Non-stationary sucrose diffusion mass flow calculation for sucrose solution cells from the «larger sugar crystal-larger sugar crystal sucrose solution-less sugar crystal sucrose solution-smaller sugar crystal-massecuite» system cells depending on the boiling sugar massecuite time

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Abstract

Introduction. In this paper is proposed one of the following steps to create the sucrose crystallization process mathematical model.

Materials and methods. To obtain the non-stationary diffusion mass sucrose solutions flow quantities for sucrose cells solved simultaneously system with 7 unsteady heat conduction problems in each separate area with constant and with variable thermophysical coefficients, and three separate unsteady diffusion mass transfer problems for four sucrose solution areas with constant and variable diffusion mass transfer coefficients applied numerical methods (method of controlling volume).

Results and discussion. The unsteady mass diffusion sucrose solutions flow distribution found for sucrose areas considered entire system cells for ten cases relative time boiling sugar massecuite τ/τ_c (τ/τ_c =0.15; 0.2; 0.3; 0.4; 0.5; 0.6; 0.7; 0.8; 0.9, 1.0) based on four simultaneous systems solution of the non-stationary parabolic type differential equations in partial derivatives (first system – for unsteady heat conduction problem; and three systems – for non-stationary diffusion mass transfer problems). For the first time based on the calculations found that the process dissolved sucrose flow from the one crystal sucrose solution cell to other crystal sucrose solution cell really is and in which direction it is going. Also for the first time were evaluated quantitative value sucrose diffusion mass flow between sucrose solutions cells areas of different sugar crystals. At the time relative boiling sugar massecuite τ/τ_c=0.15 is the substances (sucrose) transfer from the area 4 (left sucrose solution of crystal 2 cell) in the region 3 (right sucrose solution of crystal 1 cell). Approximately at τ_c=2 s is reached their minimum. Since at time τ_c=2.58 s for calculating options with constant thermophysical coefficients situation is reversed, ie the sucrose transfer is already from the area 3 in the area 4. With all variable thermal characteristics per stay system cells in the heating tube the sucrose transfer is still going on field 4 in area 3 and at the exit system cells from heating pipes approaches to zero, that is virtually absent. So in this case were clearly defined minimum diffusion mass flow. At the time relative boiling sugar massecuite τ/τ_c=1.0 were clearly defined minimum and maximum for both constant and variable for all thermal characteristics.

Conclusions. For each sucrose solution area received unsteady diffusion mass sucrose flow value depending on the contact time system cell with a heating tube. The first time the diffusion mass flow value and direction between the two regions sucrose solutions first and second sugar crystals.

Introduction

The crystalline sucrose in sugar process production is the most energy intensive part.

On the basis of literary analysis revealed that single issue crystallization process of sugar crystal, mass sucrose crystallization and related processes that directly affect these complex processes up to date engaged several authors: Tetiana Vasylenko and Sergii Vasylenko [1], Myronchuk and Dmitrenko [2], Hugot E. [3, 7], Jenkins G.H. [4], Jiahui Chen [5], Baikow V.E. [6], Lauret P. [8], Alewijn W.F. [9], Semlali Aouragh Hassani [10] and Thomas R. Gillett [11].

From the literature review we can conclude that describe the crystallization of sucrose, taking into account all factors that influence this process is extremely difficult. In addition can be said that today there is no single universally accepted approach on this issue.

Therefore, in this paper were implemented one of the next steps in the development and creation as the most complete mathematical model of mass crystallization of sucrose.

It is necessary that mathematical model is developed, more fully described process of simultaneous heat and mass transfer, which takes place between the components of the multiphase system, which is a sugar utfil.

Just note that all the above described process with all the technological, thermal and hydrodynamic characteristics that affect the sucrose mass crystallization process, practically very difficult or even impossible.

On this basis the number of simplifications was adopted. Therefore, developed a mass crystallization mathematical model is attributed to the idealized model.

So, to continue [12, 13, 14] sugar massecuite also represented as a cellular model [15, 16, 17].

Considered that each sugar crystal cell [16] surrounded by a corresponding sucrose solution cell [17] for the whole sugar massecuite boiling time.

Also assume that hydrodynamic interactions between cells occur only between crystalline sucrose solutions.

At the same time, heat exchange and mass transfer processes occurring inside the system cells and between them.

Modeling simultaneously unsteady heat and mass transfer processes for the entire system cells is very complex, so held in several stages.

In the earlier stages of constructing a mathematical model was found unsteady temperature distribution in all cells of the system.

In the first, a simple case [12] considered unsteady temperature distribution for the system cells that consisted of only one crystal sugar. In the second, more complex cases [13] system cells are made up of two sugar crystals.

In the future creation of a mathematical model of crystallization process considered system cells for two sugar crystals.

So the next step had to find the sucrose transferred value between the cells and the sugar crystal amount will crystallize (or dissolve) in each cell of crystal sugar.

The results of simultaneous unsteady temperature distribution calculation in the «larger sugar crystal–larger sugar crystal sucrose solution–less sugar crystal sucrose solution–smaller sugar crystal–massecuite» system cells and the sucrose concentration in the solutions cells of the same system have been received and are described in detail in [14] for the two sugar crystals case.

The problem in finding the value transferred between cells sucrose sucrose solution and sugar crystal amount will crystallize (or dissolve) in each the sugar crystal cell because of sufficient complexity had to be considered in several stages.

The first step is to find the diffusion mass flow value on the boundaries of each sucrose solution cell entire system cells.

That this issue and the subject of this work.

Based on these calculations finally have the opportunity to find sucrose value transferred between sucrose solution cells and the sugar crystal amount will crystallize (or dissolve) in each sugar crystal cell. What should also be implemented at a later stage to create a mathematical model of sucrose mass crystallization.

So, in this paper we find the diffusion mass flow value on the boundaries of each sucrose solution cell entire system cells which consists of two sugar crystals.

It is understood that solution to this problem is completely based on the simultaneous solution of the unsteady diffusion mass transfer problems between sucrose solution cells and unsteady heat transfer problem for the whole of the system cells.

In this work, to continue [12, 13, 14] the results of mathematical modeling at once unsteady heat process and mass transfer unsteady diffusion processes for two sugar crystals, which are surrounded by respective sucrose solution cells and which simultaneously interact with massecuite is presented.

Simulation of unsteady diffusion mass transfer process for a system with two sugar crystals was carried out based on the simultaneous solution of three separate non-stationary diffusion mass transfer problems. As a result, the distribution concentration was determined in each cell sucrose solution discussed above system.

Non-stationary heat exchange and mass transfer processes between system components to cells are considered case of system cell contact with the heating (boiling) tube surface of the heating chamber vacuum machine.

Accepted the initial time $\tau_{c,0} = 0$, when the whole cells system adjudged (included) to the bottom of a vertically oriented heating tube.

Final $\tau_{c,end}$ is the one time when the whole system comes out simultaneously with heating tubes in its upper part.

Research regarding the residence time $\tau/\tau_{c,end}$ system cells at heating tube depending on the relative boiling sugar massecuite time τ/τ_c in detail was considered in [18].

Note also that the crystal's cells, sucrose solution's cells and massecuite's cell thermal characteristics and diffusion mass transfer coefficient of sucrose solutions cells will depend on the relative boiling sugar massecuite time τ/τ_c.

Materials and methods

In this paper, the search unsteady sucrose diffusion mass flow distribution on the sucrose solutions cells boundaries entire system cells completely based on obtained mutual unsteady temperature distribution in all system cells components and simultaneously received on the between the solution cells concentrations distribution of the whole system cells.

Methods of obtaining temperature distribution in the whole system cells simultaneously interference the distribution concentration counting only in the sucrose solution the same system cells was examined in detail in [14].

Similarly in [13], first consider the volumetric case system of cells: «larger sugar crystal–larger sugar crystal sucrose solution–less sugar crystal sucrose solution–smaller sugar crystal–massecuite». By a similar method [13] made the transition from the volume cell model to equivalent one-dimensional model.

Similarly, the work [14], the non-stationary heat and mass transfer problems considered next 7 dimensional regions (Fig. 1), simultaneously pairs in contact with each other:

1 – left area larger crystal sucrose solution;

2 – larger crystal sugar;
3 – rights area larger crystal sucrose solution;
4 – sucrose solution left area smaller crystal;
5 – smaller crystal sugar;
6 – rights area smaller crystal sucrose solution
7 – massecuite.

The problem of unsteady heat transfer process simultaneously for the entire system cells has been considered by a non-stationary heat problem solution. This heat conduction problem consists of system non-stationary heat conduction problems with appropriate initial and boundary conditions [13]. Thus, the system unsteady heat transfer problems considered simultaneously for all 7 one-dimensional regions (Fig. 1), in pairs in contact with each other.

The non-stationary diffusion mass transfer process problem only for sucrose solutions has been considered by three separate unsteady mass transfer problems solution.

For three individual unsteady mass transfer problems considered one-dimensional four areas (Fig. 1): 1, 3, 4, and 6:

1 – the first unsteady diffusion mass transfer problem involved the first area, representing the left larger crystal sucrose solution region (Fig. 1);

2 – the second unsteady diffusion mass transfer problem involved the simultaneous contact areas 3 and 4 of the ideal mass transfer between them (Fig. 1);

3 – the third unsteady diffusion mass transfer problem was about the sixth area, representing the right smaller crystal sucrose solution region (Fig. 1).

Finding the unsteady heat conduction problem solution for the entire system cells and three unsteady diffusion mass transfer problems solutions for sucrose solution areas were interrelated and considered as one large system of equations.

To solve such a complex system of non-stationary differential equations with constant and with variable thermophysical characteristics of analytical methods [19] is difficult and almost impossible.

Therefore, in this case, similar to the cases examined [12, 13, 14] were applied numerical methods using well-known methods of controlling volume [20, 21].

Thus, when writing programs algorithm to obtain the unsteady heat conduction problem solution for the entire system cells and at the same time three unsteady diffusion mass transfer problems solutions for sucrose solution areas at each step conducting of the time and to coordinate calculations it was considered that they are interconnected.

The calculation of non-stationary sucrose solutions diffusion mass flow for the entire system cells also performed using numerical methods on the basis of the non-stationary concentration distribution problems solutions in their respective fields sucrose solution.

Assume the following notation. Diffusion mass flow on the boundary of two areas simultaneously in contact with each other, denoted by the value j_{mn}, ((mn) = {01; 21; 23; 34; 54; 56; 67}). Where the value of m is the number area from which comes (flowing) sucrose mass flow, and the value n is the number field which includes (enters) sucrose mass flow (Fig. 1). Region "zero" formally put in writing the algorithm calculation program for recording left boundary condition for the first sucrose solution area (Fig. 1).

Assume the following signs agreement concerning the value of sucrose mass flow j_{mn}, ((mn) = {21; 23; 34; 54; 56}):

- the substance (sucrose) comes out of the region m in region n, then the sign of the mass flow value j_{mn}, ((mn) = {21; 23; 34; 54; 56}) will be positive $j_{mn}>0$ (Fig. 1)

- if the same substance (sucrose) comes out of the field n in the region m, is a sign of the mass flow value j_{mn}, ((mn) = {21; 23; 34; 54; 56}) will be negative $j_{mn}<0$ (Fig. 1).

In Fig. 1 the footnotes show all considered in this paper cases diffusion mass flow of sucrose at appropriate boundary. Direction arrows on each of these points footnotes received positive direction of the sucrose mass flow at appropriate boundaries neighboring areas.

Setting and solution unsteady heat and mass transfer problems similar to [14] considered for future constant and variable cases selection thermal characteristics and diffusion mass transfer coefficient:

I) all the thermal characteristics in the non-stationary heat transfer problem calculation and the diffusion coefficient in the non-stationary diffusion mass transfer problem calculations is constant in all sucrose solution areas;

I) all the thermal characteristics in the non-stationary heat transfer problem calculation and the diffusion coefficient in the non-stationary diffusion mass transfer problem calculations is constant in all sucrose solution areas:

II, a) at every calculating time step all variable thermal characteristics (density, the thermal conductivity and the heat capacity) in all regions depend only on the current (variable) temperature of the corresponding cell.

In calculating non-stationary mass transfer problem the diffusion mass transfer coefficient in sucrose solution areas depended only on the current (variable) temperature corresponding cell.

A concentration of sucrose content in each area was fixed and taken an equal (steel) content of sucrose concentration in massecuite at a given relative time τ/τ_c.

II, b) as in the previous paragraph in the calculation of non-stationary heat problem at every step of calculating time-variable thermal properties (density, thermal conductivity and heat capacity) in all regions dependent only on the current (variable) temperature corresponding cell.

In calculating non-stationary mass transfer problem the diffusion mass transfer coefficient in sucrose solution areas are dependent on current (variable) temperature and the current (variable) sucrose content in each respective region at this time relative τ/τ_c.

II, c) the non-stationary heat problem calculation at every step of calculating time thermal characteristics (only density and heat capacity) in all regions dependent only on the current (variable) temperature corresponding cell. All other parameters change (factors) are dependent on the density and heat capacity were fixed for each respective region at this time relative τ/τ_c.

At every step of calculating time a variable thermal conductivity in the areas of both sugar crystals and massecuite depend only on the current (variable) temperature. Current solids content in the cell massecuite was fixed (constant) in this relative time τ/τ_c.

At every time calculating step a variable thermal conductivity in areas sucrose solution depends on the current (variable) temperature and current of dry matter content in each respective field of cell sucrose solution.

At every time calculating step a variable thermal conductivity in areas sucrose solution depends on the current (variable) temperature and current of dry matter content in each corresponding sucrose solution area cell.

In calculating non-stationary mass transfer problem at every calculating time step variable mass transfer diffusion coefficient in sucrose solution areas depends on the current (variable) temperature and the current sucrose content in each corresponding area cell sucrose solution.

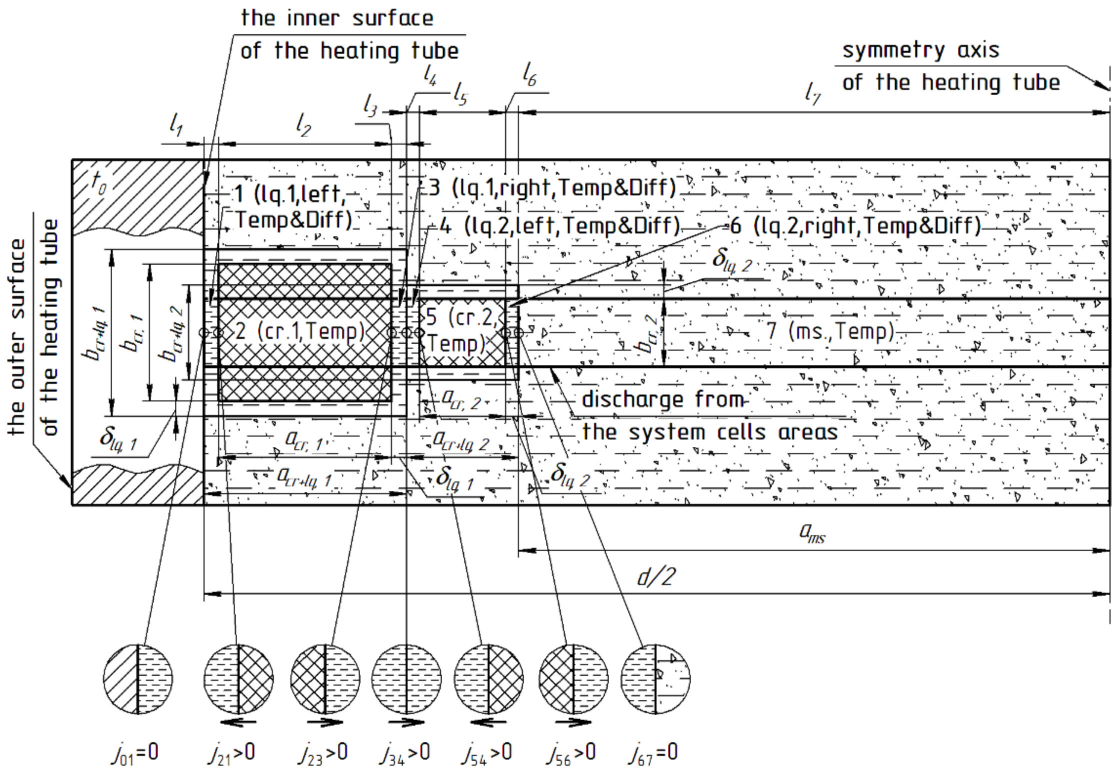

Fig. 1. One-dimensional case system cells, "the left side larger crystal sucrose solution cell–larger sugar crystal cell–right side larger crystal sucrose solution cell–left side smaller crystal sucrose solution cell–smaller sugar crystal cell–right side smaller crystal sucrose solution cell–massecuite" simultaneously take participate in non-stationary heat exchange and mass transfer diffusion processes for the calculation of unsteady sucrose diffusion mass flow.

Designation:

*) For areas: Temp— considered only unsteady heat conduction problems;
Temp&Diff— considered simultaneously unsteady heat problem and nonstationary mass diffusion transfer problem.
*) For sucrose diffusion mass flow j_{mn}, ((mn) = {21; 23; 34; 54; 56}):

Formulation of non-stationary heat conduction problems for the steady case (I) and variable cases (II, a and II, b) thermal characteristics and methods of solving such problems by numerical methods was discussed in detail in [13].

Production of non-stationary heat conduction problems for the variable case (II, c) thermal characteristics and methods of solving such problems by numerical methods was similar to the method considered in [13].

Production of non-stationary diffusion mass transfer problems in the steady case (I) and variable cases (II, a and II, b) thermal characteristics and methods of solving such problems by numerical methods similar to that discussed in detail in [14].

Formulation of unsteady three separate tasks diffusion mass transfer in the variable case (II, c) thermal characteristics and methods of solving such problems by numerical methods was discussed in detail in [14].

Remind once again, that [14] considered three separate simultaneous solution of of unsteady diffusion mass transfer problems with a solution of non-stationary heat problem.

On the basis of these three unsteady diffusion mass transfer problems [14] write the following equation to determine the diffusion mass flow in the sucrose solution boundary.

Note that previously put the diffusion mass flow value j_{mn}, ($(mn) = \{01; 21; 23; 34; 54; 56; 67\}$) calculated per unit area and its dimension in this paper is kg/(m^2·sec).

Consider the issue of boundary conditions formation for each area that reflects the appropriate cell sucrose solution.

First, we note the following. We accept that the concentration value at each border sucrose solution region in contact with the appropriate crystal sugar equals the saturation concentration sucrose solution. The value is calculated saturated concentrations at the current temperature of the crystal surface, which is in contact this sucrose solution area.

This applies to all sucrose solutions areas for all three specific unsteady diffusion mass transfer problems.

Thus, the first unsteady diffusion mass transfer problem, which concerned one area on the left border of the diffusion mass flow is absent. This fact get from a physical point of view, because there is one area in contact with the heating tubes surface, which is not leading mass.

As a result, we get the following equation:

$$j_{01} = -\rho_1(t_1, \mathrm{Pr}_1, \mathrm{DS}_1) \cdot D_1(t_1, \mathrm{Cx}_1) \left.\frac{\partial C_1}{\partial x}\right|_{x=0} = 0 \cdot \tag{1}$$

As in the previous case, it was assumed that the diffusion mass transfer area between sucrose solution region 6 and massecuite region not happens (or is so small that it can be ignored):

$$j_{67} = -\rho_6(t_6, \mathrm{Pr}_6, \mathrm{DS}_6) \cdot D_6(t_6, \mathrm{Cx}_6) \left.\frac{\partial C_6}{\partial x}\right|_{x=\sum_{k=1}^{6} l_k} = 0 \cdot \tag{2}$$

All other sucrose diffusion flows at the diffusion mass transfer processes were calculated based on the following equation:

$$j_{21} = -\rho_1(t_1, \mathrm{Pr}_1, \mathrm{DS}_1) \cdot D_1(t_1, \mathrm{Cx}_1) \left.\frac{\partial C_1}{\partial x}\right|_{x = l_1} \tag{3}$$

$$j_{23} = -\rho_3(t_3, \mathrm{Pr}_3, \mathrm{DS}_3)\cdot D_3(t_3, \mathrm{Cx}_3)\left.\frac{\partial C_3}{\partial x}\right|_{x = l_1 + l_2} \tag{4}$$

$$\begin{aligned} j_{34} &= -\rho_3(t_3, \mathrm{Pr}_3, \mathrm{DS}_3)\cdot D_3(t_3, \mathrm{Cx}_3)\left.\frac{\partial C_3}{\partial x}\right|_{x = l_1 + l_2 + l_3} = \\ &= -\rho_4(t_4, \mathrm{Pr}_4, \mathrm{DS}_4)\cdot D_4(t_4, \mathrm{Cx}_4)\left.\frac{\partial C_4}{\partial x}\right|_{x = l_1 + l_2 + l_3} \end{aligned} \tag{5}$$

$$j_{54} = -\rho_4(t_4, \mathrm{Pr}_4, \mathrm{DS}_4)\cdot D_4(t_4, \mathrm{Cx}_4)\left.\frac{\partial C_4}{\partial x}\right|_{x = l_1 + l_2 + l_3 + l_4} \tag{6}$$

$$j_{56} = -\rho_6(t_6, \mathrm{Pr}_6, \mathrm{DS}_6)\cdot D_6(t_6, \mathrm{Cx}_6)\left.\frac{\partial C_6}{\partial x}\right|_{x = l_1 + l_2 + l_3 + l_4 + l_5} \tag{7}$$

To calculate the diffusion mass flow values j_{mn}, (m= 2, 3, 5, n= 1, 3, 4, 6, $m \neq n$), the equations (3)–(7) also applied numerical simulation. In applying of numerical methods the equations (1)–(7) conducted approximation for the first and second order accuracy.

Note that in this study the results of diffusion flows calculations between sucrose solution areas using a second-order approximation.

As in [12, 13,14], the initial temperature of the system cells (Fig. 1) assumed equally to all areas simultaneously and equal 75°C.

As in [12, 13,14], the initial concentration for each area between the crystal sucrose solution calculated with a coefficient supersaturation $S = 1$. Thus, it shall be taken as in the saturation concentration state under already accepted the initial temperature and equal to 77,594%.

The temperature of the heating tube's inner wall assumed constant over the tube entire height and equal 100 °C.

Also, [14], it is assumed that between areas 3 and 4 sucrose solutions entire system cells is a perfect law of mass transfer.

Thus, in this case, as cases [12, 13, 14] were applied numerical methods using well-known methods of controlling volume [20, 21].

Discretization in time was $\Delta\tau_c = 0{,}01$ s.

The coordinate discretization for each area 1–6 was uniform, and for the area 7 (Fig. 1) massecuite was uneven.

Each region separately (Fig. 1) smashed on the corresponding control volumes number: $n_1 = n_3 = n_4 = n_6 = 10$, $n_2 = n_5 = 20$, $n_7 = 100$.

The cells values are accepted the following sizes: $a_{cr,1} = 5{,}0\cdot10^{-4}$ m, $\delta_{lq,1} = 4{,}29\cdot10^{-5}$ m, $a_{cr,2} = 2{,}5\cdot10^{-4}$ m, $\delta_{lq,2} = 3{,}73\cdot10^{-5}$ m, $a_{ms} = 4{,}83896\cdot10^{-2}$ m.

Based on the calculations [18], the end contact time of the cell system with the heating tubes wall for boiling relative time $\tau/\tau_c = 0{,}15$ is $\tau_{c,end} = 3{,}95$ *sec*, and with $\tau/\tau_c = 1{,}0$ is $\tau_{c,end} = 67{,}93$ *sec*.

Results and discussion

The calculations for the above-mentioned non-stationary heat conduction problems and three non-stationary diffusion mass transfer problems were conducted for all areas of system cells the following values relative sugar massecuite boiling time $\tau/\tau_{ц} = 0{,}15$; 0,2; 0,3; 0,4; 0,5; 0,6; 0,7; 0,8; 0,9; 1,0.

Because of limited volume in this paper are given only two cases relative boiling sugar massecuite time τ/τ_c : at the winding crystals time ($\tau/\tau_c = 0{,}15$) and complete the boiling sugar massecuite time ($\tau/\tau_c = 1{,}0$).

The unsteady sucrose diffusion mass flows distribution calculations results in an appropriate areas border 1, 3, 4, and 6 (Fig. 1) sucrose solution at $\tau/\tau_c = 0.15$ are given:

- diffusion mass flow j_{21} substances (sucrose) from region 2 (first crystal) to the sucrose solution region 1 – in Fig. 2;
- diffusion mass flow j_{23} substances (sucrose) from region 2 (first crystal) to the sucrose solution region 3 – in Fig. 3;
- diffusion mass flow j_{34} substances (sucrose) from the sucrose solution region 3 to the sucrose solution region 4 – in Fig. 4;
- diffusion mass flow j_{54} substances (sucrose) from the region 5 (second crystal) to sucrose solution region 4 – in Fig. 5;
- diffusion mass flow j_{56} substances (sucrose) with region 5 (second crystal) to sucrose solution 6 – in Fig. 6.

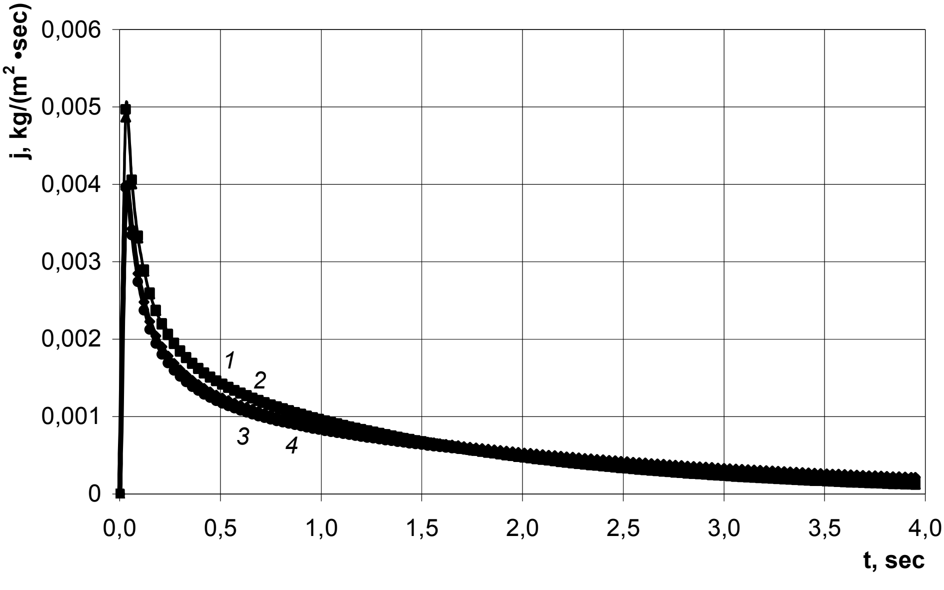

Fig. 2. Sucrose diffusion mass flow j_{21} on the boundary (Fig. 1) region 2 (the first sugar crystal) and region 1 (sucrose solution) depending on the contact time τ system cells from the inner surface of the heating tubes with a relative time massecuite boiling $\tau/\tau_c = 0{,}15$; [value $j_{21} > 0$ if the substance (sucrose) is transferred from the region 2 (the first sugar crystal) in the region 1 (sucrose solution)].

*Designations:

1 – all thermal characteristics and diffusion mass transfer coefficient are a constant (option I);

2 – thermal characteristics and diffusion mass transfer coefficient are variables (variant II, a);

3 – thermal characteristics and diffusion mass transfer coefficient are variables (variant II, b);

4 – thermal characteristics and diffusion mass transfer coefficient are variables (variant II, c);

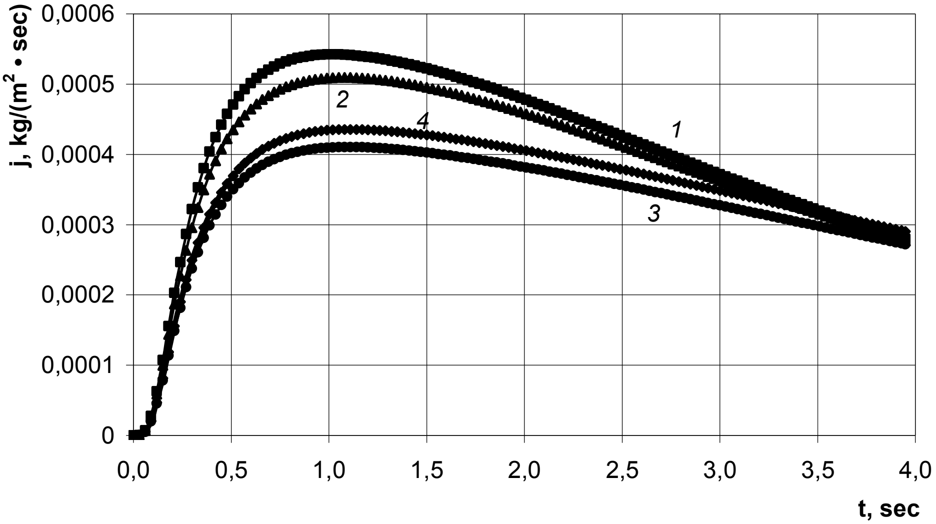

Fig. 3. Sucrose diffusion mass flow j_{23} on the boundary (Fig. 1) region 2 (the first sugar crystal) and region 3 (sucrose solution) depending on the contact time τ system cells from the inner surface of the heating tubes with a relative time massecuite boiling τ/τ_c=0,15; [value j_{23}>0 if the substance (sucrose) is transferred from the region 2 (the first sugar crystal) in the region 3 (sucrose solution)]. *Designations the same as in Fig. 2.

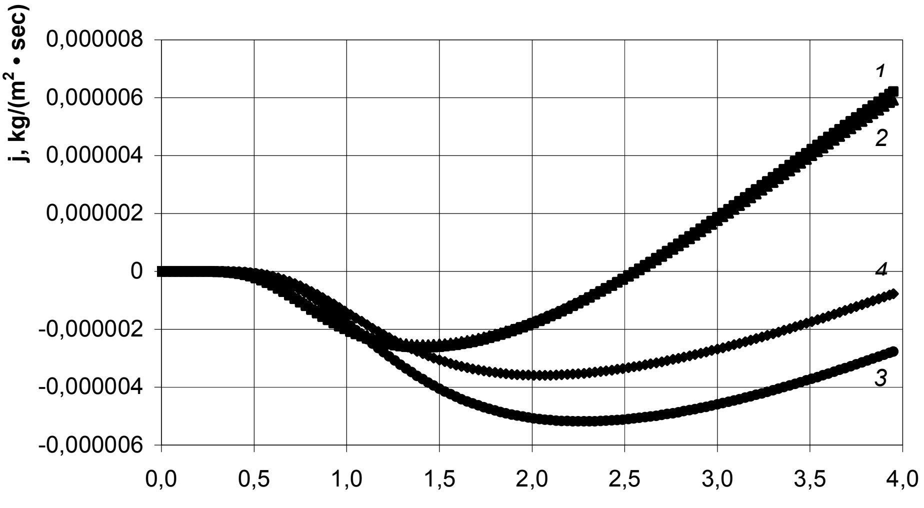

Fig. 4. Sucrose diffusion mass flow j_{23} on the boundary (Fig. 1) region 3 and region 4 (sucrose solutions both) depending on the contact time τ system cells from the inner surface of the heating tubes with a relative time massecuite boiling τ/τ_c=0,15; [value j_{34}>0 if the substance (sucrose) is transferred from the region 3 (sucrose solution) in the region 4 (sucrose solution)]. *Designations the same as in Fig. 2.

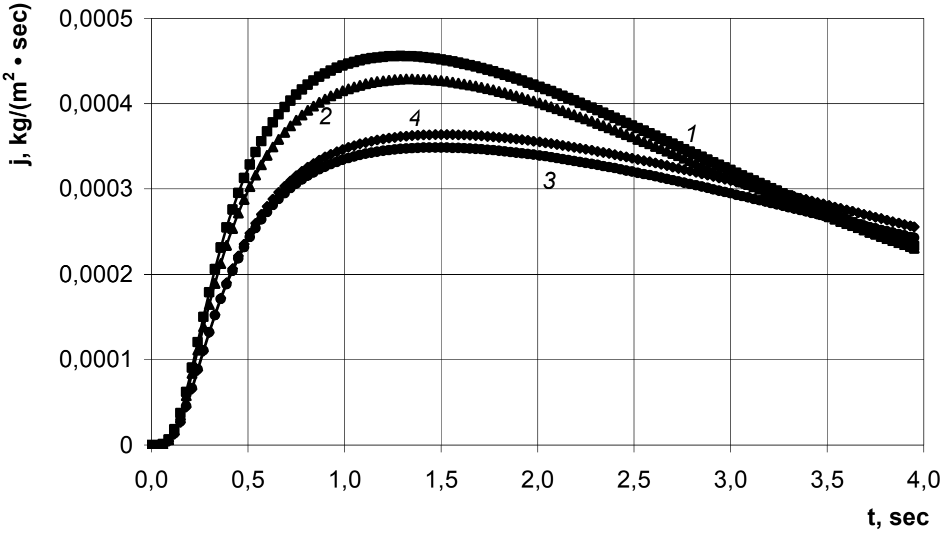

Fig. 5. Sucrose diffusion mass flow j_{54} on the boundary (Fig. 1) region 5 (the second sugar crystal) and region 4 (sucrose solution) depending on the contact time τ system cells from the inner surface of the heating tubes with a relative time massecuite boiling τ/τ_c=0,15; [value j_{54}>0 if the substance (sucrose) is transferred from the region 5 (the second sugar crystal) in the region 4 (sucrose solution)]. *Designations the same as in Fig. 2.

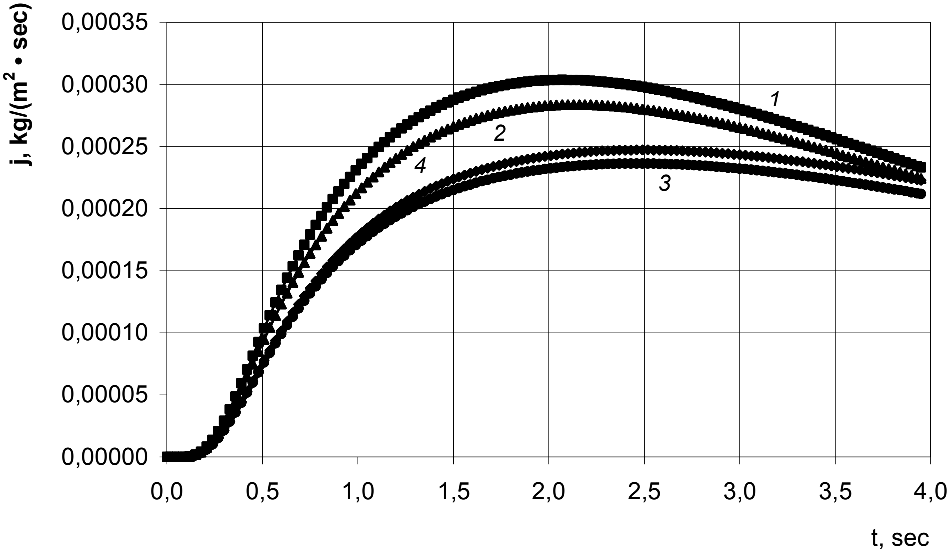

Fig. 6. Sucrose diffusion mass flow j_{56} on the boundary (Fig. 1) region 5 (the second sugar crystal) and region 6 (sucrose solution) depending on the contact time τ system cells from the inner surface of the heating tubes with a relative time massecuite boiling τ/τ_c=0,15; [value j_{56}>0 if the substance (sucrose) is transferred from the region 5 (the second sugar crystal) in the region 6 (sucrose solution)]. *Designations the same as in Fig. 2.

Finally, we present the calculations results of unsteady sucrose diffusion mass flows distribution in sucrose solutions in an appropriate areas border 1, 3, 4, and 6 (Fig. 1) sucrose solution at τ/τ_c= 1.0:

- diffusion mass flow j_{21} substances (sucrose) from region 2 (first crystal) to the sucrose solution region 1 – in Fig. 7;
- diffusion mass flow j_{23} substances (sucrose) from region 2 (first crystal) to the sucrose solution region 3 – in Fig. 8;
- diffusion mass flow j_{34} substances (sucrose) from the sucrose solution region 3 to the sucrose solution region 4 – in Fig. 9;
- diffusion mass flow j_{54} substances (sucrose) from the region 5 (second crystal) to sucrose solution region 4 – in Fig. 10;
- diffusion mass flow j_{56} substances (sucrose) with region 5 (second crystal) to sucrose solution 6 — in Fig. 11.

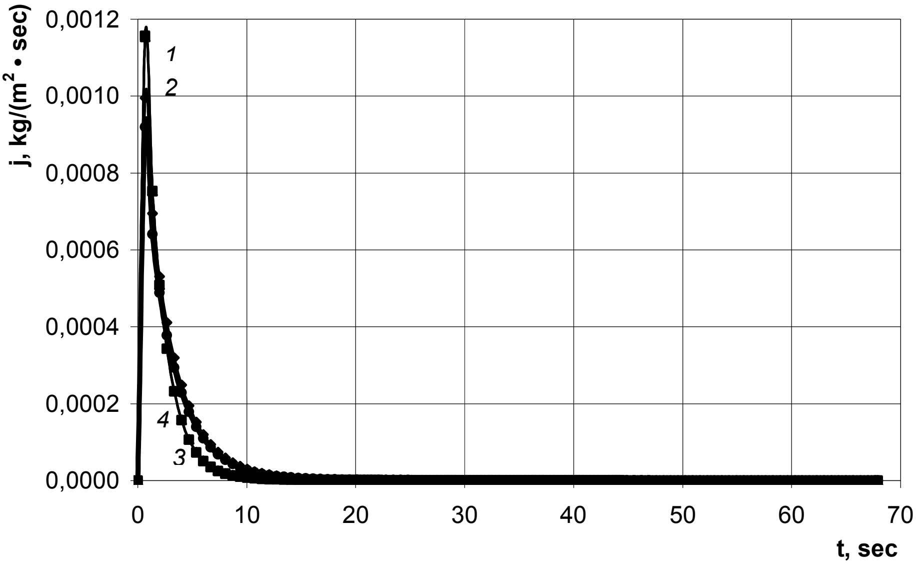

Fig. 7. Sucrose diffusion mass flow j_{21} on the boundary (Fig. 1) region 2 (the first sugar crystal) and region 1 (sucrose solution) depending on the contact time τ system cells from the inner surface of the heating tubes with a relative time massecuite boiling τ/τ_c=1,0; [value j_{21}>0 if the substance (sucrose) is transferred from the region 2 (the first sugar crystal) in the region 1 (sucrose solution)].

*Designations the same as in Fig. 2.

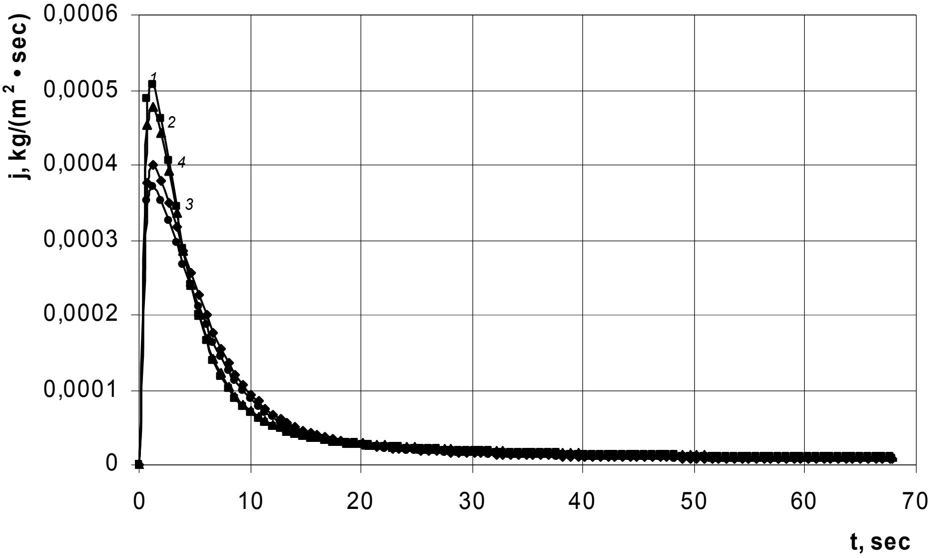

Fig. 8. Sucrose diffusion mass flow j_{23} on the boundary (Fig. 1) region 2 (the first sugar crystal) and region 3 (sucrose solution) depending on the contact time τ system cells from the inner surface of the heating tubes with a relative time massecuite boiling τ/τ_c=1,0; [value j_{23}>0 if the substance (sucrose) is transferred from the region 2 (the first sugar crystal) in the region 3 (sucrose solution)]. *Designations the same as in Fig. 2.

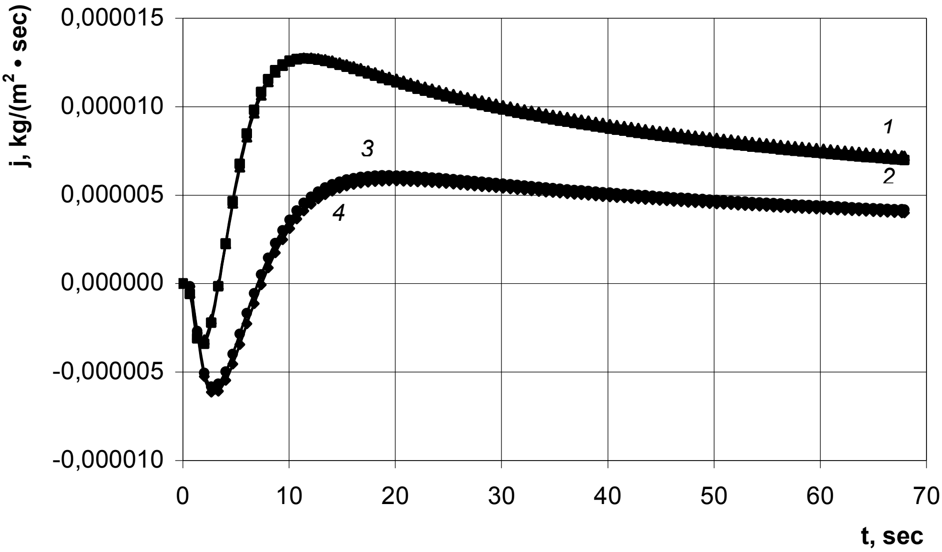

Fig. 9. Sucrose diffusion mass flow j_{23} on the boundary (Fig. 1) region 3 and region 4 (sucrose solutions both) depending on the contact time τ system cells from the inner surface of the heating tubes with a relative time massecuite boiling τ/τ_c=1,0; [value j_{34}>0 if the substance (sucrose) is transferred from the region 3 (sucrose solution) in the region 4 (sucrose solution)]. Designations the same as in Fig. 2.

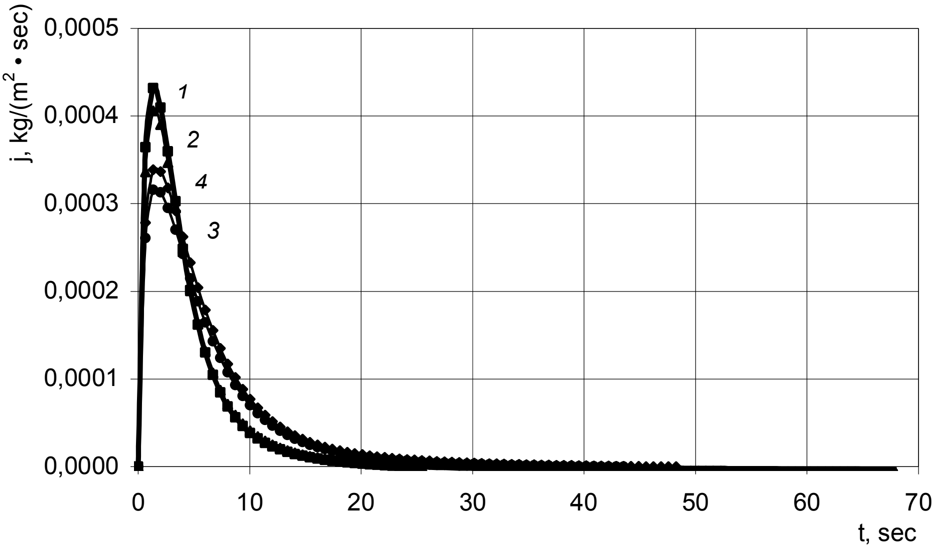

Fig. 10. Sucrose diffusion mass flow j_{54} on the boundary (Fig. 1) region 5 (the second sugar crystal) and region 4 (sucrose solution) depending on the contact time τ system cells from the inner surface of the heating tubes with a relative time massecuite boiling τ/τ_c=1,0; [value j_{54}>0 if the substance (sucrose) is transferred from the region 5 (the second sugar crystal) in the region 4 (sucrose solution)]. *Designations the same as in Fig. 2.

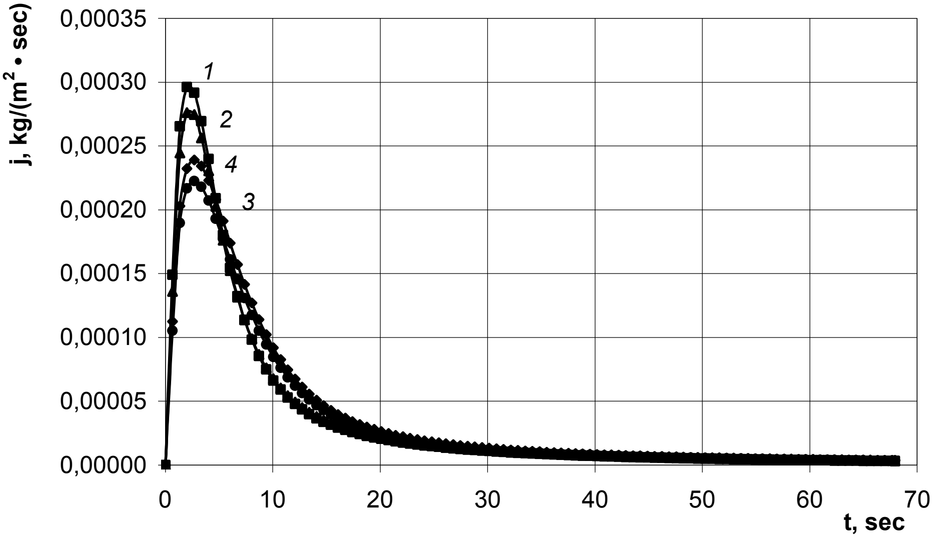

Fig. 11. Sucrose diffusion mass flow j_{56} on the boundary (Fig. 1) region 5 (the second sugar crystal) and region 6 (sucrose solution) depending on the contact time τ system cells from the inner surface of the heating tubes with a relative time massecuite boiling τ/τ_c=1,0; [value j_{56}>0 if the substance (sucrose) is transferred from the region 5 (the second sugar crystal) in the region 6 (sucrose solution)]. *Designations the same as in Fig. 2.

As can be seen from the graphs diffusion mass flows of sucrose:

- Fig. 2 and Fig. 7 the value $j_{21}>0$ for all the calculations cases (ie, constant thermal coefficient case (I) and variable thermal coefficient (II, a), (II, b) and (II, c)). That is the solute (sucrose) transfer from region 2 (the first sugar crystal) in region 1 (sucrose solution). Thus, under these conditions the first crystal dissolves. This result could be expected, because, recall, supersaturation coefficient in this study was taken to 1. Further, as shown in (Fig. 2, Fig. 7), diffusion mass flow for all versions calculations differ little between them. Also shows that in both cases τ/τ_c = 0,15 and τ/τ_c = 1,0 at the outlet of the heating tube diffusion mass flow is almost close to zero, ie, disappears.
- Fig. 3 and Fig. 8 the value $j_{23}>0$ for all the calculations cases ((I), (II, a), (II, b) and (II, c)). That is the solute (sucrose) transfer from region 2 (the first sugar crystal) in region 3 (sucrose solution). Thus, under these conditions the first crystal dissolves also. Further, as shown in (Fig. 3, Fig. 8), diffusion mass flow in variant (I) and (II, a), calculations differ little between them. As well had differing versions of calculations (II, b) and (II, c). In τ/τ_c = 0,15, as shown in Fig. 3, the output of the heating tube diffusion mass flow is 51–67% of the its maximum value. Also shows in the case τ/τ_c = 1,0 at the outlet of the heating tube diffusion mass flow is almost close to zero, ie, disappears.
- Fig. 4 and Fig. 9 diffusion mass flow j_{34} calculations in different variants ((I), (II, a), (II, b) and (II, c)) has the largest difference from each other. Note that the calculations results for options (I) and (II a) almost coincide with each other (Fig. 4, Fig. 9). Calculations for variants (II b) and (II a) and almost identical to each other.

Note that in Fig. 4 clearly shows that the first $j_{34}<0$. That is, transfer agents (sucrose) from the field 4 (the first crystal sucrose solution) in region 3 (the second crystal sucrose solution). Around the time τ_c=2 s is reached their minimum.

For calculation options (II, b) and (II, c) the sucrose diffusion mass flow value is close to zero, and the substance transfer direction remains the same.

Since at time τ_c=2,58 s for calculation options (I) and (II a) carrying direction changes to the opposite sucrose as $j_{34}>0$. So sucrose begins transferred from region 3 (the first crystal sucrose solution) in 4 (the second crystal solution sucrose).

Further, in Fig. 9 clearly shows that at first substance (sucrose) is transferred also from the region 4 (the first crystal sucrose solution) in region 3 (the second crystal sucrose solution). Approximately at time τ_c=2 s is reached their minimum.

Then the situation is reversed. In other words, sucrose begins transferred from the region 3 (the first crystal sucrose solution) in 4 (the second crystal sucrose solution). For calculation options (I) and (II a) value j_{34} reaches its maximum respectively at time τ_c=11,39 sec and τ_c=12,06 sec. For calculation options (I, b), and (II, c) the value of j_{34} reaches its maximum respectively at time τ_c=19,43 sec and τ_c=20,10 sec. Further, in all the calculations cases after reaching its maximum sucrose diffusion mass flow start to decrease to the leaving heating tube time.

The research at τ/τ_c=0,15 showed that the ratio of the minimum diffusion mass flow value $j_{34,\min}$ (between regions 3 and 4 both sucrose solutions) compared with the maximum diffusion mass flow value $j_{23,\max}$ (from region 2 (the first crystal) in region 3 (sucrose solution)) and value $j_{54,\max}$ (from region 5 (the second crystal) in region 4 (sucrose solution)) for different calculation variants is:

within 0,49-0,58% for the calculation option (I);
within 0,50–0,59% for the calculation option (II, a);
within 1,26–1,49% for the calculation option (II, b);
within 0,82–0,98% for the calculation option (II, c).

The research at τ/τ_c=0,15 showed that the ratio of the maximum diffusion mass flow value $j_{34,\max}$ (between regions 3 and 4 both sucrose solutions) compared with the maximum diffusion mass flow value $j_{23,\max}$ (from region 2 (the first crystal) in region 3 (sucrose solution)) and value $j_{54,\max}$ (from region 5 (the second crystal) in region 4 (sucrose solution)) for different calculation variants is:

within 2,52–2,95% for the calculation option (I);
within 2,68–3,14% for the calculation option (II, a);
within 1,64–1,93% for the calculation option (II, b);
within 1,45–1,71% for the calculation option (II, c).

Most importantly, in this paper for the first time found that the process dissolved sucrose flow from one solution cell to another really happening (even supersaturation factor equal to 1). Also for the first time obtained a quantitative diffusion mass flow value j_{34} between sucrose solutions cells (Fig. 4, Fig. 9).

- Fig. 5 and Fig. 10 the value $j_{54}>0$ for all the calculations cases ((I), (II, a), (II, b) and (II, c)). That is the solute (sucrose) transfer from region 5 (the second sugar crystal) in region 4 (sucrose solution). Thus, under these conditions the second crystal dissolves also. This result could be expected, because, recall, supersaturation coefficient in this study was taken to 1. As shown in (Fig. 5, Fig. 10), diffusion mass flow in variant (II, b) and (II, c) calculations differ little between them.
- Fig. 6 and Fig. 11 the value $j_{56}>0$ for all the calculations cases ((I), (II, a), (II, b) and (II, c)). That is the solute (sucrose) transfer from region 5 (the second sugar crystal) in region 6 (sucrose solution). Thus, under these conditions the second crystal dissolves also. As shown in (Fig. 6, Fig. 11), diffusion mass flow in variant (II, b) and (II, c) calculations differ little between them.

Thus, as in [14] can also be concluded that variants calculations (II, b) and (II, c) coincide with each other. Thus, based on the unsteady diffusion mass flow calculations between the system cells components also may be advisable in future to carry out calculations is the variable thermal characteristics case, variant (II, b).

Conclusions

In this work was the non-stationary sucrose diffusion mass flow calculation was conductedfor sucrose solution cells from the «larger sugar crystal–larger sugar crystal sucrose solution–less sugar crystal sucrose solution–smaller sugar crystal–massecuite» system cells depending on the boiling sugar massecuite time.

The results is obtained from the simultaneous non-stationary heat conduction problems and three of unsteady diffusion mass transfer problems system solution by numerical methods.

Calculations were made for relative boiling sugar massecuite time τ/τ_c = 0.15; 0.2; 0.3; 0.4; 0.5; 0.6; 0.7; 0.8; 0.9; 1.0.

This paper presents the results only for two cases relative boiling time: τ/τ_c = 0,15 and τ/τ_c = 1,0.

For each options of τ/τ_c in this paper considered four different computing variants: with constant (variant I) and three variants (II, a, II, b and II, c) with variable thermophysical characteristics and the mass transfer diffusion coefficient.

With further research to create a mathematical model of crystallization process according to the author should choose the formulation and solution of non-stationary heat conduction problems and mass transfer in the case of variable thermal characteristics of the option (II, b). This is the match the real physical process of mass sucrose crystallization of these variants (I), (II, a), (II, b), although it will concede variant calculations (II, c).

Most importantly, in this paper for the first time found that the dissolved sucrose flow process from one cell to another solution really is. Also for the first time were evaluated quantitative unsteady diffusion mass flow magnitude between sucrose solutions cells one and other sugar crystals (Fig. 4, Fig. 9).

The unsteady diffusion mass flow calculations results needed in the future directly for:

- the number between cells sucrose transferred values between sucrose solution for one and second sugar crystal;
- the number of sugar crystal that will crystallize (or dissolve) in each sugar crystal cell of the considered system cells.

The system consists of two sugar crystals, each of which is surrounded by a corresponding sucrose solution amount.

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