On the method for measuring piezooptic coefficients π_{25} and π_{14} in the crystals belonging to point symmetry groups 3 and $\bar{3}$

Skab I., Vasylkiv Yu., Smaga I., Savaryn V. and Vlokh R.

Institute of Physical Optics, 23 Dragomanov St., 79005 Lviv, Ukraine,
e-mail: vlokh@ifo.lviv.ua

Received: 14.12.2010

Abstract

In the present work we describe a high-accuracy torsion method for measuring piezooptic coefficients π_{14} and π_{25} in trigonal crystals. Spatial distributions of the optical indicatrix orientation and the optical birefringence induced by torsion stresses are quantitatively evaluated and analysed. It is shown that a crystal subjected to torsion stresses works as an optical lens, with the characteristics operated by a torque moment.

Keywords: trigonal crystals, piezooptic effect, torsion stresses

PACS: 78.20.Hp, 78.20.Ci
UDC: 535.55

1. Introduction

It is known that experimental determination of piezooptic coefficients yields high enough errors (~ 30%) caused by non-uniform stresses appearing in crystalline samples under their uniaxial loading [1]. This problem becomes more important in case if one measures the coefficients associated with shear stresses. Then each of the piezooptic coefficients under interest is coupled with the other coefficients by very complicated relationships [2]. For example, the stress-induced optical retardation measured with interferometric techniques along a principal axis Y in crystals belonging to the point symmetry 3 is determined as

$$\delta\Delta_2 = -\frac{1}{4} n_1^3 d_2 (\pi_{11} + \pi_{13} \pm \pi_{25}) \sigma_5 , \tag{1}$$

where σ_5 is the uniaxial stress component along the bisector between X and Z axes, n_1 the refractive index of a stress-free sample, and d_2 the sample thickness along the direction of light propagation. It is evident that determination of the coefficient π_{25} based on Eq. (1) is possible only when the coefficients π_{11} and π_{13} are known in advance, i.e. determined in the other independent experiments.

In our recent papers [3–5] we have shown that the piezooptic coefficients mentioned can be determined with high enough accuracy, when applying torsion stresses to a sample. As an example, we have also measured the piezooptic coefficient π_{14} for the lithium niobate crystals with the error not exceeding ~ 3%. In our work [5] we have developed the method for measuring different piezooptic coefficients under torsion stresses, which is valid for the crystals of all point groups of symmetry. However, in some cases this method assumes very complicated experimental geometries, e.g. those involving incident light beams propagating oblique to sample surfaces. Besides, it follows from the work [5] that the particular π_{14} and π_{25} coefficients should be determined in the geometry where the light propagates along the optic axis (i.e., the crystallographic axis *c*) and the torsion moment is applied around the same axis, whereas the orientation of *a* and *b* axes should be known in advance. In the present work we describe a simpler method for determination of the piezooptic coefficients π_{14} and π_{25} in trigonal crystals belonging to the point symmetry groups 3 or $\bar{3}$.

2. Results and discussion

Let us consider a matrix of piezooptic tensor for the symmetry groups 3 and $\bar{3}$:

$$\begin{array}{c|cccccc}
 & \sigma_{11} & \sigma_{22} & \sigma_{33} & \sigma_{32} & \sigma_{31} & \sigma_{21} \\
\hline
\Delta B_{11} & \pi_{11} & \pi_{12} & \pi_{13} & \pi_{14} & \pi_{25} & 2\pi_{62} \\
\Delta B_{22} & \pi_{12} & \pi_{11} & \pi_{13} & -\pi_{14} & \pi_{25} & 2\pi_{62} \\
\Delta B_{33} & \pi_{31} & \pi_{31} & \pi_{33} & 0 & 0 & 0 \\
\Delta B_{32} & \pi_{41} & -\pi_{41} & 0 & \pi_{44} & \pi_{45} & 2\pi_{52} \\
\Delta B_{31} & -\pi_{52} & \pi_{52} & 0 & -\pi_{45} & \pi_{44} & 2\pi_{41} \\
\Delta B_{21} & -\pi_{62} & \pi_{62} & 0 & \pi_{25} & \pi_{14} & \pi_{66} = \pi_{11} - \pi_{12}
\end{array} . \tag{2}$$

When a torsion deformation around *Z* axis is applied to a cylindrical sample, the stress tensor components are defined as [6]

$$\sigma_{\mu} = \frac{2M_z}{\pi R^4}(X\delta_{4\mu} - Y\delta_{5\mu}), \tag{3}$$

where $M_z = \int_S (r \times P)\, dS$, $\delta_{4\mu}$, $\delta_{5\mu}$ are the Kronecker deltas, *R* the cylinder radius, *S* the square of the cylinder basis, and *P* the mechanical loading. Then we deal with the two shear components of the stress tensor, σ_{32} and σ_{31}:

$$\sigma_4 = \frac{2M_z}{\pi R^4} X \tag{4}$$

and

$$\sigma_5 = \frac{2M_z}{\pi R^4} Y, \tag{5}$$

which depend linearly on the coordinates. The latter dependences enable one to determine unambiguously a spatial distribution of the shear stress components inside a sample under study. Moreover, application of torsion moments can provide pure tangential displacements (or shear stress components) alone, which otherwise (i.e., under other geometries of sample loading) are usually accompanied by normal displacements, thus leading to appearance of complementary compression and/or extension stress components.

The equation of optical indicatrix perturbed by the two shear stresses reads as

$$\begin{aligned}&(B_{11}+\pi_{14}\sigma_4+\pi_{25}\sigma_5)X^2+(B_{11}-\pi_{14}\sigma_4-\pi_{25}\sigma_5)Y^2+B_{33}Z^2\\&+2(\pi_{44}\sigma_4+\pi_{45}\sigma_5)YZ+2(\pi_{44}\sigma_5-\pi_{45}\sigma_4)XZ+2(\pi_{14}\sigma_5-\pi_{25}\sigma_4)XY=1\end{aligned} \tag{6}$$

The refractive indices in the cross section *XY* of optical indicatrix may be written as

$$\begin{aligned}n_1&=n_o-\frac{n_o^3}{2}\left(\pi_{25}\sigma_5+\sqrt{\pi_{14}^2\sigma_4^2+(\pi_{25}\sigma_4+\pi_{14}\sigma_5)^2}\right)\\&=n_o-n_o^3\frac{M_z}{\pi R^4}\left(\pi_{25}Y+\sqrt{\pi_{14}^2X^2+(\pi_{25}X+\pi_{14}Y)^2}\right),\end{aligned} \tag{7}$$

$$\begin{aligned}n_2&=n_o-\frac{n_o^3}{2}\left(\pi_{25}\sigma_5-\sqrt{\pi_{14}^2\sigma_4^2+(\pi_{25}\sigma_4+\pi_{14}\sigma_5)^2}\right)\\&=n_o-n_o^3\frac{M_z}{\pi R^4}\left(\pi_{25}Y-\sqrt{\pi_{14}^2X^2+(\pi_{25}X+\pi_{14}Y)^2}\right).\end{aligned} \tag{8}$$

Then the distribution of the birefringence in the *XY* plane is given by

$$\delta(\Delta n)_{12}=n_o^3\sqrt{\pi_{14}^2\sigma_4^2+(\pi_{25}\sigma_4+\pi_{14}\sigma_5)^2}=2n_o^3\frac{M_z}{\pi R^4}\sqrt{\pi_{14}^2X^2+(\pi_{25}X+\pi_{14}Y)^2}\ , \tag{9}$$

while the angle of optical indicatrix rotation around the *Z* axis may be represented as

$$\tan 2\zeta_Z=\frac{\pi_{25}\sigma_4+\pi_{14}\sigma_5}{\pi_{14}\sigma_4}=\frac{\pi_{25}X+\pi_{14}Y}{\pi_{14}X}\ . \tag{10}$$

Taking definitions $X=\rho\cos\varphi,\ Y=\rho\sin\varphi$ into account, we rewrite Eqs. (9) and (10) as

$$\begin{aligned}\delta(\Delta n)_{12}&=2n_o^3\frac{M_z}{\pi R^4}\sqrt{\pi_{14}^2X^2+\pi_{25}^2X^2+\pi_{14}^2Y^2+2\pi_{14}\pi_{25}XY}\\&=2n_o^3\frac{M_z}{\pi R^4}\rho\sqrt{\pi_{14}^2+\pi_{25}^2\cos^2\varphi+\pi_{25}\pi_{14}\sin 2\varphi}\end{aligned} \tag{11}$$

and

$$\tan 2\zeta_Z=\frac{\pi_{25}X+\pi_{14}Y}{\pi_{14}X}=\frac{\pi_{25}\cos\varphi+\pi_{14}\sin\varphi}{\pi_{14}\cos\varphi}\ . \tag{12}$$

It is evident that the birefringence would have a conical spatial distribution under the condition of $\pi_{25}=0$ and so would be dependent only on the distance from the centre of *XY* cross section. In other words, the birefringence induced by the torsion moment M_z becomes

$$\delta(\Delta n)_{12}=2n_o^3\frac{M_z}{\pi R^4}\pi_{14}\sqrt{X^2+Y^2}=2n_o^3\frac{M_z}{\pi R^4}\pi_{14}\rho\ , \tag{13}$$

while the optical indicatrix rotation angle is determined only by the angle φ, being twice as less:

$$\tan 2\zeta_Z=\frac{Y}{X}=\tan\varphi\ . \tag{14}$$

In fact, here we deal with the case of $LiNbO_3$ crystals (the symmetry group 3m) for which $\pi_{25}\equiv 0$ [7]. Therefore spatial distributions of the birefringence and optical indicatrix rotation for those crystals are determined by Eqs. (13) and (14).

Let us analyse Eq. (10) for the optical indicatrix rotation in crystals belonging to the symmetry groups 3 and $\bar{3}$. As follows from Eq. (10), one has

$$\tan 2\zeta_Z = \pm\infty, \quad \zeta_Z = \pm 45° \tag{15}$$

at $X = 0$ and

$$\tan 2\zeta_Z = \frac{\pi_{25}}{\pi_{14}} \text{ or } \tan 2\zeta_Z = K \tag{16}$$

at $Y = 0$. Here K is a parameter determined by the ratio of the piezooptic coefficients ($\pi_{25} = K\pi_{14}$). Thus, the angle of optical indicatrix rotation under the condition $Y = 0$ is determined by the ratio of piezooptic coefficients ($\arctan \pi_{25} / \pi_{14}$). A similar conclusion has earlier been drawn [8] when analysing a torsion method for orientation of crystals belonging to the middle-symmetry systems. A zero optical indicatrix rotation is reached under the condition $\pi_{25}\cos\varphi + \pi_{14}\sin\varphi = 0$, i.e. at the angle $\varphi = \arctan(-K)$. Dependences of the optical indicatrix rotation on the angle of clockwise rotation calculated for different ratios $K = \pi_{25} / \pi_{14}$ are shown in Fig. 1. It is seen from Fig. 1 that a gradual change in the K parameter leads to changes from a linear dependence peculiar for the case of $K = 0$ to a step-like behaviour observed at $K = 100$ (i.e., $\pi_{14} \to 0$).

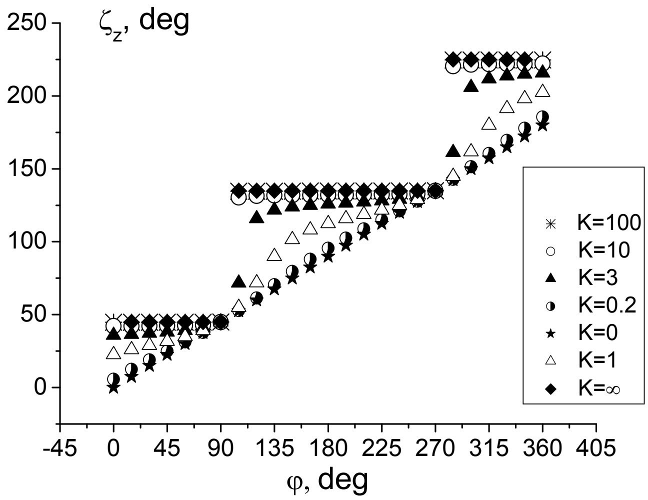
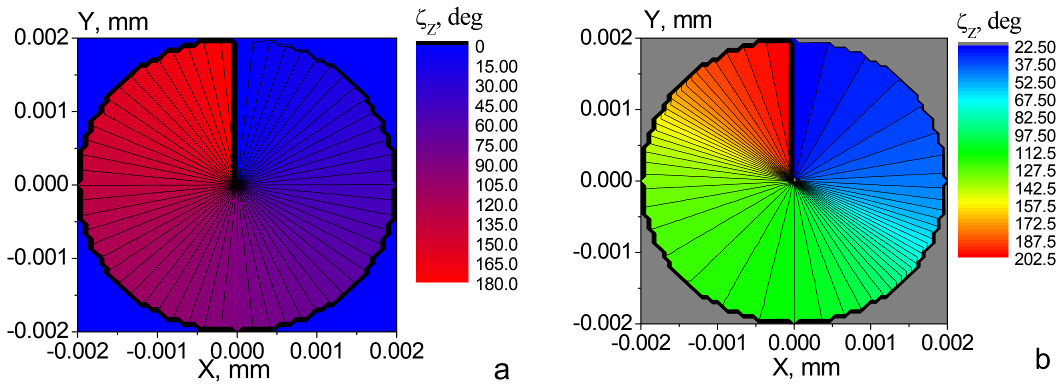


Fig. 1. Dependences of optical indicatrix rotation angle for the crystals of point symmetry groups 3 and $\bar{3}$ on the angle of clockwise rotation calculated for different ratios $K = \pi_{25} / \pi_{14}$.

Spatial distributions of the optical indicatrix rotation in the XY plane calculated for the crystals belonging to the symmetry groups 3 and $\bar{3}$ are presented in Fig. 2.

As seen from Fig. 2, the optical indicatrix rotation at $K = 0$ starts from zero and increases linearly up to $180°$ with increasing angle φ. However, the starting value of the optical indicatrix rotation at $K = 1$ is equal to $22.5°$ and ζ_Z reaches the value $200.5°$ at

$\varphi = 360°$. In this case the dependence of ζ_Z on φ is periodic. An interesting situation happens in the case of $K = 100$. In fact, a sample is divided into two parts, with the orientations of optical indicatrix given by the angles $45°$ and $135°$.



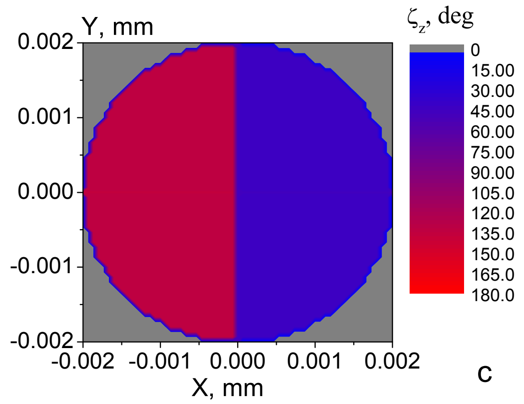


Fig. 2. Spatial distributions of optical indicatrix orientation angle in the *XY* plane calculated for the crystals belonging to the symmetry groups 3 and $\bar{3}$ at different ratios $K = \pi_{25}/\pi_{14}$: (a) $K = 0$, (b) $K = 1$, and (c) $K = 100$.

Now let us analyse a particular case of spatial distribution of the birefringence induced in the *XY* plane by the torsion moment $M_z = 0.049$ N m, while the rest of the parameters are taken to be $n_o = 2.0$, $R = 2$ mm, and $\pi_{14} = 10^{-12}\,\mathrm{m}^2/\mathrm{N}$. As follows from Eq. (11), this distribution is presented by an elliptical cone rotated in the *XY* plane. The angle β of rotation of the cone is given by the relation

$$\beta = \frac{1}{2}\arctan\frac{\pi_{14}}{\pi_{25}} = \frac{1}{2}\arctan\frac{1}{K}. \tag{17}$$

In particular, we have the angle $\beta = 22.5°$ at $K = 1$ (see Fig. 3b) and $\beta = 0$ at $\pi_{14} = 0$ (see Fig. 3c). In general, the angle β decreases with increasing K, as predicted by Eq. (17). The ellipticity of the cross section of the elliptical cone is determined by the ratio of its semi-axes a and b:

$$\frac{a}{b} = \frac{2\pi_{14}^2 + \pi_{25}^2 + \sqrt{\pi_{25}^4 + 4\pi_{14}^2\pi_{25}^2}}{2\pi_{14}^2 + \pi_{25}^2 - \sqrt{\pi_{25}^4 + 4\pi_{14}^2\pi_{25}^2}}, \tag{18}$$

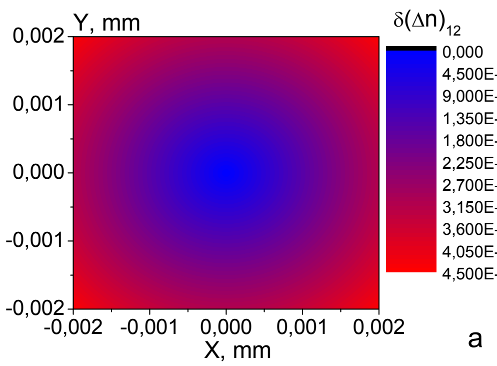
or

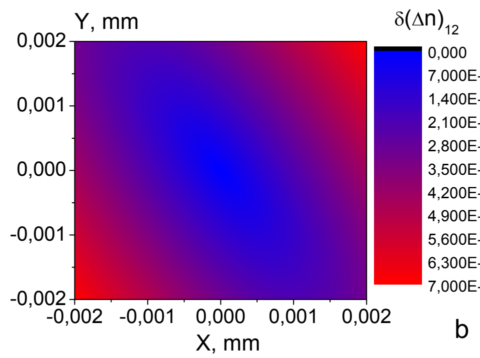
$$\frac{a}{b} = \frac{2 + K^2 + K^2\sqrt{K^2 + 4}}{2 + K^2 - K^2\sqrt{K^2 + 4}}\,. \tag{19}$$

It is seen from the above formulae that the ellipticity is equal to unity if $\pi_{25} = 0$, which corresponds to a circular cross section of the cone (see Fig. 3a), and it tends to infinity if $\pi_{14} = 0$ (see Fig. 3c). This ellipticity is equal to $(3+\sqrt{5})/(3-\sqrt{5})$ at $K = 1$. As seen from Fig. 4, the birefringence distribution along the X axis is linear in the case of $K = 100$, though the birefringence does not change its sign at $X = 0$, being equal to zero instead. In fact, the conical surface of the birefringence distribution in this case degenerates into two planes defined by the relation

$$\delta(\Delta n)_{12} = \pm 2 n_o^3 \frac{M_z}{\pi R^4} \pi_{25} X = \pm 2 n_o^3 \frac{M_z}{\pi R^4} \rho \pi_{25} \cos\varphi\,. \tag{20}$$

On either side of the line $X = 0$, the optical indicatrix is rotated by the same angle $45°$ in the opposite directions (see Fig. 2c). Notice that the induced birefringence in the case of K=100 can reach very high values (~ 3×10^{-3} – see Fig. 3c and Fig. 4), while in the other cases depicted in Fig. 3a and Fig. 3c we have the birefringences as small as 10^{-5} in the order of magnitude.





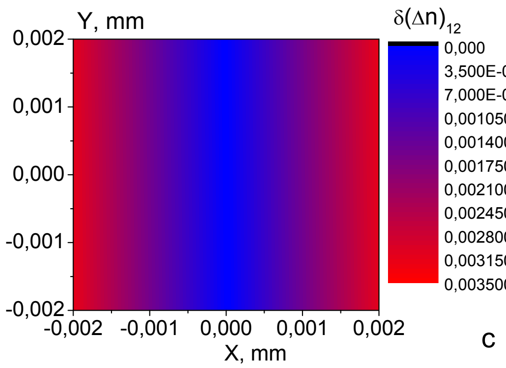


Fig. 3. Distributions of birefringence induced by the torsion moment M_z = 0.049 N m in the *XY* plane of crystals that belong to the point groups of symmetry 3 and $\bar{3}$ for different ratios $K = \pi_{25}/\pi_{14}$: (a) $K = 0$, (b) $K = 1$, and (c) $K = 100$.

Hence, one can easily determine the coefficients π_{14} and π_{25} with high enough accuracy, using experimental mapping of spatial distributions of the birefringence and the optical indicatrix orientation and solving the system of Eqs. (11) and (12). One can estimate the errors for the both coefficients as being close to that declared in the study [4] (i.e., ~ 3%).

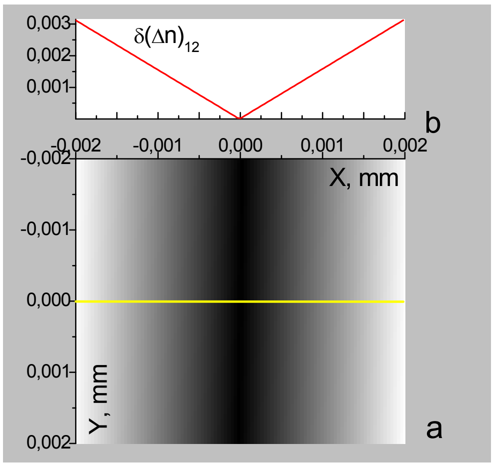


Fig. 4. Distribution of birefringence in the *XY* plane for the case of $K = 100$ (a) and dependence of birefringence on the *X* coordinate (b) along the yellow line at *Y* = 0.

Let us finally emphasise that, due to a radial distribution of the refractive indices (see Fig. 3a), a sample subjected to torsion stresses will act on a bundle of light beams as a flat lens. Depending on azimuths of linear polarisation of the incident light and the sign of mechanical torque, this lens can work as either convex or concave one (see Eqs. (7) and (8) under the condition $\pi_{25} = 0$). Reversal of the torque moment will transform a concave lens into convex one, and vice versa, whereas changing module of the torque moment will enable controlling the focal length. The birefringence distributions shown in Fig. 3c and Fig. 4 correspond to a cylindrical lens with flat faces. This lens will reveal all the properties mentioned above, i.e. its characteristics can be controlled by the torsion stresses.

3. Conclusion

In the present work we have described the high-accuracy method based on torsion loading of crystal samples and aimed at measuring the piezooptic coefficients π_{14} and π_{25} in the crystals that belong to the point symmetry groups 3 and $\bar{3}$. The spatial distributions of the birefringence and the angle of optical indicatrix rotation have been calculated theoretically and analysed. We have demonstrated that the spatial distribution of the birefringence in the sample subjected to torsion around the *Z* axis represents in general an elliptical cone, with a zero birefringence value in the centre of its cross section by the *XY* plane. The parameters of the cone have been evaluated. We have demonstrated that the elliptical cone is transformed into a circular one if $\pi_{25} = 0$, or degenerates into two planes if $\pi_{14} = 0$. We have also shown that a crystal sample subjected to torsion stresses could work as a flat lens, of which parameters can be controlled by changing the torque moment.

References

1. Vasylkiv Yu, Kvasnyuk O, Krupych O, Mys O, Maksymuk O and Vlokh R, 2009. Reconstruction of 3D stress fields basing on piezooptic experiment. Ukr. J. Phys. Opt. **10**: 22–37.
2. Mytsyk B, 2003. Methods for the studies of the piezo-optical effect in crystals and the analysis of experimental data. I. Methodology for the studies of piezo-optical effect. Ukr. J. Phys. Opt. **4**: 1–26.
3. Skab I, Vasylkiv Yu, Savaryn V and Vlokh R, 2010. Relations for optical indicatrix parameters in the conditions of crystal torsion. Ukr. J. Phys. Opt. **11**: 193–240
4. Vasylkiv Yu, Savaryn V, Smaga I, Skab I and Vlokh R, 2010. Determination of piezooptic coefficient π_{14} of $LiNbO_3$ crystals under torsion loading. Ukr. J. Phys. Opt. **11**: 156–164.
5. Skab I, Smaga I, Savaryn V, Vasylkiv Yu and Vlokh R, 2011. Torsion method for measuring piezooptic coefficients. Cryst. Res. Technol. **46**: 26–36.
6. Sirotin Yu I and Shaskolskaya M P, Fundamentals of crystal physics. Moscow: Nauka (1979).
7. Skab I, Vasylkiv Yu, Savaryn Yu and Vlokh R, 2011. Optical anisotropy induced by torsion stresses in $LiNbO_3$ crystals: appearance of an optical vortex. J. Opt. Soc. Amer. A (to be published).
8. Vlokh R, Pyatak Y and Skab I, 1992. On the method of orientation of middle syngonies crystals. Ukr. Fiz. Zhurn. **37**: 207–210.

Skab I., Vasylkiv Yu., Smaga I., Savaryn V. and Vlokh R., 2011. On the method for measuring piezooptic coefficients π_{25} and π_{14} in the crystals belonging to point symmetry groups 3 and $\bar{3}$. Ukr.J.Phys.Opt. **12**: 28 – 35.

Анотація. *В даній роботі описано високоточний торсійний метод вимірювання п'єзооптичних коефіцієнтів π_{14} і π_{25} в тригональних кристалах. Отримано і проаналізовано просторовий розподіл орієнтації оптичних індикатрис та двозаломлення, індуковані торсійними напруженнями. Показано, що кристали під дією торсійних напружень поводяться, як оптичні лінзи, керовані торсійним моментом.*