=\mathrm{A} 2 * \mathrm{x} 1+\mathrm{B} 2 * \mathrm{y} 1+\mathrm{C} 2$;
D4: =A2*x2+B2*y2+C2;
if ( $\mathrm{D} 1 * \mathrm{D} 2<0$ ) and $(\mathrm{D} 3 * \mathrm{D} 4<0)$ then peretyn:=true; end;
end;
$\{===================================\}$
This is just a software module, where, if peretyn $=$ true, the main program will exclude the part.

## 6. Research results

The procedure for the exclusion of parts is part of the developed program for the interactive formation of nesting patterns. This software is implemented in the Delphi 7 environment. This program provides interactive addition and exclusion of parts not only in the original position and returned to $180^{\circ}$, but also at an arbitrary angle. Fig. 5, $b$ shows how the program works in two modes: adding and excluding parts.

Fig. 5, 6 show two significant points:

1. The circles described around the details do not allow them to intersect.
2. The selected algorithm excludes the parts.


Fig. 5. Work of the program in the mode of interactive addition of details


Fig. 6. Work of the program in the mode of interactive exclusion of details

## 7. SWOT analysis of research results

Strengths. The above algorithm is designed to simplify the work of the cutter. Software implementation of the algorithm will reduce the number of labor resources and reduce the production time of the nesting patterns.

Weaknesses. If incorrect data arrives at the input of this program module, it can lead to a stoppage of the program, which in turn will lead, at least, to an increase in production time, and, as a maximum, to an increase in the number of labor resources.

Opportunities. Modernization of equipment at the enterprise will increase the power of computer technology, which means an increase in the efficiency of the program, which in turn will lead to an even greater reduction in the number of labor resources and a reduction in the production time of the cutting patterns.

Threats. In [6] it is proposed to use the modified method of summing angles. This method is more efficient and has a shorter lead time, but at the moment there is no cutting equipment to work using this method.

## 8. Conclusions

1. Three main algorithms for finding the point belonging to the polygon are analyzed. There are much more algorithms that solve this problem, but these algorithms have optimal
complexity parameters: $O\left(2^{*} N\right)$ against $O\left(N^{\wedge} 2\right)$ in other algorithms. This means that this algorithm is faster.
2. Based on the analysis of the three main algorithms, an algorithm for exclusion of the part from the nesting pattern is created. This algorithm is modified and adjusted to the requirements of the task - leaving 4-6 parts of one configuration in the nesting pattern
3. Based on the analysis of the three main algorithms, an algorithm for exclusion of the part from the nesting pattern is created. This algorithm is modified and adjusted to the requirements of the task - leaving 4-6 parts of one configuration in the nesting pattern. In the algorithm, instead of constructing a single point for the ray, the pole of the part is used.

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## ПРОГРАМММРОВАНИЕ АЛГОРИТМА ПОМСКА ПРИНАДЛЕЖНОСТИ ТОЧКИ МНОГОУГОЛЬНИКУ И ВЗАИМНОГО НЕПЕРЕСЕЧЕНИЯ ФИГУР

Показано, что плотность укладки деталей в схеме раскроя прямо пропорционально влияет на процент использования материала. Учитывая технологические требования к использованию листового материала, разработаны алгоритмы добавления и удаления деталей из схемы. По данным алгоритмам написаны и протестированы программные модули в интегрированной объектно-ориентированной среде Delphi

Ключевые слова: метод трассировки луча, метод Грехема, раскройная схема, добавление и удаление деталей из схемы.

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