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STOCHASTIC PERIODIC QUEUING SYSTEMS OF MARKOV TYPE AND THEIR ANALYSIS BY THE EXAMPLE OF POWER SUPPLY SYSTEMS

In this paper problems for stochastic periodic queuing systems of Markov type are reviewed. For description and research of stochastic periodicity it is suggested to use models as periodic random sequences and noises. On the example of the power supply company "Ternopilmiskenergo" the methods of statistical analysis of the graphs of electro-consumption are considered. For description and analysis of Markov properties models are considered as periodic Markov processes, periodic Markov chains and methods of estimation of their transition matrices.

The queuing system, input stream, model, periodic noise, periodic Markov chain, statistical estimation, simulation.

1. Introduction

During a long time, from the works of A.K. Erlang, which mainly are on 1908–1922 y.y. and refer to organization of telephone systems, scientists have the interest in researches of the queuing systems. It appeared that the tasks of processing requirements arise in the different areas of researches – in technique, economy and transport. The problems of queuing systems become interesting for mathematicians, and the methods of their researches were formed in scientific field which got the name "theory of queuing systems". About the level of the personal interest of the queuing systems and according to theory of queuing systems testifies the statistic given below [1]. The scientific studies, in which simultaneously there are words "queue", "randomness", form – among the mathematical articles in 1980-1995 – 13 percent's; – among dissertations in 1980-1995 – 24 percent's; – among the works, published at scientific and engineering magazines and collections in the areas of physics, electronics, calculable methods and IT – 60 percent's (data according to the index of INSPEC, which is developed by American and German societies of the electronic engineering).

Among the various queuing systems, there are the systems which have two properties – Markov property and stochastic periodicity. It is considered that the system has Markov property, if the state of the system in the future depends on its position, in which it is in the fixed moment of time t, and does not depend on position in which the system was before this moment. Sometimes The Markov property is described more briefly: the future of the system depends on its present state and does not depend on the past. The term "stochastic periodicity" means that for the signals, got as a result of observation after the system, the determined periodicity is absent, but it is supposed that certain probabilistic descriptions change periodically.

For the study of the queuing systems the one of the appropriate approach is framed in a triad **"model-algorithm-program"**. On the first stage the model of the researched object is built; on the second – on the base of model the algorithms, methods of their researches and analysis, are developed; on the third stage the proper software is created, which is used for processing of experimental data, simulation of work of the queuing systems in general.

The first two stages of triad can be realized in the theory of queuing systems, it is the stages "model" and "algorithm". As to the models which are researched in the theory of queuing systems, they can be divided into four categories [2]:

- models **of input stream**;
- models, which describe **service expenses** (in the most cases the index of expenses is the time of service);
- models, which describe the **rules** of service in accordance with the sequence of choice of requests is set from a turn on service;
- models **of quality indexes of** service and **efficiency** of the work of the whole system.

Among the models given above, the models of the input stream deserve the most attention. It's naturally, because the mode of functioning of the queuing systems depends on the input stream and its properties. The input stream influences on forming of service expenses, on the rules of service and on the indexes of quality.

The analysis of achieving of the theory of queuing systems shows that the queuing systems (in particular, systems of Markov type) that function in stationary mode are researched closely enough. Certain results are received for the queuing systems, which function in the rhythmic mode. At the same time several tasks of selection of the models of input stream of requests for the stochastic-periodic queuing systems of Markov type remain unsolved. Little attention is given to different types to the statistical problems, especially to the methods of statistical analysis and prognosis of stochastic periodicity in the queuing systems, and also to the methods of simulation of the queuing systems, which function in the rhythmic mode.

Objective: to prove the possibility of the use of periodic Markov chains for the description of the electroenergy systems, systems of water- and gas-supplying, and to research the perspective ways of their research using the methods of mathematical statistics.

2. Stochastic periodicity electroenergy systems

The previous analysis shows that to the queuing systems, which have two properties – Markov property and stochastic periodicity, we can refer the electroenergy systems, their regional subsystems, water systems, gas-transport systems, ambulance stations, global and local networks etc. Markov property of the queuing systems form the input streams of requests and duration of their service [2], and stochastic periodicity is generated by the daily, weekly, seasonal and annual rhythms of business life on the world [3,4]. As the subject of research of this work, let's examine the queuing systems on the example of the electro-supply system of the power supply company "Ternopilmiskenergo". To the basic indexes which characterize this system, belong the input streams of requests and signals of consumption of electric power.

At first let's consider a question which refers to the stochastic periodicity of the electro-supply system. For this purpose we have to analyze the graphs of energy consumption and their properties. On the figure 1 there is a graph of consumption of electric power on the example of the power supply company "Ternopilmiskenergo" with the weekly data in January, 2010, the days from 1st to 5th – are workings days, and 6th and 7th days are weekends.

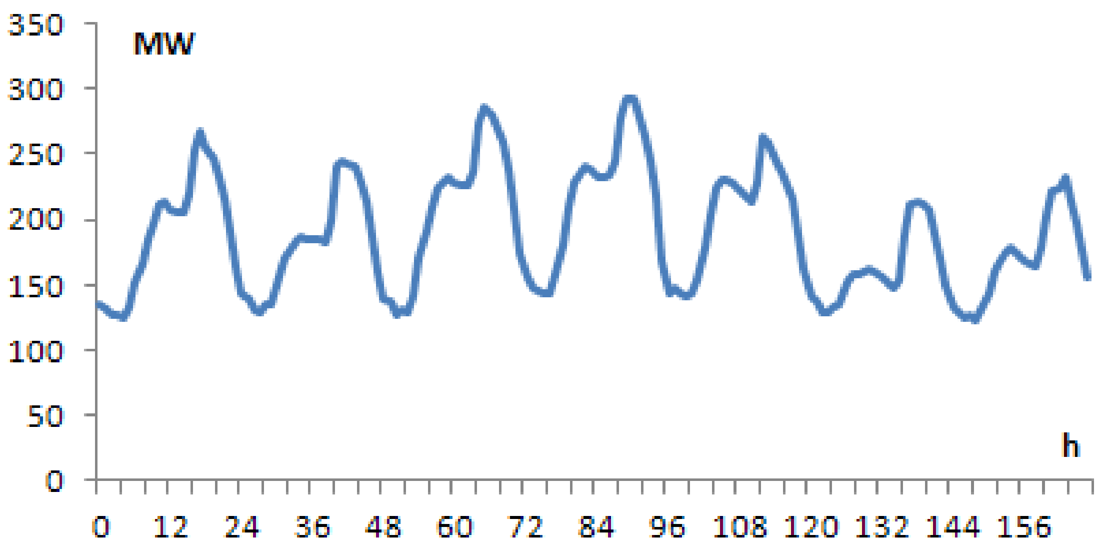

Fig.1. The graph of consumption of electric power of the electro-supply system in Ternopil during a week in January, 2010

The visual analysis of the graph allows to make previous conclusions. We can observe "approximate" repetition of values of loading in a period $T = 24\,h$; the speeds of increasing and decreasing of loadings "repeat" oneself on the proper time domains; the value of loadings at night hours also similar to each other. At the same time a certain difference between the loadings in working days and weekends attracts the attention. Thus the loading in workings days approximately from 7 a.m. to 8 p.m. exceed the proper loadings in the weekends. Among the workings days we have to emphasize on the graphs of loadings in Monday and Tuesday. In a certain degree they occupy intermediate position between the graphs for the followings three workings days from one side and by the graphs for weekends from another. Besides that we can notice the considerable growth of loading which takes place at 9 p.m. every day. But giving the description of loadings, a question, how these properties of loadings can be described quantitatively, remains actual.

Nowadays there are the models [3] which allow to take into account stochastic periodicity of signals, in our case – energy consumption. First of all, it's periodic and the periodically correlated processes and sequences, linear periodical processes. On the basis of these models the methods of their statistical analysis and prognosis are developed with the use of only one realization [3]. Using the developed statistical methods, the processing of electro-consumption of the power supply company "Ternopilmiskenergo" in January, 2010 was made. The graph of estimation of mathematical expectation is shown on the figure 2, estimations of standard deviation – on the figure 3.

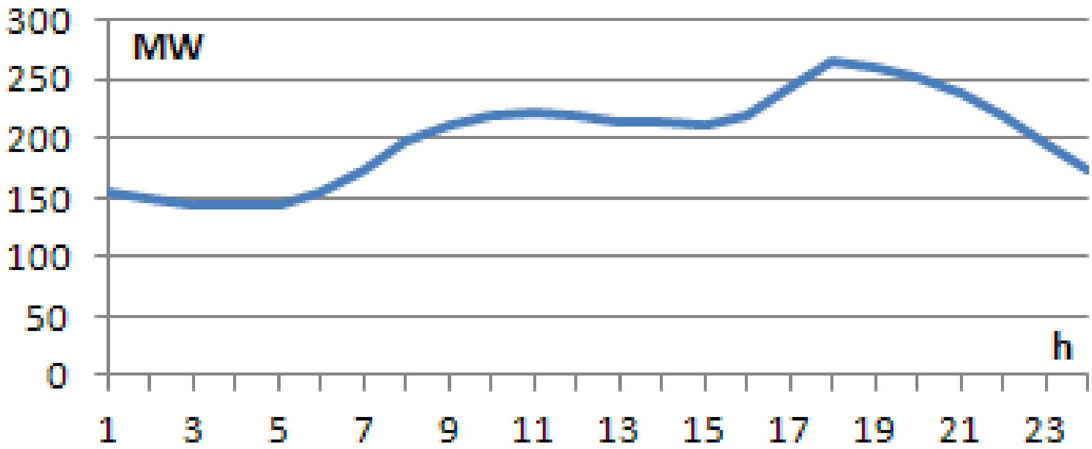

Fig. 2. The estimation of mathematical expectation $M\xi(t)$, $t\in[0, 24]$, of the consumed electric power of the power supply company "Ternopilmiskenergo" in January, 2010.

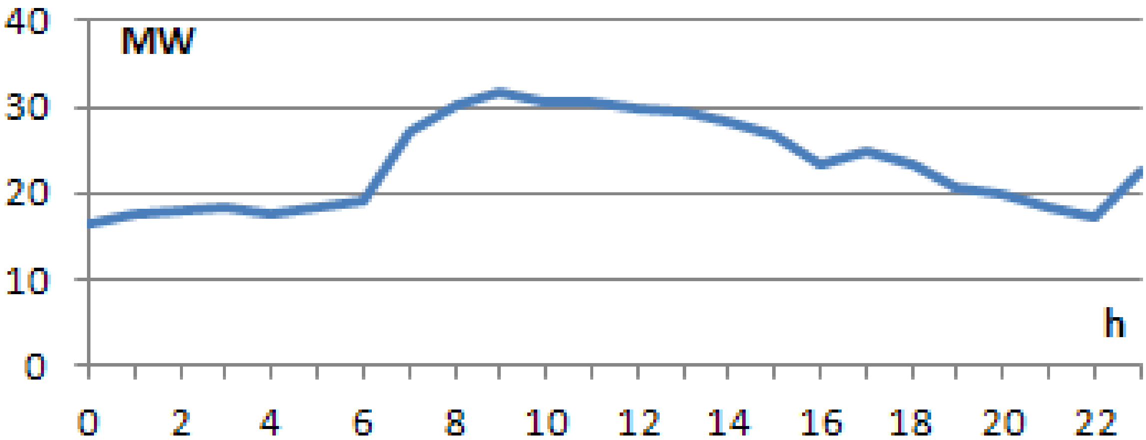

Fig. 3. An estimation of standard deviation, of the used electric power of the power supply company "Ternopilmiskenergo" in January, 2010

The analysis of evaluation results shows that the estimation of mathematical expectation of electro-consumption from 11p.m to 7 a.m. is less, than the value of the estimations from 8 a.m. to 10 p.m. A maximal electro-consumption refers to a period from 5 p.m. to 7 p.m.. From the estimation of standard deviation it is possible to make conclusion, that the consumption of electric power is more regular at nighttime, the biggest dispersion of electro-consumption, concerning to average values, takes place from 9 a.m. to 11a.m.

3. Markov property of electric power supplying systems

Now let's consider a question about the possibility of classification of electric power supplying systems as the queuing systems of Markov type. As mentioned above, the streams of inputs of requests and its duration form Markov property [2].

The stream of requests can be described by the one of three ways:

1) by a random saltatory process $\xi(t)$, $t \geq 0$ which gets single hops in the probable time moments t_i, i=0,1,2,… , t_0=0, receiving of requests;

2) directly by the sequence of probable time moments, t_i, i=0,1,2,… , t_0=0, in which the requests is received;

3) by the sequence of intervals τ_i, $i = 1,2,...$, where $\tau_i = t_i - t_{i-1}$ an interval between (i–1)th and ith requests.

The durations of processing of requests can be described with consistency s_i, i =1,2,…, where s_i is duration of processing of ith order. If the sequences of intervals τ_i, $i = 1,2,...$, and s_i, i =1,2,…, have the exponential distribution with some parameters of λ_1 and λ_2, such queuing systems can be referred to the systems of Markov type. As to the definition of Markov property of the electro supplying systems, on the example of the power supply company "Ternopilmiskenergo" we will examine only the question of input stream of this system.

It is clear, that it's hard to study the input stream of electro-supply system (as well as the other similar queuing systems) in all details. But it is possible to use another approach – firstly, study this process on the small subsystem of electro-supply system. If the received results will satisfy the certain conditions (will be discussed below), for making a final decision it will be enough to use one of the Khinchin's theorems about the sum of large number of streams with certain properties.

For realization of this approach, the local electric system of the power supply company "Ternopilmiskenergo" was chosen as a subsystem, namely one of Ternopil educational institution. The proper researches were made, which showed that the input stream of requests satisfy the conditions of absence of consequences and ordinariness [2], and on the certain intervals of time (duration of which, depending on the hour of the day, can change from 5-10 minutes to 2–3 hours) – and to the condition of stationarity. It's naturally to assume that all local electric systems of the power supply company "Ternopilmiskenergo" meet the same conditions.

To draw conclusions for the input stream of the whole system, let's use one of Khinchin's theorems [2,7], the essence of which consists in the following. If a stream $\xi(t)$ is the sum of a large number of independent between each other stationary and ordinary streams, each of which brings a small contribution in a lump sum, then at the one condition of analytical character a stream $\xi(t)$ will be near to the simplest.

It is known [2,7] that for the simplest stream the sequence of intervals τ_i, $i = 1,2,...$ which describes the input stream, has the exponential distribution

$$F_i(x) = P(\tau_i < x) = 1 - e^{-\lambda_i x}, \tag{1}$$

which, is a restricted aftereffects distribution. In the partial case all intervals are distributed uniformly and their distribution function is $F(x) = 1 - e^{-\lambda x}$. By this we proved, that the model of input stream is Markov process ξ(t), $t \geq 0$. If we investigate time intervals τ_i then the sequence τ_i, $i = 1,2,...$ is a restricted aftereffects sequence. This, as a model of power supply system of "Ternopilmiskenergo" and of queuing systems in general with similar properties.

For the model of the queuing system, which takes into account their stochastic periodicity and simultaneously Markov property, Markov processes can be used, and in the case of discrete argument we can use the periodic Markov chains. Let's recall their definitions [3,8].

4. Periodic Markov processes and chains as models of stochastic periodic queuing systems of Markov type

As the basis of the concept of Markov process is an idea about processes "without consequences". Let's imagine the system which can be in the different states. Possible states of the system form some set *X* which is called phase space. Let the system evolve in time. Its state in the moment of time *t* we will designate as x_t. If $x_t \in B$, $B \subset X$, it is considered that the system in a moment *t* is in the set *B*. Let us assume, that the evolution of the system has stochastic nature, i.e. the state of the system in the moment of time t is not determined unambiguously through the state of the system in the previous moments of time s, where $s < t$, but is random and described by the probabilistic law. Let's designate the probability of event $x_t \in B$ as $P(s, x, t, B)$ for the condition $x_s = x,\ s < t$. The probabilistic measure $P(s, x, t, B)$ is named the probability of transition (sometimes – a transitional function; a transitional probability; a conditional probability of transition) of the given system.

As the restricted aftereffects system we understand the system, for which the probability when the moment of time *t* gets caught in set *B* with fully known changing the system before the moment of time s, as well as before, equals $P(s, x, t, B)$ and, thus, depends only on the state of the system in the last known moment of time. In other words, the state of the system in the present moment of time *s* determines the probability of future evolution of process at $t > s$, and additional information about the past conduct of the process in moments $t < s$ does not influence on this probability, i.e. that remains without consequences.

The property which characterizes the conduct of the stochastic restricted aftereffects systems is named the property of absence of consequences or Markov property, and the probabilistic measure $P(s, x, t, B)$ is named Markov function of transition (the probability).

A random process $\{\xi(t), t \in (-\infty, \infty)\}$ is named Markov, if for two random moments of time t_0 and t_1, $t_0 < t_1$, a conditional distribution $\xi(t_1)$ provided that all values $\xi(t)$ are given at $t \le t_0$ depends only on $\xi(t_0)$.

The definitions of periodic Markov process and periodic Markov chain [3].

Definition 1. Markov process $\{\xi(t), t \in (-\infty, \infty)\}$ is named **a periodic Markov process**, if its conditional probability of transition is periodic by totality of the time variables, i.e. there is the number *T* that

$$P(s, x; t, B) = P(s + T, x; t + T, B) \qquad (2).$$

If for transitional probability $P(s, x, t, B)$ the set $B=(-\infty, y)$, then a function $F(s, x, t, y) = P(s, x, t, B)$ is named the distribution function of transition. For the periodic Markov process its distribution function is periodic, that is

$$F(s, x; t, y) = F(s + T, x; t + T, y) \qquad (3).$$

Let's come to the concept of periodical Markov chain. The sequence of integer of random variables $\{\xi_n, n = 0,1,2,...\}$ which take the values from phase space is named Markov chain, if for all $n \ge 0$ the conditional probability is

$$\begin{aligned} &P\{\xi_{n+1} = j | \xi_0 = h, \ldots, \xi_{n-1} = k,\ \xi_n = i\} = \\ &= P\{\xi_{n+1} = j | \xi_n = i\} \overset{df}{=} p_{ij}(n) \end{aligned} \qquad (4).$$

Conditional probabilities $p_{ij}(n)$ in totality form matrices $\Pi(n) = \|p_{ij}(n)\|,\ i, j \in X,\ n = 0,1,2,\ldots,$ which is named the matrices of transitions (transitional probabilities, probabilities of transitions) of the chain.

Definition 2. Markov chain $\{\xi_n\}, n = 0,1,2,...$, is named **periodic**, if its probabilities of transitions are periodic, i.e. there is the integer L > 1, that $p_{ij}(n) = p_{ij}(n+L), i, j \in X$ where *i,j* – are the states, $i, j \in X$.

It's clear, that for a periodic Markov chain its matrices of transitions $\Pi(n) = |p_{ij}(n)|$ also change periodically with the same period L:

$$\Pi(n) = \Pi(n+L), n = 0,1,... \qquad (5).$$

We can see from the definition, that a periodic chain is determined by the first L matrices of transitions

$$\{\Pi(0),..., \Pi(k),..., \Pi(L-1)\}. \qquad (6).$$

For periodic Markov chains the methods of their statistical analysis are developed, in particular, method of estimation of matrices of transitions (6), the properties of these estimations are explored, the criteria for determination of accuracy of these estimations are offered. Also we developed the method of the simulation of periodic Markov chains [3]. The method of estimations of matrices of transitions is tested with the use of realization of periodic chains, got by their simulation [8].

Periodic Markov processes, periodic Markov chains enable to use them as the models of real stochastic periodic queuing systems, and methods of statistical analysis of periodic chains enable to design such systems and solve various tasks of their optimization. On the base of analysis of the power consumption we can make a conclusion that the power consumption processes can be described with the offered model, and, thus, periodic Markov processes and chains can be applied for descriptions of the power supply systems.

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