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MODELLING OF DYNAMICAL REGIMES OF TWO-MOTOR ELECTRICAL DRIVES WITH ELASTICITIES AND GAPS OF THE MECHANICAL TRANSDUCERS

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A mathematical model is constructed for the two-motor electrical drive with one speed controller and two thyristor converters, by taking into account elasticities and gaps of mechanical transducers. Using state-space methods, the transfer functions of the object are defined. Recommendations for tunings the regulators are obtained. Applied modelling in the frame of MatLab yields in the near-optimal dynamical characteristics.
Keywords: dynamics, two-motor electrical drive, modelling.

Introduction. Advanced Electric Companies (Siemens, AEG, Harland etc.) with the purpose of performance reliability in the technological processes and to improving quality of manufactured products for various machines suggest several constructions of control systems of two-motor electric drives, e.g.: the systems with one regulator of speed (RS) and one (common) thyristor converter (TC); the systems with individual channels of control for each of motors, i.e. systems with two speed regulators and individual thyristor converters for each motors; the systems with common converter and speed regulable via motor's excitement, etc. Above mentioned control systems of two-motor electrical drives does not protect proportional distribution of loadings between the motors without auxiliary contours of regulations, i.e. without auxiliary sensors and regulators, but their presence in the control circuit complicates tuning of the whole system and reduces its reliability [5, 6].

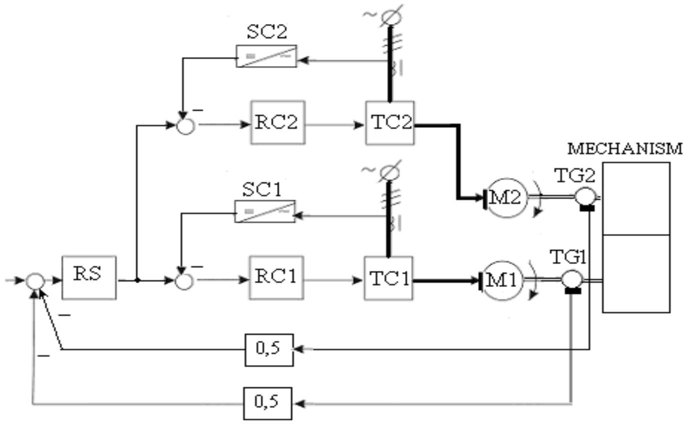

Fig.1. Functional scheme of two-motor electrical drive with one RS and two TC

On the Fig.1 we suggest scheme of two-motor electrical drive, which provides proportional distribution loadings between the motors without any complementary elements and control circuits [1, 2]. Here we use the following notations: M1 and M2 denotes different motors of the electrical drive, respectively; TC1 and TC2 are abbreviations of the thyristor converters; RC1 and RC2 are regulators of currents of anchors of the motors; SC1 and SC2 –sensors of the currents of motors anchors; TG1 and TG2 –tachogenerators at the motors (i.e. sensors of speeds); RS-regulator of speed.

Mathematical model of the electrical drive in the related increments (from their values in static) by taking into account elasticities of shafts and gaps we describe via following equations of movements:

$$\begin{cases} \mu_1 - \mu_{e1} = T_{M1}\dfrac{d\nu_1}{dt}; \\ \mu_2 - \mu_{e2} = \mathrm{T}_{M2}\dfrac{d\nu_2}{dt}; \\ k_{L1}\mu_{e1} + k_{L2}\mu_{e2} - \mu_M = T_{MM}\dfrac{d\nu_M}{dt}; \\ \mu_{e1} = \begin{cases} \dfrac{1}{T_{c1}}\int(\nu_1-\nu_M)dt + \dfrac{T_{d1}}{T_{c1}}(\nu_1-\nu_M), & \text{if } \Delta\Psi_{S1} \in \left]-\infty;\ -(1+\Psi_{G1})\right[\bigcup\left]-1;\ +\infty\right[\\ -1, & \text{if } \Delta\Psi_{S1} \in [-(1+\Psi_{G1});\ -1]; \end{cases} \\ \mu_{e2} = \begin{cases} \dfrac{1}{T_{c2}}\int(\nu_2-\nu_M)dt + \dfrac{T_{d2}}{T_{c2}}(\nu_2-\nu_M), & \text{if } \Delta\Psi_{S2} \in \left]-\infty;\ -(1+\Psi_{G2})\right[\bigcup\left]-1;\ +\infty\right[\\ -1, & \text{if } \Delta\Psi_{S2} \in [-(1+\Psi_{G2});\ -1] \end{cases} \end{cases} \quad (1)$$

Where: $\mu_1, \mu_2, \mu_{e1}, \mu_{e2}, \mu_M, \nu_1, \nu_2$ and ν_M -relative increments of rotational moments of the motors, elastic moments of mechanical transducers (long connecting shafts) to mechanism, also angular speeds of inertional masses (particularly, of the motors and mechanism); $k_{L1} = \dfrac{M_{M1}}{M_{MC}}$ and $k_{L2} = \dfrac{M_{M2}}{M_{MC}}$ are coefficients of loadings for each motor from total static loading of the drive; M_{MC} -total statical moment of resistance of the drive; T_{M1}, T_{M2} and T_{MM} are mechanical constants of time of the inertional masses of the drive; T_{d1} and T_{d2} are time constants, that characterize process of attenuation of elastic fluctuations, acting by adherent dragging in the shafts; T_{c1} and T_{c2} -time constants, characterizing processes of deformation of swirl of the long shafts; $\Delta\Psi_{S1} = \dfrac{\Delta\phi_1}{\phi_{c1}}$ and $\Delta\Psi_{S2} = \dfrac{\Delta\phi_2}{\phi_{c2}}$ are relative over patching corners of swirl of long connecting shafts; $\phi_{c1} = \dfrac{M_{M1c}}{c_1}$ and $\phi_{c2} = \dfrac{M_{M2c}}{c_2}$ - corners of swirl of long shafts during action of the resistance moments M_{M1c} and M_{M2c}; c_1 and c_2 -coefficients of rigidity of long shafts between motors and mechanism; $\Delta\phi_1 = \phi_1 - \phi_M$ and $\Delta\phi_2 = \phi_2 - \phi_M$ - corners of swirl of connecting shafts; ϕ_{G1} and ϕ_{G2} -reduced gaps of mechanical transducers;

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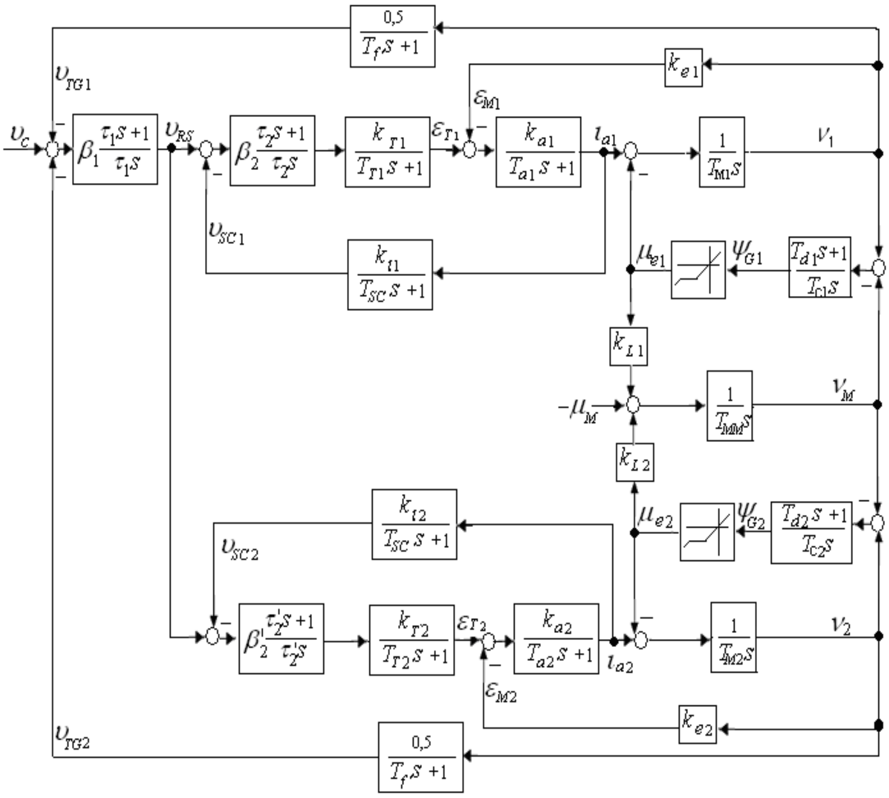

Fig. 2. Structural scheme of elastic two-motor thyristor electric drive involving gaps in the kinematics

On the Fig. 2. are used following notations: β_1 and τ_1 - dynamic gain and time constant of RS; β_2, τ_2, β_2', τ_2' -parameters of the RC_1 and RC_2; k_{T1}, T_{T1}, k_{T2}, T_{T2} - gains and time constants TC_1 and TC_2; $k_{\iota 1}$, $k_{\iota 2}$, T_{sc} -gains and time constants of the SC1 and SC2; k_{a1}, T_{a1}, k_{a2}, T_{a2} -coefficients of transducers and time constants anchoral circuits of the motors M_1 and M_2; T_f - time constant of the filters after TG1 and TG2; Ψ_{S1} и Ψ_{S2} -gaps of mechanical connections; υ_c - relative increment of control signal of the drive system; υ_{TG1} and υ_{TG2} - relative increments output voltages from the TG1 and TG2; υ_{SC1} and υ_{SC2} - relative increments of output voltages from SC1 and SC2; ε_{T1} and ε_{T2} - relative increments of the electromotances from TC1 and TC2; ε_{M1} and ε_{M2} - relative increments of electromotances from the motors; ι_{a1} and ι_{a2} - relative increments of the currents of the anchors (i.e. running torques) of the motors.

To obtain transfer functions from the signal μ_1 to ν_1, also from μ_2 to ν_1, of mechanical part of the drive system ignoring gaps in the reductors we write the system of equations in Cauchy's form:

$$\begin{cases} \dfrac{dx}{dt} = Ax + Bu \\ y = Cx, \end{cases} \tag{2}$$

Where: $x^T = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 \end{bmatrix}$; x_1, x_2, x_3, x_4, x_5 -states of the control system, particularly: x_1 and x_2 are angular speeds of the motors, x_3 is angular speed of the mechanism; x_4 and x_5 are elastic moments of mechanical connecting shafts; u - input signal of object of the drive system (μ_1 and μ_2); y-output signal of the system (i.e. ν_1);

$$A = \begin{bmatrix} 0 & 0 & 0 & -\dfrac{1}{T_{M1}} & 0 \\ 0 & 0 & 0 & 0 & -\dfrac{1}{T_{M2}} \\ 0 & 0 & 0 & \dfrac{k_{L1}}{T_{MM}} & \dfrac{k_{L2}}{T_{MM}} \\ \dfrac{1}{T_{c1}} & 0 & -\dfrac{1}{T_{c1}} & -\dfrac{T_{d1}}{T_{c1}}\left(\dfrac{1}{T_{M1}} + \dfrac{k_{L1}}{T_{MM}}\right) & -\dfrac{T_{d1}}{T_{c1}} \cdot \dfrac{k_{L2}}{T_{MM}} \\ 0 & \dfrac{1}{T_{c2}} & -\dfrac{1}{T_{c2}} & -\dfrac{T_{d2}}{T_{c2}} \cdot \dfrac{k_{L1}}{T_{MM}} & -\dfrac{T_{d2}}{T_{c2}} \cdot \left(\dfrac{1}{T_{M2}} + \dfrac{k_{L2}}{T_{MM}}\right) \end{bmatrix}; \; B = \begin{bmatrix} \dfrac{1}{T_{M1}} \\ \dfrac{1}{T_{M2}} \\ 0 \\ 0 \\ 0 \end{bmatrix}; \; C^T = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{3}$$

It is well known that to obtain transfer function of the system described by equ-ations (2), we need to apply the expression:

$$W(s) = C \cdot (sE - A)^{-1} \cdot B, \tag{4}$$

where: $s = d/dt$; E – identity matrix.

Substituting the expressions (3) in the (4), we define transfer functions of the mechanical part of the drive with respect to angular speed of first motor as follows:

$$W_{01}(s) = \frac{\nu_1(s)}{\mu_1(s)} = \frac{k_{H1}\left[1 + \dfrac{T_1^2 s^2 (T_2^2 s^2 + 2\xi_1 T_2 s + 1)}{k_{L1}(T_{d1}s + 1)(T_{d2}s + 1)}\right]}{T_{M\Sigma} s N(s)}; \tag{5}$$

$$M_{12}(s) = \frac{\nu_1(s)}{\mu_2(s)} = \frac{k_{L2}}{T_{M\Sigma} s N(s)}, \tag{6}$$

where: $T_1 = \sqrt{k_{L1} T_{M2} T_{c2} + (k_{L2} T_{M2} + T_{MM}) T_{c1}}$; $T_2 = \dfrac{\sqrt{T_{M2} T_{MM} T_{c1} T_{c2}}}{T_1}$;

$$\xi_1 = \frac{k_{L1}T_{M2}T_{c2}T_{d1} + (k_{L2}T_{M2} + T_{MM})T_{c1}T_{d2}}{2T_1^2 T_2};$$

$$N(s) = 1 + \frac{T_{M1}T_{M2}T_3 s^2 (T_4 s + 1)}{T_{M\Sigma}(T_{d1}s + 1)(T_{d2}s + 1)} \times \left[1 + \frac{T_{MM}T_5^2(T_6^2 s^2 + 2\xi_2 T_6 s + 1)}{T_{M1}T_{M2}T_3(T_4 s + 1)}\right];$$

$$T_3 = k_{L1}T_{c2} + k_{L2}T_{c1}; \quad T_4 = \frac{k_{L1}T_{c2}T_{d1} + k_{l2}T_{c1}T_{d2}}{T_3};$$

$$T_5 = \sqrt{T_{M1}T_{c1} + T_{M2}T_{c2}}; \quad T_6 = \frac{\sqrt{T_{M1}T_{M2}T_{c1}T_{c2}}}{T_5}; \quad \xi_2 = \frac{T_{M1}T_{c1}T_{d2} + T_{M2}T_{c2}T_{d1}}{2T_5^2 T_6};$$

$T_{M\Sigma} = k_{L1}T_{M1} + k_{L2}T_{M2} + T_{MM}$ -total mechanical time constant of the drive.

Below we use following parameters of the electric drive system: $P_{M1} = 300$ kw.; $P_{M2} = 100$ kw.; $k_{L1} = 0,7$; $k_{L2} = 0,3$; $T_{M1} = 1,5$ sec.; $T_{M2} = 0,7$ sec.; $T_{MM} = 10$ sec.; $T_{d1} = T_{d2} = 0,002$ sec.; $T_{c1} = 0,0004$ sec.; $T_{c2} = 0,00035$ sec..

Transfer function of the controlled object of total system, i.e. the system from output voltage of RS to the signal of angular speed of first (basic) motor, with optimized contours of currents we can write as follows:

$$W_{Obj.}(s) = \frac{\nu_1(s)}{\upsilon_{pc}(s)} = \frac{W_{01}(s) + M_{12}(s)}{k_\iota (T_{\Sigma 2}s + 1)}, \tag{7}$$

where υ_{RS} -relative increment of output voltage of the RS; k_ι - coefficient of transmission of the sensors of currents of anchors of the motors $(k_\iota = 0,1)$; $T_{\Sigma 2}$ -small time constant of optimized current circuits $(T_{\Sigma 2} = 0,01$ sec.).

Apply above mentioned parameters of the electric drive we rewrite transfer function (7) in numerical form as expression:

$$W_{obj.}(s) = \frac{0,48s^4 + 9,84s^3 + 4150s^2 + 7000s + 300000}{s(0,00072s^5 + 0,2s^4 + 8,9s^3 + 786,6s + 15560s + 1110000}. \tag{8}$$

Synthesis of RS of the drive system will be obtained by methods of frequency characteristics, i.e. Bode Plots via MatLab.

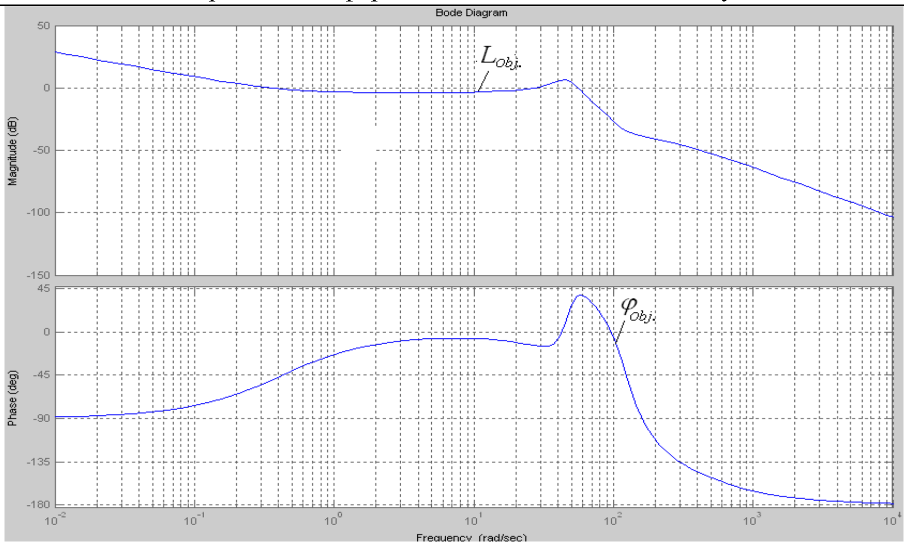

Fig. 3. Logarithmic magnitude ($L_{Obj.}$) and phase ($\phi_{Obj.}$) frequency characteristics of the object with optimized current circuits

On Fig.3 are presented logarithm magnitude ($L_{Obj.}$) and phase ($\phi_{Obj.}$) frequency characteristics of the object with optimized current circuits for drive system described by transfer function (8). At the frequency $\omega_e = \frac{1}{\sqrt{T_{M1}T_{c1}}} = 42, \text{sec.}^{-1}$ magnitude has moderate resonance peak. Consequently, on a first step of approximate tuning of regulators might be realized similarly as recommended for the drives with hard connecting shafts. By tuning of regulators of currents during computer modeling of the electric drive system were applied method of "modulus optimum", but for regulator of speed "symmetric optimum" [3;4].

For realization of characteristics of gaps (III-characteristic on Fig. 4) during modeling by representation of the variables in relative increments were used characteristics of the blocks of a gap (II-characteristic on Fig. 4) and a saturation (I-characteristic on Fig. 4). These blocks were in-parallel included in drive scheme during computer modeling and their signals added.

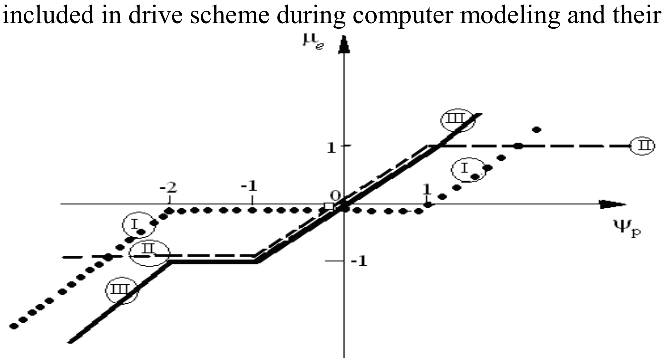

Fig. 4. Nonlinearity characteristics of gaps

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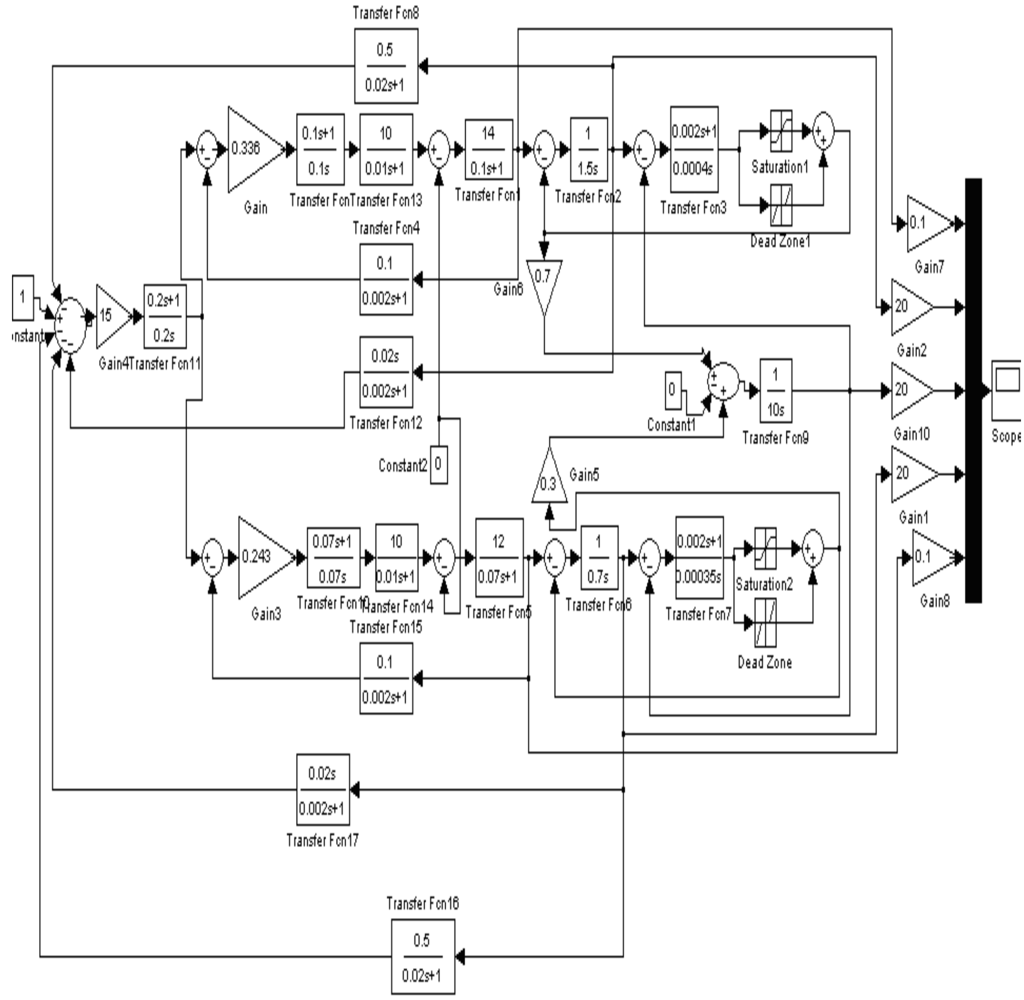

Fig. 5. Computer modeling of two-motor elastic thyristor drive with presence of gaps in the kinematics pairs

Investigations on computer models of considered system of two-motor elastic electrical drive with gaps in the mechanical transmissions (Fig. 5) shows that without augmental flexible feedbacks from speed were inadequate (highly) swingable (Fig. 6,a)

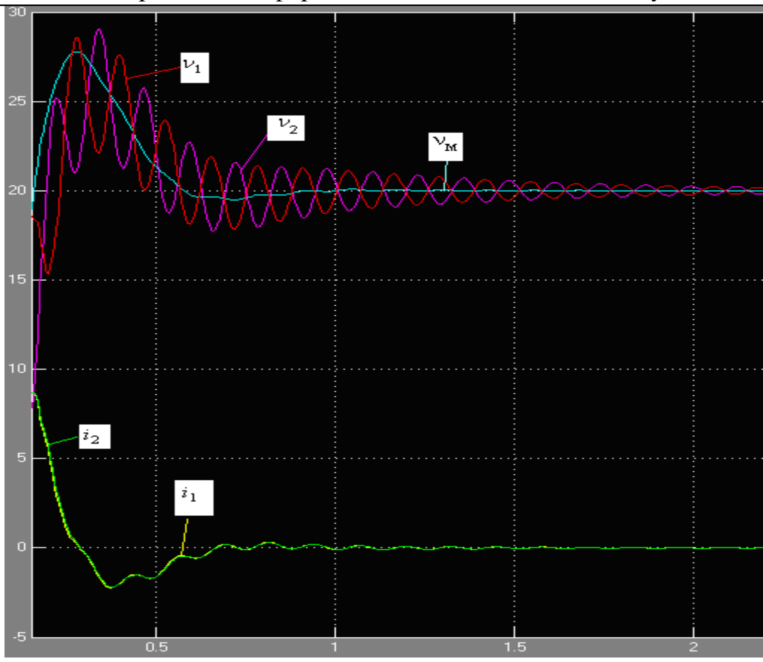

Fig. 6, *a*

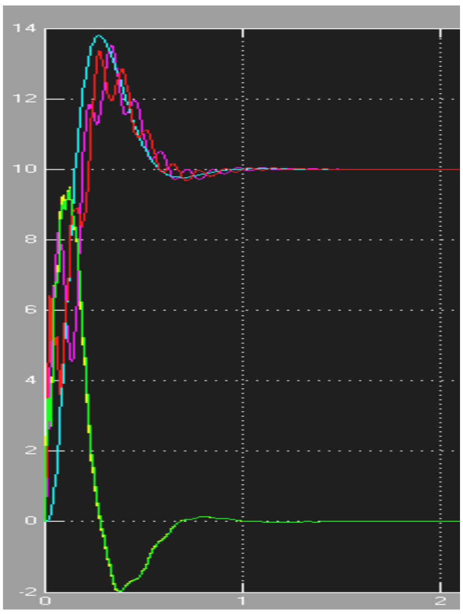

Fig. 6, *b*

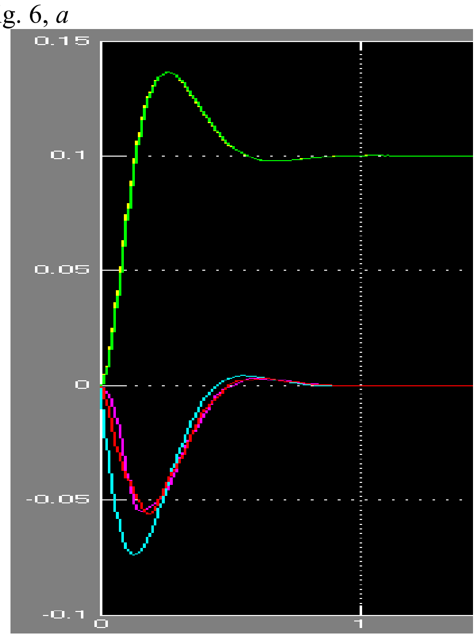

Fig. 6, *c*

Fig. 6. Transient processes by step influence on the system: a from control signal, without corrector; b-from control signal with corrector; c-from loading with corrector

Apply flexible feedbacks for separate motors on input of RS with the following transfer function:

$$W_{flex.\,feedb.}(s) = \frac{0,02s}{0,002s+1}, \qquad (9)$$

Dynamic characteristics of the electric drive substantially improved (Fig. 6, b,c). Pointers of quality (performance is approximately equal to1sec., overshot $\sigma = 35\%$ and dynamic decreasing of a speed $\Delta\nu \le 0,15$) are near to optimal [4;6].

By investigations we establish that for proportional distribution of loadings between the motors for considered control system it is necessary to choose coefficients of transmissions of the sensors of currents of motor's anchors inversely-proportional to powers of the electric motors.

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МОДЕЛИРОВАНИЕ ДИНАМИЧЕСКИХ РЕЖИМОВ ДВУХДВИГАТЕЛЬНОГО ЭЛЕКТРОПРИВОДА С УЧЁТОМ УПРУГОСТЕЙ И ЗАЗОРОВ МЕХАНИЧЕСКИХ ПЕРЕДАЧ

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Составлена математическая модель двухдвигательного электропривода с одним регулятором скорости и двумя тиристорными преобразователями при учете упругостей и зазоров механических передач. Методом пространства состояний переменных введены передаточные функции объекта. Даны рекомендации по

настройке регуляторов системы. В процессе моделирования в MATLAB получены близкие к оптимальным динамические характеристики.
Ключевые слова: динамика, двухдвигательный электропривод, моделирование.

МОДЕЛЮВАННЯ ДИНАМІЧНИХ РЕЖИМІВ ДВОДВИГУНОВОГО ЕЛЕКТРОПРИВОДУ З УРАХУВАННЯМ ПРУЖНОСТЕЙ І ШПАРУВАТОСТЕЙ МЕХАНІЧНИХ ПЕРЕДАЧ

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Створено математичну модель дводвигунового електроприводу з одним регулятором швидкості і двома тиристорними перетворювачами з урахуванням пружностей і шпаруватостей механічних передач. За допомогою методу простору станів змінних введено передаточні функції об’єкта. Дано рекомендації по налаштуванню регуляторів системи. В процесі моделювання в MATLAB отримано динамічні характеристики, близькі до оптимальних.
Ключові слова: динаміка, двохдвигуновий електропривід, моделювання.

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