

17. Property Process.ProcessorAffinity [Electronic resource] // Microsoft Developer Network. – 2016, October. – Available at: \www/URL: [https://msdn.microsoft.com/ru-ru/library/system.diagnostics.process.processoraffinity\(v=vs.110\).aspx](https://msdn.microsoft.com/ru-ru/library/system.diagnostics.process.processoraffinity(v=vs.110).aspx)
18. Intel® Math Kernel Library – Documentation [Electronic resource] // Intel Software Developer Zone. – 2017, September 13. – Available at: \www/URL: <https://software.intel.com/en-us/articles/intel-math-kernel-library-documentation>
19. Grama, A. Introduction to Parallel Computing [Text] / A. Grama, G. Karypis, V. Kumar, A. Gupta. – Ed. 2. – Boston, MA, US: Addison-Wesley, 2003. – 656 p.
20. Richter, J. CLR via C# [Text] / J. Richter. – Ed. 4. – Redmond, WA, US: Microsoft prePress, 2012. – 813 p.
21. Phantom 4 Pro: specifications [Electronic resource] // DJI. – 2017. – Available at: \www/URL: <https://www.dji.com/ru/phantom-4-pro/info>

ПОВЫШЕНИЕ ЭФФЕКТИВНОСТИ ИСПОЛЬЗОВАНИЯ БЕСПИЛОТНЫХ ЛЕТАТЕЛЬНЫХ АППАРАТОВ ЗА СЧЕТ РАСПРЕДЕЛЕНИЯ ЗАДАЧ МЕЖДУ ЯДРАМИ ВЫЧИСЛИТЕЛЬНОГО ПРОЦЕССОРА

Рассмотрены алгоритмы распараллеливания задач между ядрами 4-ядерных процессоров вычислительных систем беспилотных летательных аппаратов (БПЛА). Показано, что высвобождение 1-го ядра и распределение задачи между 2–4-м ядрами на 10,3 % уменьшает энергопотребление процессора.

Применение разработанного алгоритма в комплексе с предложенным дополнительным питанием от энергии измерительного сигнала датчиков повышает эффективность использования БПЛА.

Ключевые слова: вычислительные системы беспилотных летательных аппаратов (БПЛА), 4-ядерный процессор, имитационное моделирование.

Zhuravska Iryna, PhD, Associate Professor, Department of Computer Engineering, Petro Mohyla Black Sea National University, Mykolaiv, Ukraine, e-mail: irina.zhuravska@chmmu.edu.ua, ORCID: <http://orcid.org/0000-0002-8102-9854>

Borovylova Svitlana, Senior Lecturer, Department of Software Engineering, Petro Mohyla Black Sea National University, Mykolaiv, Ukraine, e-mail: svetlana.borovylova@chmmu.edu.ua, ORCID: <http://orcid.org/0000-0003-1994-0556>

Kostyria Mykhailo, Department of Intelligent Information Systems, Petro Mohyla Black Sea National University, Mykolaiv, Ukraine, e-mail: assassin19741@gmail.com, ORCID: <http://orcid.org/0000-0001-9537-6374>

Koretska Oleksandra, Postgraduate Student, Department of Computer Engineering, Petro Mohyla Black Sea National University, Mykolaiv, Ukraine, e-mail: alex.koretska@chmmu.edu.ua, ORCID: <http://orcid.org/0000-0002-1240-1472>

UDC 519.853.33

DOI: 10.15587/2312-8372.2017.118338

**Raskin L.,
Sira O.,
Parfeniuk Y.**

ANALYSIS AND DEVELOPMENT OF COMPROMISE SOLUTIONS IN MULTICRITERIA TRANSPORT TASKS

Розглянуто метод розв'язання багатокритеріальних транспортних завдань. Запропонована ітераційна процедура, в якій початковий план завдання є оптимальним за основним з критеріїв. На наступних ітераціях реалізується уступка за основним з критеріїв з метою поліпшення значення додаткових. Процедура триває до отримання компромісного рішення. Розглянуто приклади розв'язання задачі.

Ключові слова: багатокритеріальна транспортна задача, ітераційне рішення, формування Парето-безлічі рішень.

1. Introduction

In the practice of planning and organization of transportation of goods, two different mathematical models are traditionally used:

- the transport task by the cost criterion (at the same time the average total cost of transportation is minimized);
- the transport task by the time criterion (the maximum of the traffic durations is minimized).

These tasks are alternative in the sense that their optimal plans, as a rule, do not coincide (the shortest route is not necessarily the cheapest one). The technologies for solving these problems have been well worked out [1–3] and constructively take into account the different specifics and features of the productions of each of them. For this reason, they are fundamentally different and their

integration into a single computational procedure is very problematic. At the same time, when solving the practical problems of transport logistics, there is a need to solve compromise problems, for example, such:

- a) to find a transportation plan that minimizes the average total cost of transport, provided that the longest of them does not exceed the prescribed one;
- b) to find a transportation plan that minimizes the maximum of the transportation duration, provided that their average total cost does not exceed the specified value.

It should be noted that when solving practical tasks of transportation planning, it is expedient to take into account one more criterion – the probability of successful implementation of the transportation plan for the aggregate of routes from suppliers to consumers, which are determined by the selected plan. The development of a method for solving this problem is of theoretical and practical interest.

2. The object of research and its technological audit

The object of research is the multicriteria transport problem of linear programming.

Simultaneous consideration of several criteria for optimal solutions is a problem. The fact is that the optimal solutions for different criteria do not coincide as a rule. This circumstance motivates the search for compromise solutions to the multicriteria problem. In this connection, the subject of research is the method of constructing a compromise solution.

3. The aim and objectives of research

The aim of research is development of a technology for finding compromise solutions for linear programming transport problems by the criteria: average total cost, the probability of implementing a transportation plan, the maximum duration of transportation.

To achieve this aim, it is necessary to solve the following tasks:

1. To analyze the general principles of compromise for a set of criteria and the choice of rational.

2. Development of a computational procedure for constructing a compromise solution.

4. Research of existing solutions of the problem

Several different approaches to the solution of multicriteria problems are known and used in practice. One of the most commonly used is the formation of the Pareto optimal set of solutions [4–8]. Let X_j , $j=1,2,\dots,n$ be the set of possible solutions of the problem, and let F_i , $i=1,2,\dots,m$ be the set of criteria for evaluating the quality of the solution. Let's introduce the concept of a dominant solution. The decision is dominant if:

$$F_i(X_k) \geq F_i(X_s), \quad s=1,2,\dots,n, \quad s \neq k, \quad i=1,2,\dots,m,$$

and for each i at least one of these inequalities holds strictly. A Pareto-optimal set is a subset of the set of all solutions X_1, X_2, \dots, X_n , such that none of its elements is dominant. The shortcomings of this method are obvious. First, there is no constructive way to form a Pareto-set (except for the search of all solutions). Secondly, it is not clear how to choose the best from the set of unmodified decisions.

Another frequently used method is scalarization [9–11]. The simplest way to implement the basic idea is additive convolution. At the same time, for a set of criteria F_1, F_2, \dots, F_m , its scalar mapping is formed by the formula:

$$F(X) = \sum_{i=1}^m \alpha_i F_i(X), \quad \alpha_i \geq 0, \quad \sum_{i=1}^m \alpha_i = 1.$$

Another way is to convolve using the «ideal point» [11]. In this case, for each criterion F_i , the best value $F_i^{(0)}$ is chosen, and then the quality level of a particular solution X_j is estimated by a number:

$$L_j = \sum_{i=1}^m \alpha_i [F_i(X_j) - F_i^{(0)}]^2.$$

The general disadvantage of this approach is the arbitrariness in the choice of numerical values of weighting coefficients α_i , $i=1,2,\dots,m$.

Finally, when solving multicriteria problems, the so-called method of concessions is often used [12, 13]. Thus from a set of criteria the basic criterion is chosen, the others are defined as additional. Then the problems are solved according to the main criterion and then the values for all additional criteria are calculated for the obtained solution. If these values are satisfied by the person making the decision, then the solution of the problem is over. Otherwise, a concession is made: a new solution is found that will be worse than the previous one by the main criterion, but somewhat better by additional ones. The procedure continues until a compromise solution is found that satisfies all the criteria. The advantage of this approach: simplicity, interactivity and ability to control the breakpoint. The disadvantage – there is no general method that describes the procedure for moving from an existing solution to an alternative one. At the same time, if in accordance with the features and nature of the problem this transition is easily realized, then the proposed method is expedient to use.

5. Methods of research

Let there be m centers – suppliers of cargo and n centers of its consumption. In this case:

a_i – volume of cargo to be transported from the i -th supplier;

b_j – volume of cargo to be transported to the j -th customer;

c_{ij} – average cost of transporting a unit of cargo from the i -th supplier to the j -th consumer;

p_{ij} – probability of successful realization of cargo transportation from i -th supplier to j -th consumer;

t_{ij} – duration of the corresponding transportation.

Let's introduce the set:

$$X = (x_{ij}),$$

where x_{ij} – the volume of cargo planned for transport from the i -th supplier to the j -th consumer, $i=1,2,\dots,m$, $j=1,2,\dots,n$.

Let's formulate the criteria for the effectiveness of the transportation plan X :

$$L(X) = \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij}, \quad (1)$$

$$T(X) = \max_{i,j} \{t_{ij} \cdot \delta(x_{ij})\}, \quad (2)$$

$$P = \prod_{i=1}^m \prod_{j=1}^n P_{ij} \delta(x_{ij}), \quad (3)$$

where $\delta(x_{ij}) = \begin{cases} 0, & \text{if } x_{ij} = 0, \\ 1, & \text{if } x_{ij} > 0. \end{cases}$

The required transportation plan must satisfy the following constraints:

$$\sum_{j=1}^n x_{ij} = a_i, \quad i=1,2,\dots,m, \quad (4)$$

$$\sum_{i=1}^m x_{ij} = b_j, j = 1, 2, \dots, n, \tag{5}$$

$$x_{ij} \geq 0, i = 1, 2, \dots, m, j = 1, 2, \dots, n. \tag{6}$$

It is assumed that the balance condition $\sum_{i=1}^m a_i = \sum_{j=1}^n b_j$ is

met, and in addition, the duration of transportation from the i -th supplier to the j -th consumer does not depend on the volume of transportation $x_{ij}, i = 1, 2, \dots, m, j = 1, 2, \dots, n$.

Let's set the task of developing a method for finding a transportation plan $X = (x_{ij})$ that satisfies the constraints (4)–(6) and optimizing the criteria (1)–(3) optimally.

6. Research results

First let's consider the two-criterion problem by the criteria (1), (3), using the following iterative procedure.

Iteration 1. Using standard methods, the transport problem is solved by the cost criterion: to find a plan $X = (x_{ij})$ that minimizes (1), satisfying constraints (4)–(6).

Let $X^{(1)} = (x_{ij}^{(1)})$ be the solution of the problem. Using this plan, let's find a non-zero set $\{p_{ij}\delta_j(x_{ij}^{(1)})\}$. In this case only those elements of the matrix $P = (p_{ij})$ to which the plan's non-zero supplies $X^{(1)}$ correspond will be highlighted. Let's find a pair of indices:

$$(i_1, j_1) = \arg \min \{p_{ij}\delta_j(x_{ij}^{(1)})\}. \tag{7}$$

It is clear that the route from the supplier i_1 to the consumer j_1 is the least reliable of the plan's routes $X^{(1)}$. Let's call it critical.

Now let's modify the initial matrices C and P , as follows:

$$c_{ij}^{(1)} = \begin{cases} c_{ij}, & \text{if } p_{ij} > p_{i_1 j_1}, \\ M, & \text{if } p_{ij} \leq p_{i_1 j_1}; \end{cases} \tag{8}$$

$$p_{ij}^{(1)} = \begin{cases} p_{ij}, & \text{if } p_{ij} > p_{i_1 j_1}, \\ 0, & \text{if } p_{ij} \leq p_{i_1 j_1}. \end{cases} \tag{9}$$

Operations (8) and (9) exclude the possibility of using routes whose reliability is worse than critical. Iteration 1 is complete. The first stage of the assignment procedure is carried out.

Iteration 2. Using the resulting matrix $C^{(1)} = (c_{ij}^{(1)})$, let's again solve the transport problem (1), (4)–(6). Let $X^{(2)} = (x_{ij}^{(2)})$ be the solution of this problem. It is clear that the plan $X^{(2)}$ will not be better than the plan $X^{(1)}$ by criterion (1), but it is certainly better than this plan by criterion (3). Thus, the assignment by criterion (1) in favor of criterion (3) is realized.

Next, let's select the set $\{p_{ij}\delta_j(x_{ij}^{(2)})\}$ again, find $(i_2, j_2) = \arg \min \{p_{ij}\delta_j(x_{ij}^{(2)})\}$ and modify the matrices $C^{(1)}, P^{(1)}$:

$$c_{ij}^{(2)} = \begin{cases} c_{ij}^{(1)}, & \text{if } p_{ij}^{(1)} > p_{i_2 j_2}^{(1)}, \\ M, & \text{if } p_{ij}^{(1)} \leq p_{i_2 j_2}^{(1)}; \end{cases}$$

$$p_{ij}^{(2)} = \begin{cases} p_{ij}^{(1)}, & \text{if } p_{ij}^{(1)} > p_{i_2 j_2}^{(1)}, \\ 0, & \text{if } p_{ij}^{(1)} \leq p_{i_2 j_2}^{(1)}. \end{cases}$$

Iteration 2 is complete.

Carrying out of each iteration reduces the number of possible routes. The solution of the problem is continued

until the next solutions of problem (1), (4)–(6) give an admissible plan.

Let's consider an example. For a system of three manufacturers and four consumers with numerical characteristics:

$$a_1 = 10, a_2 = 30, a_3 = 20,$$

$$b_1 = 18, b_2 = 8, b_3 = 12, b_4 = 22,$$

let's introduce the value matrix C and the probability P :

$$C^{(0)} = \begin{vmatrix} 4 & 7 & 8 & 9 \\ 7 & 10 & 3 & 5 \\ 2 & 6 & 5 & 4 \end{vmatrix}, P^{(0)} = \begin{vmatrix} 0.25 & 0.9 & 0.75 & 0.8 \\ 0.5 & 0.45 & 0.2 & 0.8 \\ 0.3 & 0.7 & 0.8 & 0.4 \end{vmatrix}.$$

Iteration 1. The solution of problem (1), (4)–(6) gives the transportation plan:

$$X^{(1)} = \begin{vmatrix} 0 & 8 & 0 & 2 \\ 0 & 0 & 12 & 18 \\ 18 & 0 & 0 & 2 \end{vmatrix}.$$

The values of the criteria (1) and (3) for the plan are:

$$L(X^{(1)}) = 244; P(X^{(1)}) = 0.0138.$$

Let's obtain a nonzero set of probabilities:

$$\{p_{ij}\delta_j(x_{ij}^{(1)})\} = \{0.9; 0.8; 0.2; 0.8; 0.3; 0.4\}.$$

In this case, $P_{\min} = 0.2 = P_{23}$. the modification of the original matrices C and P in accordance with (8), (9):

$$C^{(1)} = \begin{vmatrix} 4 & 7 & 8 & 9 \\ 7 & 10 & M & 5 \\ 2 & 6 & 5 & 4 \end{vmatrix}, P^{(1)} = \begin{vmatrix} 0.25 & 0.9 & 0.75 & 0.8 \\ 0.5 & 0.45 & 0 & 0.8 \\ 0.3 & 0.7 & 0.8 & 0.4 \end{vmatrix}.$$

Iteration 1 is complete.

Iteration 2. Acting like the previous one, let's obtain the plan:

$$X_2 = \begin{vmatrix} 8 & 0 & 2 & 0 \\ 0 & 8 & 0 & 22 \\ 10 & 0 & 10 & 0 \end{vmatrix}.$$

The value of criteria (1) and (3) for the plan are equal:

$$L(X^{(2)}) = 308; P(X^{(2)}) = 0.0144.$$

In this case, $L(X^{(2)}) > L(X^{(1)}), P(X^{(2)}) > P(X^{(1)})$. The non-zero probability set, corresponding $X^{(2)}$, has the form:

$$\{p_{ij}\delta_j(x_{ij}^{(2)})\} = \{0.25; 0.7; 0.45; 0.8; 0.3; 0.8\},$$

from where $P_{\min} = 0.25$. Modification of matrices $C^{(1)}$ and $P^{(1)}$:

$$C^{(2)} = \begin{vmatrix} 7 & 8 & 9 \\ 7 & 10 & M & 5 \\ 2 & 6 & 5 & 4 \end{vmatrix}, P^{(2)} = \begin{vmatrix} 0 & 0.9 & 0.75 & 0.8 \\ 0.5 & 0.45 & 0 & 0.8 \\ 0.3 & 0.7 & 0.8 & 0.4 \end{vmatrix}.$$

Iteration 2 is complete.

All calculations on subsequent iterations are performed in a manner similar to the previous one, let's give their results without explanation.

Iteration 3.

$$X^{(3)} = \begin{vmatrix} 0 & 8 & 2 & 0 \\ 10 & 0 & 0 & 20 \\ 8 & 0 & 10 & 2 \end{vmatrix},$$

$$L(X^{(3)}) = 316; P(X^{(3)}) = 0.0324,$$

$$L(X^{(3)}) > L(X^{(2)}); P(X^{(3)}) > P(X^{(2)}),$$

$$\{p_{ij}\delta(x_{ij}^{(3)})\} = \{0.9; 0.75; 0.5; 0.8; 0.3; 0.8\}, P_{\min} = 0.3,$$

$$C^{(3)} = \begin{vmatrix} 7 & 8 & 9 \\ 7 & 10 & M & 5 \\ M & 6 & 5 & 4 \end{vmatrix}, P^{(3)} = \begin{vmatrix} 0 & 0.9 & 0.75 & 0.8 \\ 0.5 & 0.45 & 0 & 0.8 \\ 0 & 0.7 & 0.8 & 0.4 \end{vmatrix}.$$

Iteration 4.

$$X^{(4)} = \begin{vmatrix} 0 & 8 & 2 & 0 \\ 18 & 0 & 0 & 12 \\ 0 & 0 & 10 & 10 \end{vmatrix},$$

$$L(X^{(4)}) = 348; P(X^{(4)}) = 0.0864,$$

$$L(X^{(4)}) > L(X^{(3)}); P(X^{(4)}) > P(X^{(3)}),$$

$$\{p_{ij}\delta(x_{ij}^{(4)})\} = \{0.9; 0.75; 0.5; 0.8; 0.8; 0.4\},$$

$$P_{\min} = 0.4,$$

$$C^{(4)} = \begin{vmatrix} 7 & 8 & 9 \\ 7 & 10 & M & 5 \\ M & 6 & 5 & M \end{vmatrix}, P^{(4)} = \begin{vmatrix} 0 & 0.9 & 0.75 & 0.8 \\ 0.5 & 0.45 & 0 & 0.8 \\ 0 & 0.7 & 0.8 & 0 \end{vmatrix}.$$

Iteration 5.

$$X^{(5)} = \begin{vmatrix} 0 & 0 & 0 & 10 \\ 18 & 0 & 0 & 12 \\ 0 & 8 & 12 & 0 \end{vmatrix},$$

$$L(X^{(5)}) = 384; P(X^{(5)}) = 0.1792,$$

$$L(X^{(5)}) > L(X^{(4)}); P(X^{(5)}) > P(X^{(4)}),$$

$$\{p_{ij}\delta(x_{ij}^{(5)})\} = \{0.8; 0.5; 0.8; 0.7; 0.8\}, P_{\min} = 0.5,$$

$$C^{(5)} = \begin{vmatrix} 7 & 8 & 9 \\ M & 10 & M & 5 \\ M & 6 & 5 & M \end{vmatrix}, P^{(5)} = \begin{vmatrix} 0 & 0.9 & 0.75 & 0.8 \\ 0 & 0.45 & 0 & 0.8 \\ 0 & 0.7 & 0.8 & 0 \end{vmatrix}.$$

Further solution of the problem is impossible, since all routes to the first consumer are blocked.

A joint analysis of all the results of solving the problem allows to draw the following conclusions.

1. The realized iterative procedure of successive concessions ensured the receipt of a set of plans, none of which is majorizing, that is, for no pair:

$$(L(X^{(s)}), P(X^{(s)})), (L(X^{(r)}), P(X^{(r)})),$$

the inequalities:

$$(L(X^{(s)}) < L(X^{(r)})), (P(X^{(s)}) > P(X^{(r)}))$$

aren't satisfied simultaneously.

2. The obtained results make it possible to choose a compromise solution of the problem. Let the criterion (1) be chosen as the main one, and it must be minimized, and the criterion (3) is constrained (for example, $P(x) \geq P_{all} = 0.05$). Then the plan $X^{(4)}$ is a compromise, since this plan minimizes (1), satisfying the constraint ($P(X^{(4)}) = 0.084 > 0.05$).

Let's note now that the formulated multicriteria transport problem can be solved in a similar way in the other case, when the main criterion is the reliability of the transportation plan implementation, and the average total cost should not exceed a given threshold. At the same time, as the initial transportation plan, the resulting plan of the previous task can be chosen, for which the maximum probability of its realization is achieved.

Iteration 1. So the initial plan has the form:

$$X^{(1)} = \begin{vmatrix} 0 & 8 & 0 & 10 \\ 18 & 0 & 0 & 12 \\ 0 & 8 & 12 & 0 \end{vmatrix},$$

$$L(X^{(1)}) = 384; P(X^{(1)}) = 0.1792.$$

Let's find a non-zero set $\{c_{ij}\delta(x_{ij}^{(1)})\}$ and define a pair of indices $(i_1, j_1) = \arg \max_{i,j} \{c_{ij}\delta(x_{ij}^{(1)})\}$.

The corresponding route from the supplier i_1 to the consumer j_1 will be the most expensive:

$$\{c_{ij}\delta(x_{ij}^{(1)})\} = \{9; 7; 5; 6; 5\}, c_{ij \max} = c_{14} = 9.$$

The operation of modifying the matrices C and P is as follows:

$$c_{ij}^{(1)} = \begin{cases} c_{ij}, & \text{if } c_{ij} < c_{14}, \\ M, & \text{if } c_{ij} \geq c_{14}; \end{cases}$$

$$p_{ij}^{(1)} = \begin{cases} p_{ij}, & \text{if } c_{ij} < c_{14}, \\ 0, & \text{if } c_{ij} \geq c_{14}. \end{cases}$$

Then

$$C^{(1)} = \begin{vmatrix} 4 & 7 & 8 & M \\ 7 & M & 3 & 5 \\ 2 & 6 & 5 & 4 \end{vmatrix}, P^{(1)} = \begin{vmatrix} 0.25 & 0.9 & 0.75 & 0 \\ 0.5 & 0 & 0.2 & 0.8 \\ 0.3 & 0.7 & 0.8 & 0.4 \end{vmatrix}.$$

Iteration 1 is complete.

Iteration 2. Using the method of the maximum element of the matrix $P^{(1)}=(p_{ij}^{(1)})$, let's obtain a plan:

$$X^{(2)} = \begin{vmatrix} 0 & 8 & 2 & 0 \\ 18 & 0 & 0 & 12 \\ 0 & 0 & 10 & 10 \end{vmatrix}.$$

The values of the criteria (1) and (3) for the plan $X^{(2)}$ are:

$$L(X^{(2)})=384; P(X^{(2)})=0.0864;$$

$$L(X^{(2)})<L(X^{(1)}), P(X^{(2)})<P(X^{(1)}).$$

Then the set is formed:

$$\{c_{ij}\delta(x_{ij}^{(2)})\}=\{7; 8; 7; 5; 5; 4\}, c_{ij\max}=8.$$

Modification of matrices $C^{(2)}$ and $P^{(2)}$ gives:

$$C^{(2)} = \begin{vmatrix} 4 & 7 & M & M \\ 7 & M & 3 & 5 \\ 2 & 6 & 5 & 4 \end{vmatrix}, P^{(2)} = \begin{vmatrix} 0.25 & 0.9 & 0 & 0 \\ 0.5 & 0 & 0.2 & 0.8 \\ 0.3 & 0.7 & 0.8 & 0.4 \end{vmatrix}.$$

Iteration 2 is complete.

Iteration 3. Acting similarly, let's obtain a plan:

$$X^{(3)} = \begin{vmatrix} 2 & 8 & 0 & 0 \\ 8 & 0 & 0 & 22 \\ 8 & 0 & 12 & 0 \end{vmatrix}.$$

The values of the criteria (1) and (3) for the plan $X^{(2)}$ are:

$$L(X^{(3)})=306; P(X^{(3)})=0.0216;$$

$$L(X^{(3)})<L(X^{(2)}), P(X^{(3)})<P(X^{(2)}).$$

The set of values corresponding to non-zero deliveries has the form:

$$\{c_{ij}\delta(x_{ij}^{(3)})\}=\{4; 7; 7; 5; 2; 5\}, c_{ij\max}=7.$$

Modification of matrices $P^{(2)}$ and $C^{(2)}$ gives:

$$C^{(3)} = \begin{vmatrix} 4 & M & M & M \\ M & M & 3 & 5 \\ 2 & 6 & 5 & 4 \end{vmatrix}, P^{(3)} = \begin{vmatrix} 0.25 & 0 & 0 & 0 \\ 0 & 0 & 0.2 & 0.8 \\ 0.3 & 0.7 & 0.8 & 0.4 \end{vmatrix}.$$

Iteration 3 is complete.

Iteration 4. Again, using the maximum element method for the matrix $P^{(3)}$, let's obtain the plan:

$$X^{(4)} = \begin{vmatrix} 10 & 0 & 0 & 0 \\ 0 & 0 & 8 & 22 \\ 8 & 0 & 4 & 0 \end{vmatrix}.$$

The values of the criteria (1) and (3) for the plan $X^{(3)}$ are:

$$L(X^{(3)})=258; P(X^{(3)})=0.00672;$$

$$L(X^{(3)})<L(X^{(2)}), P(X^{(3)})<P(X^{(2)}).$$

Let's form the set:

$$\{c_{ij}\delta(x_{ij}^{(4)})\}=\{4; 3; 5; 2; 6; 5\}, c_{ij\max}=6.$$

Modification of matrices $P^{(3)}$ and $C^{(3)}$ gives:

$$C^{(4)} = \begin{vmatrix} 4 & M & M & M \\ M & M & 3 & 5 \\ 2 & M & 5 & 4 \end{vmatrix}, P^{(4)} = \begin{vmatrix} 0.25 & 0 & 0 & 0 \\ 0 & 0 & 0.2 & 0.8 \\ 0.3 & 0 & 0.8 & 0.4 \end{vmatrix}.$$

Further solution of the problem is impossible, since all routes leading to the second consumer are blocked.

Let's note that an independent solution of the problem formed in the example using two different basic criteria yields partially coincident partial solutions, but this, apparently, is an effect of small dimension of the original problem. On the other hand, the joint use of all the particular solutions obtained allows to realize important advantages of the resulting Pareto-set solutions, since these solutions do not majorize each other. Analysis of this set gives a wide scope for choosing a compromise solution.

The proposed method is easily generalized to the case of a larger number of criteria. Let the duration of transportation be chosen as the third criterion. Let's now construct the matrix $T=(t_{ij})$, as before, the problem is solved according to the basic, cost criterion (1). Let $X^{(1)}=(X_{ij}^{(1)})$ be the solution of the problem. Further, let's find nonzero sets $\{p_{ij}\delta(x_{ij}^{(1)})\}$ and $\{t_{ij}\delta(x_{ij}^{(1)})\}$, and pairs of indices:

$$(i_1, j_1) = \arg \min \{p_{ij}\delta(x_{ij}^{(1)})\},$$

$$(i_2, j_2) = \arg \max \{t_{ij}\delta(x_{ij}^{(1)})\}.$$

It is clear that the route (i_1, j_1) is the least reliable of the route of the plan $X^{(1)}$, and the route (i_2, j_2) is the most durable. Let's call them critical. We modify the matrices C, P, T by the rule:

$$C_{ij}^{(1)} = \begin{cases} c_{ij}, & \text{if } (p_{ij} > p_{i_1 j_1}) \wedge (t_{ij} < t_{i_2 j_2}), \\ M, & \text{if } (p_{ij} \leq p_{i_1 j_1}) \vee (t_{ij} \geq t_{i_2 j_2}); \end{cases}$$

$$p_{ij}^{(1)} = \begin{cases} p_{ij}, & \text{if } (p_{ij} > p_{i_1 j_1}) \wedge (t_{ij} < t_{i_2 j_2}), \\ 0, & \text{if } (p_{ij} \leq p_{i_1 j_1}) \vee (t_{ij} \geq t_{i_2 j_2}); \end{cases}$$

$$t_{ij}^{(1)} = \begin{cases} t_{ij}, & \text{if } (p_{ij} > p_{i_1 j_1}) \wedge (t_{ij} < t_{i_2 j_2}), \\ M, & \text{if } (p_{ij} \leq p_{i_1 j_1}) \vee (t_{ij} \geq t_{i_2 j_2}). \end{cases}$$

These operations exclude the possibility of further using routes that are worse than critical ones.

Further actions are performed as described above.

The direction of further research is connected with the consideration of the situation when the initial data of the problem are not defined exactly. The real variants of the emerging uncertainty are described in [14–18]. Possible approaches to solving transport problems with the uncertainty of the initial ones are discussed in [19–21].

7. SWOT analysis of research results

Strengths. The proposed method for solving the transport problem of linear programming, in contrast to the known ones, allows one to obtain a compromise solution that

provides the best value for the main criterion, provided that the additional criteria take values not worse than those given. The achievement of this effect by traditional methods is not feasible.

Weaknesses. The application of this method increases the total time for solving the problem.

Opportunities. Application of the proposed method opens up the prospects for solving transportation problems in conditions when the initial data on the cost of transportation, the likelihood of the plan and the duration of its implementation contain uncertainty.

Threats. The application of the method of concessions provides the possibility of obtaining a compromise solution that will inevitably be worse than the optimal solution by the main criterion. At the same time, the more stringent the requirements will be on additional criteria, the more tangible the deterioration of the decision will be on the basis of the main criterion.

8. Conclusions

1. Based on the results of the analysis of known methods for solving multicriteria problems (Pareto-set formation, scalarization of the vector criterion, concession method), the last is justified. Important advantages of the proposed method: simplicity of the computational procedure, grounded technology of forming a new solution at each iteration, realizing the concept of assignment, quality control of the solution obtained at each step.

2. To implement the method, an iterative procedure is proposed in which the initial plan is optimal by the main criterion. At subsequent iterations, an assignment is made to the main criterion in order to improve the values of the additional criteria. The solution of the problem is continued until a compromise solution is obtained, ensuring the best value for the main criterion, provided that the values for the remaining criteria are no worse than those given. The application of the proposed method opens the prospect of its generalization to the case when the initial data for the solution of the problem contain uncertainty.

References

1. Yudin, D. B. Zadachi lineinogo programmirovaniia transportnogo tipa [Text] / D. B. Yudin, E. G. Golshtein. – Moscow: Nauka, 1969. – 384 p.
2. Sira, O. V. Mnogomernnye modeli logistiki v usloviiah neopredelennosti [Text] / O. V. Sira. – Kharkiv: FOP Stetsenko I. I., 2010. – 512 p.
3. Raskin, L. G. Mnogoindeksnye zadachi lineinogo programmirovaniia [Text] / L. G. Raskin, O. I. Kirichenko. – Moscow: Radio i sviaz, 1982. – 240 p.
4. Steuer, R. Multiple Criteria Optimization: Theory, Computation and Application [Text] / R. Steuer. – New York: John Wiley, 1986. – 546 p.
5. Savaragi, Y. Theory of Multiobjective Optimization [Text] / Y. Savaragi, H. Nakayama, T. Tanin. – Orlando: Academic Press Inc., 1985. – 296 p.
6. Keeney, R. L. Decisions with Multiple Objectives [Text] / R. L. Keeney, H. Raiffa. – Cambridge University Press, 1993. – 570 p. doi:10.1017/cbo9781139174084
7. Ehrgott, M. Multicriteria Optimization [Text] / M. Ehrgott. – Heidelberg: Springer, 2005. – 323 p. doi:10.1007/3-540-27659-9
8. Craft, D. L. Approximating convex Pareto surfaces in multiobjective radiotherapy planning [Text] / D. L. Craft, T. F. Halabi, H. A. Shih, T. R. Bortfeld // Medical Physics. – 2006. – Vol. 33, No. 9. – P. 3399–3407. doi:10.1118/1.2335486
9. Lotov, A. V. Mnogokriterial'nye zadachi priniatiia reshenii [Text] / A. V. Lotov, I. I. Pospelova. – Moscow: MAKS Press, 2008. – 197 p.
10. Intrilligator, M. Matematicheskie metody optimizatsii i ekonomicheskaiia teoriia [Text] / M. Intrilligator. – Moscow: Antrispres, 2002. – 553 p.
11. Cohon, J. L. Multiobjective Programming and Planning [Text] / J. L. Cohon. – New York: Dover Publ, 2004. – 352 p.
12. Luque, M. Global formulation for interactive multiobjective optimization [Text] / M. Luque, F. Ruiz, K. Miettinen // OR Spectrum. – 2008. – Vol. 33, No. 1. – P. 27–48. doi:10.1007/s00291-008-0154-3
13. Panda, S. Multi-objective evolutionary algorithm for SSSC-based controller design [Text] / S. Panda // Electric Power Systems Research. – 2009. – Vol. 79, No. 6. – P. 937–944. doi:10.1016/j.epsr.2008.12.004
14. Zadeh, L. A. Fuzzy sets [Text] / L. A. Zadeh // Information and Control. – 1965. – Vol. 8, No. 3. – P. 338–353. doi:10.1016/s0019-9958(65)90241-x
15. Negoitse, K. Primenenie teorii sistem k problemam upravleniia [Text] / K. Negoitse. – Moscow: MIR, 1981. – 219 p.
16. Orlovskii, S. A. Problemy priniatiia reshenii pri nechetkoi informatsii [Text] / S. A. Orlovskii. – Moscow: Nauka, 1981. – 264 p.
17. Diubua, D. Teoriia vozmozhnostei. Prilozhenie k predstavleniiu znanii v informatike [Text] / D. Diubua, A. Prad. – Moscow: Radio i sviaz, 1990. – 286 p.
18. Raskin, L. G. Nechetkaia matematika. Osnovy teorii. Prilozheniia [Text] / L. G. Raskin, O. V. Sira. – Kharkiv: Parus, 2008. – 352 p.
19. Raskin, L. Method of solving fuzzy problems of mathematical programming [Text] / L. Raskin, O. Sira // Eastern-European Journal of Enterprise Technologies. – 2016. – Vol. 5, No. 4 (83). – P. 23–28. doi:10.15587/1729-4061.2016.81292
20. Pawlak, Z. Rough sets [Text] / Z. Pawlak // International Journal of Computer & Information Sciences. – 1982. – Vol. 11, No. 5. – P. 341–356. doi:10.1007/bf01001956
21. Raskin, L. Fuzzy models of rough mathematics [Text] / L. Raskin, O. Sira // Eastern-European Journal of Enterprise Technologies. – 2016. – Vol. 6, No. 4 (84). – P. 53–60. doi:10.15587/1729-4061.2016.86739

АНАЛІЗ І РОЗРОБКА КОМПРОМІСНИХ РЕШЕНЬ МНОГОКРИТЕРІАЛЬНИХ ТРАНСПОРТНИХ ЗАДАЧ

Рассмотрен метод решения многокритериальных транспортных задач. Предложена итерационная процедура, в которой начальный план задачи является оптимальным по основному из критериев. На последующих итерациях реализуется уступка по основному из критериев с целью улучшения значения дополнительных. Процедура продолжается до получения компромиссного решения. Рассмотрены примеры решения задачи.

Ключевые слова: многокритериальная транспортная задача, итерационное решение, формирование Парето-множества решений.

Raskin Lev, Doctor of Technical Sciences, Professor, Head of the Department of Distributed Information Systems and Cloud Technologies, National Technical University «Kharkiv Polytechnic Institute», Ukraine, e-mail: topology@ukr.net, ORCID: <http://orcid.org/0000-0002-9015-4016>

Sira Oksana, Doctor of Technical Sciences, Professor, Department of Computer Monitoring and Logistics, National Technical University «Kharkiv Polytechnic Institute», Ukraine, e-mail: chime@bk.ru, ORCID: <http://orcid.org/0000-0002-4869-2371>

Parfeniuk Yurii, Postgraduate Student, Department of Distributed Information Systems and Cloud Technologies, National Technical University «Kharkiv Polytechnic Institute», Ukraine, e-mail: parfuriy.l@gmail.com, ORCID: <https://orcid.org/0000-0001-5357-1868>