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APPROXIMATIONS AND FORECASTING QUASI-STATIONARY PROCESSES WITH SUDDEN RUNS

Об'єктом дослідження є гетероскедастичні процеси при формуванні та оцінці офсетної політики держав на міжнародних ринках, зокрема, при укладенні та виконанні офсетних контрактів. Процес укладення та виконання умов офсетного контракту є слабо нестаціонарним, оскільки при його укладенні можуть мати місце найрізноманітніші раптові події, форс-мажорні обставини, які неможливо докладно описати та передбачити, з припустимою точністю, в повному обсязі. Показано, що для опису таких процесів більш придатним є термін «квазістаціонарний процес», що має деяке наближення до стаціонарного процесу. Найбільш перспективним підходом до побудови математичних моделей таких процесів є застосування комбінованих фрактальних моделей авторегресії та інтегрованого ковзного середнього. В ході дослідження використовувались методи теорії нестаціонарних випадкових процесів та теорії викидів випадкових процесів, показано, що комбінація квазістаціонарного процесу з послідовністю випадкових викидів цілком задовільно моделюється так званим фрактальним або самоподібним процесом. У якості універсальної математичної моделі самоподібних процесів з повільно убуваючими залежностями використовують модель фрактальної інтегрованої авторегресії та ковзного середнього FARIMA. Однак в цій моделі не враховується вплив викидів випадкових процесів на коефіцієнти чисельника та знаменника дрібно-раціональної функції, якою апроксимується сам процес авторегресії та ковзного середнього. Тому в проведеному дослідженні є зв'язок у параметрів викиду та коефіцієнтів апроксимуючої функції. Розглянуті математичні моделі дискретних часових рядів, які характеризуються квазістаціонарністю та наявністю раптових викидів. Показано, що для апроксимації таких рядів доволі придатними є моделі типу авторегресії та ковзного середнього, модифіковані до класів моделей авторегресії та інтегрованого ковзного середнього. А для самоподібних (фрактальних) процесів модифіковані до класів моделей авторегресії та фрактального інтегрованого ковзного середнього. Співвідношення між довжиною зсуву при обчисленні коефіцієнтів автокореляції та загальною довжиною вибірки може служити прийнятним індикатором коректності рішення.

Ключові слова: *гетероскедастичність, дискретні часові ряди, авторегресія, ковзне середнє, фрактальне інтегроване ковзне середнє.*

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1. Introduction

In world practice on the market of military goods (MG), the inclusion in commercial offers, as components, of offset agreements has been used for a relatively long time. Despite global economic downturns, the military offset market is showing strong momentum, which is accumulating significant defense procurement programs in the countries of the Asia-Pacific region and the Middle East. In recent years, more stringent conditions for importing countries have appeared in offset contracts in respect of military compensation and an increase in penalties for failure to fulfill obligations. This situation will contribute to the growth of the market for military goods, but it can also negatively affect the income of exporting countries.

The presented work is devoted to a very relevant and specific area of financial and economic activity – the problems of the formation and evaluation of the offset policy of states in international markets, in particular, the conclusion and implementation of offset contracts [1, 2]. The process of concluding and fulfilling the conditions of an offset contract in a statistical sense is, of course, non-stationary. In addition, when concluding an offset contract, a wide variety of sudden events, force majeure circumstances, etc. can take place – phenomena that can't be described in detail and predicted with acceptable accuracy in full.

An unsteady random process is an adequate mathematical model of real processes occurring in technical systems, in the economy and society. With a high degree of abstraction and universality of such a model, the results obtained when using it, of course, will also be quite abstract, separated from specific applications. Therefore, first of all, it is necessary to specify the subject area of industrial or agricultural production, the economy and the like. The characteristics of the subject area will almost completely depend on the characteristics of the processes occurring in the system – dynamics (in particular, statistical dynamics), current and final probabilistic characteristics, type and asymptotically transient processes, etc. Returning to the subject of offset contracts, it is possible to state that in a certain sense, they are unsteady (possibly slightly unsteady) random processes with short-term or medium-term runs [3].

The work will not consider regional or global development processes, an exhaustive analysis of this problem has been made, in [1]. Let's consider only the statistical aspects of the topic. Very important parts of scientific and practical research in this field of activity are the following:

– construction of mathematical model of the analyzed process;
– construction of a method for predicting the development of this process;
– estimates (at the previous stage – at least qualitative) of the asymptotic characteristics of the process.

Therefore, the solution of these problems is urgent, and this work is dedicated to this.

Thus, *the object of research is* heteroskedastic processes in the formation and evaluation of the offset policy of states in international markets, in particular, in the conclusion and execution of offset contracts.

The aim of research as a whole is to develop an effective methodology for choosing mathematical models of discrete time series of various classes for quasi-stationary processes with runs, which are used in the practical forecasting of processes.

2. Methods of research

Mathematical models of sudden changes can be built, for example, on the basis of the theory of runs of random processes [1] or by methods of the theory of Markov processes [2]. In the study, methods of the theory of non-stationary random processes and the theory of runs of random processes are used [4].

The theory of runs is well known and worked out, but has not been investigated in cases of unsteady random processes. Existing run theory allows one to determine the probabilities of finding a random parameter outside level tolerances.

To describe the time series, mathematical models are used that can take various forms. Among them, one can distinguish autoregressive models, moving average models, and integral models. Based on them, models of autoregressive moving average (ARMA), autoregressive integrated moving average (ARIMA) and fractal integrated moving average (FARIMA) are built.

3. Research results and discussion

3.1. The mathematical model of the offset process. In economics and econometrics, they often operate with the concept of heteroskedastic, that is, a weakly unsteady process [4, 5]. However, the notions of non-stationary in general and weak non-stationary in particular also need to be specified.

Determining the stationarity of a random process is one of the key in probability theory. It is given in many works on statistical theory, for example, [6, 7]. Let's only recall that in applied statistics and in econometrics the concept of stationarity is most often used in the broad sense. In addition, a process stationary in the narrow sense is a process stationary in the broad sense (but not vice versa. This remark will be needed later).

The non-stationary process is such that it does not satisfy the stationary conditions, even in a broad sense. There is such a classification of non-stationary processes:

– non-stationary process with constant mathematical expectation and variable dispersion in time;
– an unsteady process with constant dispersion and a time-variable mathematical expectation;
– non-stationary process with time-variable mathematical expectation and dispersion.

Widely used, especially in econometrics, the term «weakly unsteady process» is not specific enough, because it does not provide a regular way to determine quantitative and, accordingly, comparative estimates, the term «quasi-stationary process», which has some approximation to the stationary process, is more suitable. This term is usually used in those cases when the characteristic time of equilibrium in a stochastic system is much less than the characteristic time of the change in the equilibrium parameters of the system, which are determined by the effect on the system. A quasi-stationary process proceeds in a conservative system and spreads in it so quickly that its state does not have time to change during the spread of this process within the system. By «system» let's mean both technical and economic systems, taking into account their characteristic features.

When assessing the stability of economic systems under the influence of random factors, methods of the theory of runs of random processes are widely used. Runs of a random process (stationary or quasi-stationary) are a separate section of the theory of random processes [8, 9]. Guided by the methods of the theory of non-stationary random processes and the theory of runs of random processes, it can be argued that the combination of a quasi-stationary process with a sequence of random runs is quite satisfactorily modeled by the so-called fractal or self-similar process [10, 11]. The latter has this property: with relatively slow changes in the mathematical expectation and/or variance of the process on the observation interval T_s, short-term runs (dispersion jumps) with an average length δt_σ, $\delta t_\sigma << T_s$ can occur. Self-similar processes also have the property of a slowly decreasing dependence of the correlation and spectral characteristics of the process.

As a universal mathematical model of self-similar processes with slowly decreasing dependencies, the model of fractally integrated autoregressive moving average (FARIMA) is used [11]. However, this model does not take into account the effect of runs of random processes on the coefficients of the numerator and denominator of the finely rational function, which approximates the process of autoregression and the moving average. Therefore, in the next subsection, let's consider the relationship between the parameters of the runs and the coefficients of the approximating function.

3.2. Properties of coefficients of models of different classes for quasi-stationary processes with runs. The mixed process of autoregression and the moving average is a discrete time series of the form:

$$y(n)=\sum_{l=1}^{L} b_l y(n-l)+\sum_{k=0}^{K} a_k u(n-k), \qquad (1)$$

where $u(n-k)$, $k=0,1,2,\ldots,K$ – external excitation factors (random noise signals); a_k, b_l responses of the system at previous points in time; a_k, b_l – weighting factors. In general $K \neq L$.

Taking z-transformations from expression (1), let's obtain a system function $H(z)$. Let's write the general expression for the autoregression model and the moving average on the z-transformation plane:

$$H_{ARMA}(z)=\frac{\sum_{k=0}^{K} a_k z^{-k}}{1-\sum_{l=1}^{L} b_l z^{-l}}. \quad (2)$$

Based on the expression (1), let's deal with a model of a stationary discrete time series, which is described by a difference equation with constant coefficients. Accordingly, expression (2) is a system function of a given time series.

A mixed model of autoregression and moving average (*ARMA*) with correspondingly selected coefficients a_k, b_l can quite satisfactorily describe real processes. With a rational choice of coefficients a_k, b_l, it is possible to build more economical models; they give an acceptable approximation with a minimum number of parameters.

For such specific types of unsteadiness of random processes as a process with a linearly increasing mathematical expectation, it is possible to modify the model *ARMA* by integrating the terms by which a moving average is formed. This class of models is autoregressive model and integrated moving average (*ARIMA* process).

The discrete process of autoregression and integrated moving average is formed by successive m-multiple summation of process (1), which, in essence, is an *ARMA* process. If in the *ARIMA* process the order of the moving average equation is equal K, the order of the autoregressive equation is L, and a multiplicity of integration is M, let's obtain the model $ARIMA(K,M,L)$. Thus, the model $ARIMA(K,M,L)$ encompasses classes of stationary processes $(M=0)$ and unsteady processes of M-th $(M+1)$ order.

It is known [11] that abstract fractal processes can have small and even mixed dimensions. If the multiplicity of integration M lies in the range $M+m$, $m \in [-1/2, 1/2]$, $M=1,2,\ldots$, then let's proceed to the autoregressive model and the fractal integrated moving average $FARIMA[K,(M+m),L]$. Transformation (2) includes decompositions of the numerator $A(z)$ in the binomial series:

$$[1-A(z)]^d=\sum_{n=0}^{\infty}(-1)^n \binom{m}{n} [A(z)]^n, \quad (3)$$

where

$$\binom{m}{n}=\frac{m(m-1)(m-2)\ldots(m-n-1)}{n!}.$$

A very useful property of this expansion is the simplicity of regulation of autoregression coefficients and moving average depending on the sign of the parameter m. In particular, the signs of the autocorrelation coefficients of the process described by equations (1)–(3) coincide with the sign of the parameter m. Due to this property, rapid changes in process parameters (in fact, runs) can be monitored and predicted with acceptable accuracy. On the other hand, when applying the $FARIMA[K,(M+m),L]$ model, an incorrect determination of the parameters K, L leads to disagreements in the estimation of the coefficients of the autoregressive models and the moving average and, as a result, to model identification errors. To ensure the stability of the model, estimates of the coefficients of the polynomial polynomials and the denominator of the rational function should be based on a relatively small part of the sample data. In other words, the maximum values of the time shifts should be less than the length of the sample, at least an order of magnitude. If this condition is not met, the estimation errors at the ends of the sample will grow unlimitedly.

4. Conclusions

The analysis of the time structure of discrete series of various natures plays a key role in predicting heteroskedastic processes that are subject to sudden runs. Mathematical models of discrete time series are considered, which are characterized by quasi-stationary and the presence of sudden runs. It is revealed that for the approximation of such series, the most promising approach is the use of combined fractal autoregression models and an integrated moving average.

The advantages of using such models and the difficulties arising from inaccurate estimation of model parameters are analyzed. It is shown that attention should be paid to the accuracy of determining the expansion coefficients of the polynomials of the numerator and denominator of the finely rational function, is used. The relationship between the shear length when calculating autocorrelation coefficients and the total sample length can serve as an acceptable indicator of the correctness of the solution.

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