Theoretical aspects of non-newtonian fluids flow simulation in food technologies

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Abstract

Introduction. The problems of simulating viscoplastic longitudinal and cross-sectional flow of non-Newtonian fluids are overviewed.

Materials and methods. For the first time the superposition method by expressing the components of the stress tensor for building flow fields with higher dimension from flow fields with lower dimension with various boundary conditions when rheological parameters change depending on pressure was used. The flows in the channel are categorized by velocity and pressure values in each point of the section.

Results. The theoretical methods for simulating flows of non-Newtonian fluids in channels of different geometry with moving bounds and pressure drop on channel edges with respect to functional connections between main process parameters are described using the superposition method. It is shown that longitudinal and cross-sectional are reduced to the collection of one-dimensional longitudinal flows of the same type which allow to describe three-dimensional isothermal in rectangular channel and two-dimensional flows in flat channels with different channel aspect ratio. The received theoretical two- and three-dimensional model of viscous flows in channels with basic geometry allow to research main regularities of the process and to establish optimal macro-kinetic and macro-dynamic flow characteristics of non-Newtonian materials which are aimed at reducing energy costs and material consumption of food processing equipment.

Conclusion. The developed and theoretically reasonable three-dimensional models flows of non-Newtonian fluids in channels allow to perform qualitatively new design of food processing equipment which allows to reduce energy costs and material consumption.

Introduction

The problems of viscous fluids flow simulation are overviewed. The method for building flow fields of higher dimension from flow fields of lower dimension with different boundary conditions and with changes in parameters of rheological state based on the pressure is suggested.

Materials and methods

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Theoretical analysis

All the variety of flows inside equipment can be divided into two major classes: flows with Reynolds numbers higher than 1 and flows with Reynolds numbers lower than 1. The flows of the first class are realized in practice for liquids with low viscosity. These flows allow easy sharing of heat and mass thus reallocating target substance [1]. Flows of the second class are realized for liquids with high viscosity. These flows allow developing high shear rates and internal friction powers as well as high pressure values. Pressure and internal friction allow affecting and changing internal structure of such fluids. While for the flows with Reynolds number higher than 1 the more important are the characteristics which provide effective interchange of heat and mass, and less important is the rheological state of the fluid; for the flows with Reynolds number lower than 1 the most important if the equation of substance state in rheological aspect. Therefore studying the fluid flow using only Newtonian model is not sufficient. The main difference between flows with high Reynolds number and flows with low Reynolds number lies in the fact that for the first type of liquids rheological parameters are fixed and constant during the flow while for the second type rheological parameters may change during the flow. This fact is especially true for chemical technology. The driving force for flows with Reynolds number higher than 1 is the pressure difference on both ends of the flow region. The driving force of the flows with Reynolds number lower than 1 is the movement of flow region bounds. Moving of liquids with high viscosity demands more energy. The pressure therein is the result of flow region bounds movement [2]. The typical regions of flows with high Reynolds number is tube segments with predefined pressure difference on their ends. Such flows appear in technological equipment of various constructions and also in conduits [3, 4]. Typical flow regions for high viscous fluids are segments of channels and their branches. These channels form working chambers for various screw machines in which high viscous fluids move and change their properties. When describing viscous fluids flow some principles which apply for both Newtonian and non-Newtonian fluids can be used.

Results and discussion

In this paper the methods for simulating viscoplastic longitudinal flow in flat and rectangular channels are discussed. The channel bounds are movable. This movement can occur both along and across the channel. The channel with rectangular cross-section is considered standard. The flow in the channel is characterized by velocity and pressure values in each point of the flow region. Information about the flow may be condensed (pressure and consumption only) and full, or local (pressure and velocity) in each point. Movement of liquid in the channel can be straight and curved. The latter does not affect the results because inertia does not matter for the flows with Reynolds number lower than 1 [4].

The equations for stokes flows have the following general form:

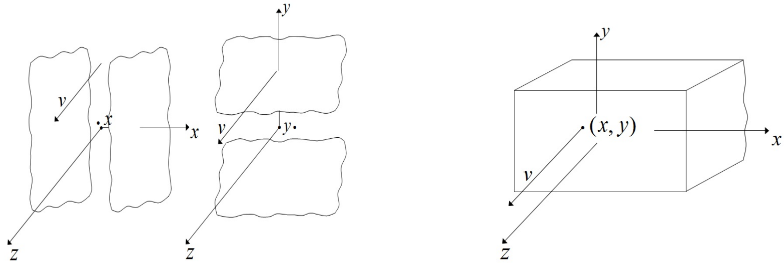
$$
\begin{gathered}
-\widetilde{N}P+\widetilde{N}\widehat{t}=0\,,\vec{\upsilon}=\left(\upsilon_x,\upsilon_y,\upsilon_z\right), \\
\nabla\rho\vec{\upsilon}=0\,,\hat{\tau}=\tau_{ik}\,,i,k=x,y,z\,, \\
\hat{\tau}=\hat{\tau}\left(\hat{\dot{\varepsilon}},P,T\right),\hat{\dot{\varepsilon}}=\dot{\varepsilon}_{ik}\,,\rho=\rho\left(P\right),
\end{gathered}
\qquad (1)
$$

where P – pressure in the fluid, Pa; ρ – density of the fluid, kg/m^3; $\hat{\tau}$ – stress tensor, Pa; $\vec{\upsilon}$ – flow velocity vector, m/s; T – temperature, K; $\hat{\dot{e}}$ – strain rate tensor, 1/s; *x, y, z* – coordinates of point in the flow region, m.

All flows described by equations (1) can be divided into two groups. First group includes flows with velocity vector which has only one component. This component may depend on one or two coordinates but these coordinates should be transverse. If coordinate *Z* is chosen as the longitudinal coordinate (along *OZ* axis), then coordinates x and y will be transverse. Longitudinal flows have velocity component υ_z which can depend either on x or on y separately or on both these coordinates. Longitudinal flows have only one velocity component which depends only on transverse coordinates and can not contain any values which depend on pressure and temperature except for pressure gradient. In these longitudinal flows В таких продольных течениях картина the distribution of velocity is the same in all cross-sections. The second group contains flows with velocity vector which has two or three components each of which depends on two or three coordinates.

The flows of the second group can be ordered like this: two-dimensional longitudinal flows; two-dimensional transverse flows with zero longitudinal velocity, three-dimensional трехмерные twist-and-steer flows which contain all three velocity vector components each of which depends on all three coordinates.

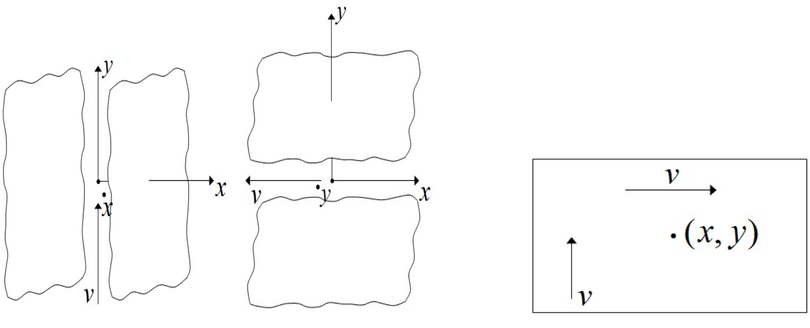
The reasons for various types of flows are the boundary conditions and dependency (or independency) of rheological characteristics on pressure and temperature. Such dependency of reasons and consequences can be illustrated by the example of Newtonian or non-classical non-Newtonian fluid with properties that do not depend on strain rate tensor. If some flow depends only on longitudinal coordinate and has longitudinal component only than this is the flow in flat channel which has only one pair of bounds – and velocities of these bounds are also longitudinal. At the same time only pressure varies along the channel. If rheological parameters depend on pressure than component of stress tensor in equation (1) will also depend on pressure. In this case longitudinal velocity depends on longitudinal coordinate. Due to the equation of matter conservation (1) the second – transverse – velocity component appears, although boundary conditions are purely longitudinal. For Newtonian fluid the longitudinal flows with one velocity component which depends on two transverse coordinates are possible. These flows demand for one additional pair of bounds with longitudinal boundary conditions to be available. Complication of this problem and addition of dependency for rheological characteristics leads the solution of this problem beyond the two-component flow. The flow obtains additional component and yet another coordinate as an argument. Thus adding another pair of bounds adds new coordinate while adding dependency from the pressure adds both component and coordinate. This is illustrated on fig. 1.



a b

Fig. 1. Longitudinal fluid flow:
a – in channels with mutually perpendicular bounds;
b – in the rectangular channel

There are also purely transverse flows for Newtonian fluids. If the channel is flat and the velocities of its bounds are purely transverse then the flow will be purely transverse and will depend on one transverse coordinate only. In the practical aspect these flows are of interest to the small channels with large width. These channels may be approximately considered as flat. The flow consumption in wide closed channel has the value of zero. Hence in order for transverse flow in flat channel to adequately represent the transverse flow in rectangular channel, the purely longitudinal flow with zero consumption should be considered. If rheological characteristics of the flow depend on pressure then the transverse flow in flat channel obtains additional velocity component and additional coordinate as a variable. This is illustrated on fig. 2.



a **b**

Fig. 2. Transverse fluid flow:
a – in flat channels;
b – in the rectangular channel.

The flow in rectangular channel with bounds that move both longitudinally and transversally has three velocity components. If the fluid is Newtonian then all these components depend on two transverse coordinates only. If equation of rheological state includes pressure than the third – longitudinal – coordinate is added, and the flow itself has the highest complexity level.

When connection between bounds count, the type of boundary velocities and rheological characteristics is established, then the method of building velocity field of two- and three-dimensional flows on the velocity field of the flow with lower dimension can be suggested. This method involves representation of transverse flow in rectangular channel as the superposition of two transverse flows in two flat channels which are perpendicular to each other and have zero consumption. This method can be applied to both Newtonian and non-Newtonian fluids. This superposition is represented on fig. 3.

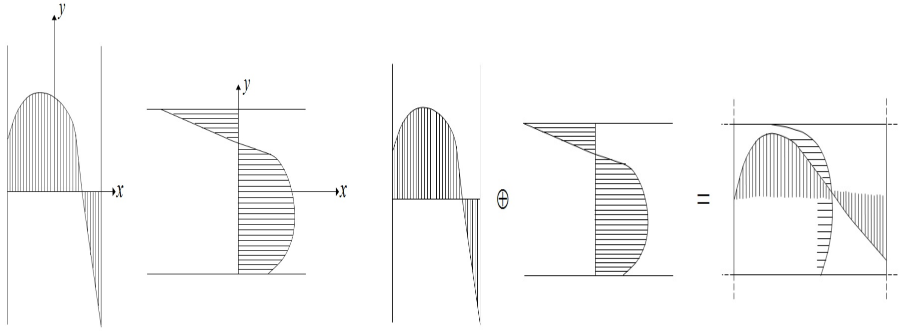


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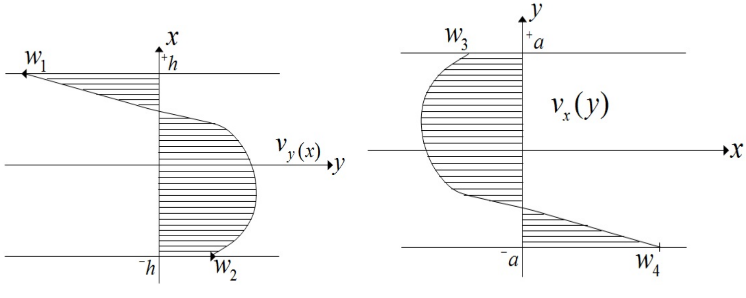
Fig. 3. Fluid flow in flat channels:
(a) – velocity profiles in transverse flows; (b) – superpositions of transverse flows

The superposition lies in the fact that for each flat channel with bound that are perpendicular to each other the longitudinal flow is considered. The equations of this flow contains terms related to another channel. The easiest way to see this is to consider the transverse flow of Newtonian fluid. Suppose there is a transverse flow on the oy axis direction which depends on x coordinate (see fig.4.).



a b

Fig. 4. Velocity profile and boundary velocities:
a – in longitudinal flow which depends on x coordinate;
b – in longitudinal flow which depends on y coordinate

In this case the equations for stress balance have the following form:

$$\frac{\partial P}{\partial y}=\frac{\partial \tau_{yx}}{\partial x}+\frac{\partial \tau_{yy}}{\partial y}, v_{y(+h)}=W_1,$$

$$v_y=v_y\left(x\right), v_{y(-h)}=W_2\ v_y\left(-h\right)=W_2, \qquad (2)$$

where h – if the half of flat channel width, W_1, W_2 – are the channel boundaries velocities.

In order to solve the problem (1) the connection between τ_{yx} and τ_{yy} should be specified. This can be done in several different ways, however the connection between stresses and velocities of deformations in the $\tau_{ik}=\mu\dot{\varepsilon}_{ik}$ form for Newtonian fluid should be known. Thus knowing the boundary conditions the derivatives with respect to x coordinate can always be expressed in terms of derivatives with respect to y coordinate. The derivatives with respect to x coordinate are related to the flow in channel with sides that are perpendicular to the channel from problem (2). This can be done as follows: $\frac{\partial v_y}{\partial x}\sim\left(W_1-W_2\right)/2h$; $\frac{\partial v_x}{\partial y}\sim\left(W_3-W_4\right)/2a$ in case when $W_{1\_}W_2\neq 0$, $W_3-W_4\neq 0$. Otherwise the estimates of the following form should be used: $\frac{\partial v_y}{\partial x}\sim\left(v_m-W_1\right)/\Delta_x^+$; $\left(v_m-W_2\right)\Delta_x^-$; $\frac{\partial v_x}{\partial y}\sim\left(v_m-W_3\right)/\Delta_y^+$; $\left(v_m-W_4\right)\Delta_y^-$, where $\Delta_x^\pm$, $\Delta_y^\pm$ characterize the extremum position of velocity of the respective longitudinal flow: $\Delta_y^+ +\Delta_y^- =2h$; $\Delta_x^+ +\Delta_x^- =2a$. In the first case the estimates lead to $\frac{\partial \tau_{yy}}{\partial y}$ expressed in terms of $\frac{\partial \tau_{yx}}{\partial x}$. In the second case values v_m and $\Delta_x^\pm$, $\Delta_y^\pm$ act as unknown parameters which are determined after solution of the problem. In both cases the problem (2) is reduced to the longitudinal problem with one transverse coordinate. Then the same problem, but for the flat channel which is perpendicular to the first one is considered. This problem can be represented in the following form:

$$\frac{\partial P}{\partial x}=\frac{\partial \tau_{xy}}{\partial y}+\frac{\partial \tau_{xx}}{\partial x}, \qquad v_x\left(+a\right)=W_3,$$

$$v_x=v_x\left(y\right), \qquad v_x\left(-a\right)=W_4. \qquad (3)$$

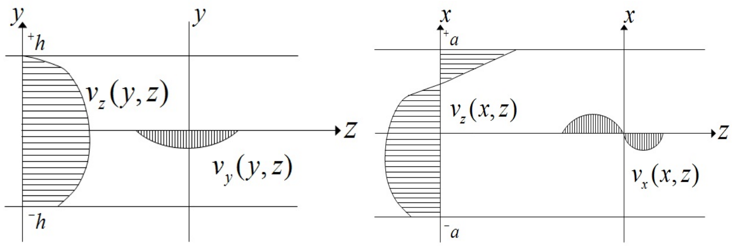
The solution to this problem has the same form as the solution of problem (2). The estimates for velocity derivatives allow expressing $\frac{\partial \tau_{xx}}{\partial x}$ in terms of $\frac{\partial \tau_{xy}}{\partial y}$. The solution (2) and (3) should consider the zero-consumption condition. This condition leads to the equations for $\frac{\partial P}{\partial y}$ and $\frac{\partial P}{\partial x}$ in a way that values of these pressure become dependant on $W_1 - W_2\pi$ and $W_3 - W_4$.

Applying the method described below to non-Newtonian fluid does not lead to any fundamental changes but make the solution for problems (2) and (3) core complicated. Here the following cases are possible: viscosity depends on the second invariant of the strain velocity tensor, or viscosity depends on pressure.

In the first case all terms of the second invariant should be expressed in terms of corresponding derivative of the longitudinal velocity: for problem (2) it is $\frac{\partial \upsilon_y}{\partial x}$; for problem (3) – it is $\frac{\partial \upsilon_x}{\partial y}$. In the second case the longitudinal flows $\upsilon_y(x)$ and $\upsilon_x(y)$ obtain additional component $\upsilon_x(x)$ – for problem (2), and $\upsilon_y(y)$ – for problem (3). Hence in this case the problem for longitudinal flow with one transverse component should be considered. This problem is based on the following equations:

$$\begin{gathered}\frac{\partial P}{\partial z} = \frac{\partial \tau_{zy}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z},\\ \upsilon_z = \upsilon_z(y,z),\upsilon_z(+h,z) = W_5,\\ \upsilon_x = \upsilon_x(y,z),\upsilon_z(-h,z) = W_6,\end{gathered} \tag{4}$$

where W_5, W_6 – values for longitudinal velocities of the bounds (see fig. 5).



a b

Fig. 5. Longitudinal flow with transverse component for the fluid, properties of which depend on pressure:
a – transverse component is directed along *ox* axis;
b – transverse component is directed along *oy* axis

The solution for this problem is based in reducing $\frac{\partial \tau_{zz}}{\partial z}$ to $\frac{\partial \tau_{zy}}{\partial y}$ with the aid of described estimates. The solution for problem (4) describes longitudinal flow along the channel axis with the following boundary conditions:

$$P(z=L)=P_L\,,\quad P(z=0)=P_0\,, \tag{5}$$

where L – channel length, m; P_0 and P_L – pressure values on the channel bounds, Pa.

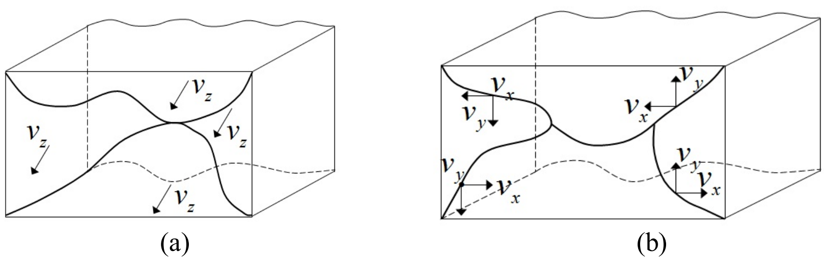
If the zero-consumption condition is taken into account then the solution for the problem describes the transverse flows in channel which are perpendicular to each other and have two flat bound.

The problem for longitudinal flow for another pair of bounds looks similar to the problem (4):

$$\begin{gathered}
-\frac{\partial P}{\partial z}=\frac{\partial \tau_{zx}}{\partial x}+\frac{\partial \tau_{zz}}{\partial z}=0\,,\\
\upsilon_z=\upsilon_z(x,z),\ \upsilon_z(+a,z)=W_7\,,\\
\upsilon_y=\upsilon_x(x,z),\ \upsilon_z(-a,z)=W_8\,,
\end{gathered} \tag{6}$$

where W_7 and W_8 – longitudinal velocities, m (see fig. 5).

Problems (2), (3), (4), (5) lead to velocity fields which consist of two velocity fields in the intersection of flat channels. Thus the question of choosing one or another field arises. This can be done in two ways. The first method: the velocity fields obtained as the solutions for different problems are attributed to the bounds for which they are obtained. Herewith four fields are obtained for longitudinal velocity: two for each pair of bound. The condition for continuity of longitudinal velocity values, which are calculated based on two different expressions, leads to four equations for four lines which divide the rectangle of channel cross-section into sub-areas, each of which is described by its own expression for longitudinal velocity (fig. 6a).



(a) (b)

Fig. 6. Partition of cross-section of rectangular channel into sub-areas:
(a) – for longitudinal flow; (б) – for transverse flow

The same procedure is applied for transverse flow. In this case there are four sub-areas as well. However the condition for their fixation is not the condition for velocity continuity but the condition for continuity of the absolute velocity. Velocity vectors are tested on the lines which separate one area from another by rotating by the angle of $\frac{\pi}{2}$ (see fig. 6b). This method lies in the fact that for the rectangular channel with different sides a and h the main velocity field is the field which has longer bounds. This field is corrected by the multipliers in order to match boundary conditions for the second pair of bounds. Also the experimentally tested statement about the fact that the influence of the pair of bounds extends inside the flow region for the distance approximately equal to the bounds length is used. Thus for large values of this length the influence of shorter pair of bounds weakens. In order to illustrate this statement the channel in which $a >> h$ can be considered. In this case the cross-section has two sub-areas which abut the sides with length h and extend deep for a distance of h, in which expression for velocity flow in flat channel with the width of $2h$ should be corrected in the abovementioned sense. Outside of these two regions the flow is the same as without these two bounds. Applying this method to the transverse flow leads to these two velocity fields, each obtained for its own pair of bounds, are attributed to the entire area of rectangular cross-section, but corrected by multipliers which consider the missing pair of bounds. The method described above is more precise but also more complicated. The complexity of its application is in the fact that bounds influence extends inside the flow region for a distance of bound area and is valid for Newtonian fluids only. This rule is also valid for non-Newtonian fluid but the bound length should be multiplied by some multiplier which depends on parameters of rheological state equation.

Conclusion

The method for reducing problems of flows with higher dimension to the problems of flows with lower dimension described in this paper can be applied to the wide variety of non-linear fluids with various boundary conditions which are based not only on adhesion. This method can be extended to the flows with slipping and to the non-isothermal flows. Herein one should consider the fact that the flow of fluids with high viscosity is accompanied by significant dissipative heat release which is described by distributed source. The presence of slipping apart from mentioned source indicates the necessity for accounting of surface source which is localized on the flow region bound. Sliding contact on the border is similar to the contact of two solid surfaces. Heat release in this contact depends both on pressure normal to the contact and on the magnitude of the slip. In the first case the heat release occurs on the Coulomb type, while in the second case – on the hydrodynamic type. The task for the future is to extend method of solving three-dimensional problems to the problems with surface heat sources.

Having the possibility of non-Newtonian fluid to slide at the region bounds allows dividing all flows into two groups. First group includes flows throughout which the first sliding conditions are complied. The second group includes flows which partially formed by adhesion conditions and partially – by sliding conditions. For the flows of the last group the number of velocity vector components and the number of coordinates are changed in the cross-section of the channel which has longitudinal coordinate that matches the coordinate along which the change of boundary condition form may occur. For such flows the problem of "bonding" several flows of two different types should be solved. Such "bonding" should be subordinated to the conditions of continuity of all velocity and pressure components. Here first derivatives of the velocity in the coordinates will experience a leap. Considering connections between the components of stress tensor and strain velocities the leap of velocity derivatives means the leap of components of stress tensors. Thus the imposition of velocity components continuity conditions is not fully consistent, because it leads to the leap of stress components and the continuity of pressure (fig. 7).

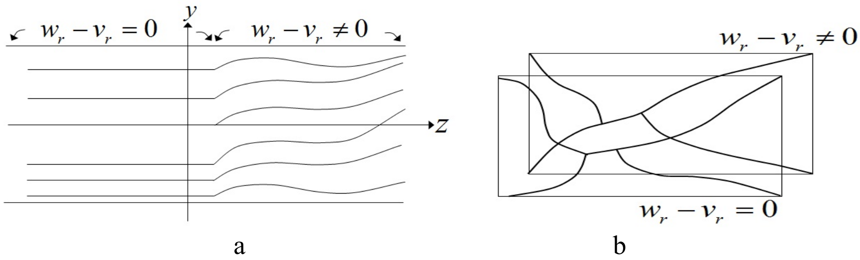


Fig. 7. Flow of the fluid with combined bounds:
a – "bonding" lines for the fluid with sliding and adhesion conditions at the part of the bounds
b – partition of channel; cross-section into sub-areas when one of the sections belongs to the area with sliding and the other section belongs to the area with adhesion

The more consistent is the extension of continuity conditions on the partial derivatives of velocity vector in cross-section, where flows with different dimensions are linked. Everything said above applies to the longitudinal components of the velocity field and to the transverse components providing that partition of channel cross-section rectangle into sub-areas from different sides of the cross-section of transition from one boundary conditions to another – is the same. In fact it is not so, thus the problem of "bonding" and partitioning arises. This problem requires additional study. Therefore the method for building three-dimensional velocity and pressure fields described in this paper has certain potential of development and extension on the flows which appear during the description of large amount of practical situations in food technological processes. The method described in this paper was applied to isothermal flows without sliding for the three-dimensional problems of the flow of Newtonian, power-law, generalized and Bingham fluids in the rectangular channel with arbitrary piecewise constant distribution of bound velocities [6]. Herein in some cases it was possible to consider fluid compressibility and the dependence of the parameters of rheological state equation of the pressure.

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