Measurements of piezooptic coefficients π_{14} and π_{25} in $Pb_5Ge_3O_{11}$ crystals using torsion induced optical vortex

Vasylkiv Yu., Skab I. and Vlokh R.

Institute of Physical Optics, 23 Dragomanov St., 79005 Lviv, Ukraine, vlokh@ifo.lviv.ua

Received: 05.04.2011

Abstract

We show how a singularity of optical field can be used for determination of piezooptic coefficient of crystals under their torsion, in particular in the situation when the natural optical activity makes direct polarimetric experiments too complicated. Experimental studies of an optical vortex appearing under torsion of lead germanate crystals have enabled us to determine so-called non-principal coefficients (π_{14} and π_{25}). A relationship between the spin angular momentum of light beam and the piezooptic coefficients has been derived.

Keywords: torsion stress, optical vortex, spin angular momentum, piezooptic coefficients, lead germanate crystals

PACS: 78.20.Hp, 78.20.Ek, 74.25.Uv, 42.50.Tx
UDC: 535.551

1. Introduction

Recently we have suggested a new high-accuracy torsion method for studies of piezooptic effect in solids [1–5]. As shown in our works, the method generally provides fairly high accuracy for piezooptic coefficients. In particular, the error for so-called non-principal coefficients has been reduced to a few percents [2, 3] (Concerning the piezooptic effect itself, we refer reader to Ref. [6]). As a result, we have successfully determined the piezooptic coefficient π_{14} for $LiNbO_3$ crystals belonging to the point symmetry group 3m and α-BaB_2O_4 crystal (the point group $\bar{3}m$) [2–4]. It has been demonstrated that the sign of the piezooptic coefficients, e. g. the coefficient π_{14}, can also be determined using the torsion method [4].

The authors have found that the method mentioned above enables measuring, easily enough, the piezooptic coefficients for the crystals whose symmetry groups include a three-fold symmetry axis, i.e. the groups 3m, $\bar{3}m$, 3, $\bar{3}$, 32, 432, m3, 23, and $\bar{4}3m$ [3, 5, 7]. Let us notice that, in the case of cubic crystals, one can determine a combination of the coefficients π_{11}, π_{12}, π_{13} and π_{44} under torsion torque applied around {111} direction (see [7]). For the crystals of 3 and $\bar{3}$ symmetry groups, the coefficients π_{14} and π_{25} are measurable when the torsion is applied around the optic axis and the light propagates along the same direction [5]. The experimental geometries needed for crystals of the other symmetries remain more complicated.

It is worthwhile that, due to a lack of inversion symmetry, some of the trigonal or cubic crystals (namely, those belonging to the point groups 3, 32, 432, and 23) permit a natural optical activity. Then the optical eigenwaves propagating along the three-fold axis would become elliptically polarised. Moreover, a circular birefringence associated with the optical activity can even turn out to be higher than its linear counterpart induced by the torsion stresses due to the piezooptic effect. In this case the relations for the quantities determined experimentally, e. g. for the azimuth of polarisation ellipse of the emergent light and the ellipticity of the latter, become nonlinear and very cumbersome [8–10]. As a consequence, one has very serious difficulties in finding unique solutions of appropriate systems of equations for the induced linear birefringence and the orientation of optical indicatrix. This would lead to inevitable difficulties with determination of the piezooptic coefficients.

Nonetheless, it is known [5, 7, 11] that the torsion of crystals belonging to the both trigonal and cubic symmetry groups is accompanied by appearance of an optical vortex, whenever the crystal sample is placed between circular polarisers. Then the intensity distribution in a wide cross section of a nearly plane-wave optical beam transmitted through the output circular polariser is very peculiar: the intensity remains equal to zero in the beam centre and a bright ring is observed around a central dark region.

In the present work we will show that the latter peculiarity could, in principle, be used for determining the piezooptic coefficients of optically active trigonal crystals. This will be done on the example of lead germanate crystals, $Pb_5Ge_3O_{11}$, belonging to the point symmetry group 3 under normal conditions [12–14]. Moreover, as far as we know the piezooptic coefficients π_{14} and π_{25} for these crystals have not yet been reported in the literature.

2. Basic relations, experimental procedure, and results

If the crystals under analysis are twisted, the distribution of the shear stress components σ_4 and σ_5 around Z axis may be written as [15]

$$\sigma_4 = \frac{2M_Z}{\pi R^4} X \ , \qquad (1)$$

$$\sigma_5 = \frac{2M_Z}{\pi R^4} Y \ , \qquad (2)$$

where M_Z denotes the torque moment, R the radius of cylindrical crystalline rod, and the generatrix line is parallel to Z, with X, Y and Z being the principal axes of the Fresnel ellipsoid. As shown in the study [5], the spatial distribution of the birefringence in the XY plane for the crystals belonging to the group 3 is determined by the relation

$$\Delta n_{XZ} = n_o^3 \sqrt{\pi_{14}^2 \sigma_4^2 + (\pi_{25}\sigma_4 + \pi_{14}\sigma_5)^2} = 2n_o^3 \frac{M_Z}{\pi R^4} \sqrt{\pi_{14}^2 X^2 + (\pi_{25} X + \pi_{14} Y)^2} \ , \qquad (3)$$

while the angle of optical indicatrix rotation around the Z axis may be represented as

$$\tan 2\zeta_Z = \frac{\pi_{25}\sigma_4 + \pi_{14}\sigma_5}{\pi_{14}\sigma_4} = \frac{\pi_{25}X + \pi_{14}Y}{\pi_{14}X} . \qquad (4)$$

As one can easily see, Eq. (3) represents equation of an elliptical cone. Then the spatial distribution of the light intensity behind the quarter-wave plate in the system 'right-handed polariser, crystalline sample, left-handed polariser' (see Fig. 1) should be written in the following form:

$$I = I_0 \sin^2 \frac{\Delta\Gamma}{2} \cos^2 2\varepsilon , \qquad (5)$$

where

$$\Delta\Gamma^2 = \Delta\Gamma_L^2 + \Delta\Gamma_C^2 = \left(\frac{2\pi\Delta n_{XZ} d_Z}{\lambda}\right)^2 + (2\rho d_Z)^2 \qquad (6)$$

is the total phase difference, $\Delta\Gamma_L$ and $\Delta\Gamma_C$ the partial phase differences caused respectively by the linear (Δn_L) and circular (Δn_C) birefringences, $d_Z = 14.5$ mm the sample thickness along the direction of light propagation, $\rho = 97$ rad/m the specific rotation of polarisation plane, and ε the ellipticity angle of the eigenwaves defined by the relation

$$\mathrm{tg}\varepsilon = \frac{1}{g_{33} n_0}\left(n_0 \Delta n_{XY} - \sqrt{4 n_0^2 \Delta n_{XY}^2 + g_{33}^2 n_0^2}\right) . \qquad (7)$$

Here λ = 632.8 nm is the light wavelength, $R = 2.55$ mm the radius of cross section of the sample, $M_Z = 0.248$ N×m the torsion moment, $g_3 = 4.02 \times 10^{-5}$ and $n_o = 2.115$ respectively the gyration tensor component and the ordinary refractive index [9].

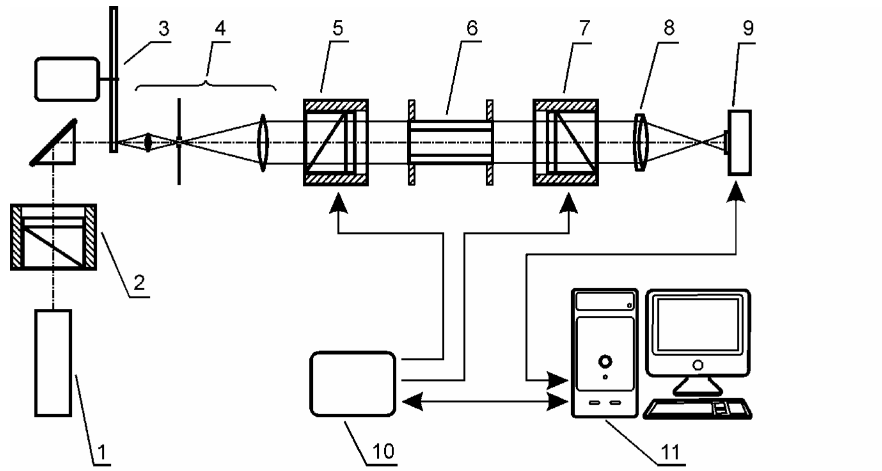

Fig. 1. Experimental setup: 1 – He-Ne laser, 2 – incident light intensity adjuster, 3 – coherence scrambler, 4 – beam expander, 3 – linear polariser, 5 – right-handed circular polariser, 6 – sample, 7 – left-handed circular polariser, 8 – objective lens, 9 – CCD camera, 10 – controller of step motors, 11 – personal computer.

The experimental procedure is as follows. A wide incident (nearly plane-wave) beam with the linear polarisation shaped by a beam expander 4 propagates through an input right-handed circular polariser 5. It becomes circularly polarised and then propagates through a sample 6 and a next left-handed circular polariser 7. The intensity distribution is detected by a CCD camera 9. As shown in our recent works [7, 11], an optical vortex should appear behind an output circular polariser (7), which in general would have an elliptical intensity distribution and a topological charge equal to ±1. Here the optical vortex appears owing to a process similar to that observed in the case of conical refraction in biaxial crystals [16–18], while the elliptical intensity distribution is caused by a mixed screw-edge dislocation of the wave front [7, 19].

As seen from Eq. (3), the elliptical conical distribution of the induced birefringence would appear only when the magnitudes of π_{25} and π_{14} are comparable. Otherwise, when π_{25} is much smaller than π_{14} or equal to zero, Eq. (3) may be rewritten as

$$\Delta n_{XY} = 2n_0^3 \frac{M_Z}{\pi R^4} \pi_{14} \sqrt{X^2 + Y^2} = 2n_0^3 \frac{M_Z}{\pi R^4} \pi_{14} r \,, \qquad (8)$$

where the r coordinate refers to the polar system. Notice that Eq. (8) describes a canonical cone with a circular basis. In fact, we have observed an almost circular intensity distribution in the vortex beam obtained on the twisted $Pb_5Ge_3O_{11}$ crystal (see Fig. 2). This means that the wave front contains almost a pure screw dislocation, while the π_{25} coefficient for the $Pb_5Ge_3O_{11}$ crystals is negligibly small.

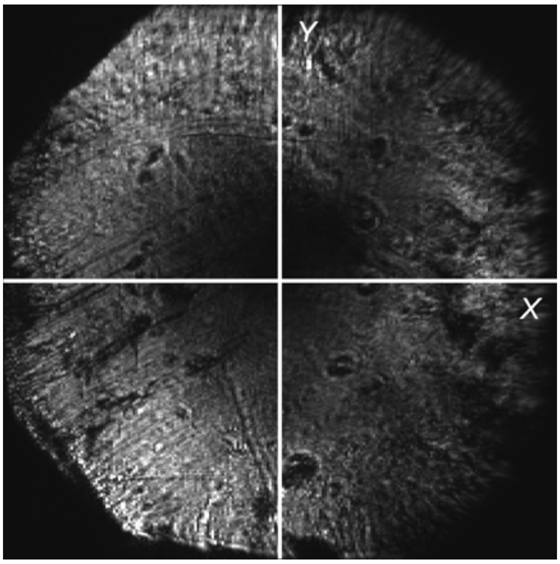

Fig. 2. Intensity distribution in the optical vortex created using a twisted $Pb_5Ge_3O_{11}$ crystal (λ = 6.328 nm, M_Z = 0.248 N×m).

After fitting, along different diameters, the image presented in Fig. 2 with Eq. (5) and accounting for Eq. (8), one can evaluate the coefficient π_{14}. As an example, Fig. 3 shows the intensity distribution obtained experimentally along the X axis and the result of its fitting. Following from the results obtained, we have determined the module of the piezooptic coefficient π_{14} for the lead germanate: $|\pi_{14}| \simeq 0.35\times10^{-12}\,\mathrm{m^2/N}$.

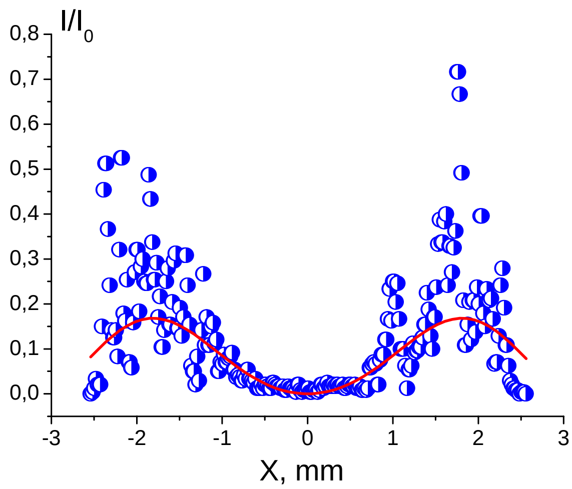

Fig. 3. Intensity distribution along X axis (see text): points denote the experimental data and solid curve the result of fitting with Eq. (5).

The contrast of the bright ring against the background of the central dark ring depends on the torque moment and the magnitude of the piezooptic coefficient. It increases when the mentioned quantities do. At the same time, the central dark spot becomes narrower (see [11]). Such a behaviour suggests that increasing torque moment (i.e., the piezooptic coefficient) induces decrease in the total spin angular momentum of the outgoing beam, which is given by the relation $S_Z^{out} = (I_r^{out} - I_l^{out})/(I_r^{out} + I_l^{out})$ (with I_r and I_l standing for the total intensities of the right-hand and left-hand polarised beam components, respectively) [11]. Hence, both the orbital angular momentum of the emergent beam and the mechanical momentum transferred to the sample could increase with increasing torque moment, owing to the momentum conservation law:

$$L_Z^{inc} + S_Z^{inc} = L_Z^{out} + S_Z^{out} + M_Z^{mech}, \tag{9}$$

where $L_Z^{inc} = 0$ and $S_Z^{inc} = 1$ are respectively the orbital angular and spin angular momentums of the incident beam, L_Z^{out} and S_Z^{out} respectively the orbital angular and spin angular momentums of the outgoing beam, and M_Z^{mech} the mechanical angular momentum transferred to the sample due to the Beth effect [20].

In our particular case, we rewrite Eq. (9) as

$$S_Z^{inc} = M_Z^{mech} + L_Z^{out} + \frac{\left(I_r^{out} - I_l^{out}\right)}{\left(I_r^{out} + I_l^{out}\right)} = 1, \tag{10}$$

so that the spin angular momentum of the outgoing beam is expressed by

$$S_Z^{out} = \frac{\left(I_r^{out} - I_l^{out}\right)}{\left(I_r^{out} + I_l^{out}\right)} = \cos^2\frac{\Delta\Gamma}{2} - \sin^2\frac{\Delta\Gamma}{2}\cos 4\varepsilon . \qquad (11)$$

Using the experimental image (see Fig. 2), one can determine the S_Z^{out} values for different X and Y coordinates (Table 1). Then one can obtain the piezooptic coefficient π_{14} basing on Eq. (11) and Eqs. (6)–(8).

Table 1. Modules of the piezooptic coefficient π_{14} that correspond to different spin angular momentums S_Z^{out} and different coordinates X (Y=0).

Coordinate X, mm	Spin angular momentum S_Z^{out}	Piezooptic coefficient $\lvert\pi_{14}\rvert, 10^{-12}\,\mathrm{m^2/N}$
0.7	0.58	0.45
1.0	0.76	0.21
1.5	0.20	0.40
1.7	0.17	0.35
2.0	0.37	0.25
–0.5	0.80	0.40
–1.0	0.67	0.28
–1.5	0.49	0.25
–1.7	0.36	0.46
–2.0	0.54	0.43

Using the data presented in Table 1, we are now able to calculate the average value of the piezooptic coefficient and the error of its determination: $\lvert\pi_{14}\rvert=(0.35\pm0.09)\times10^{-12}\,\mathrm{m^2/N}$. It is evident that the error for the piezooptic coefficient is equal to 26%. Therefore the π_{25} coefficient should be still smaller than that error ($\lvert\pi_{25}\rvert\leq0.09\times10^{-12}\,\mathrm{m^2/N}$). When compared to the relative piezooptic errors achieved earlier for the both cases of $LiNbO_3$ (3.1% [3]) and α-BaB_2O_4 crystals (9.6% [4]), the error for the π_{14} coefficient of $Pb_5Ge_3O_{11}$ is high enough. As already explained, this should be associated with the fact that the lead germanate crystals possess the natural optical activity which hinders precise direct polarimetric measurements of the optical retardation.

While observing the changes in the conoscopic fringes that appear when the light propagates through the twisted $Pb_5Ge_3O_{11}$ crystal (this method has been in detail described in our recent work [4]), we have determined the sign of the π_{14} coefficient. Since the coefficient is negative, we have the final result $\pi_{14}=-(0.35\pm0.09)\times10^{-12}\,\mathrm{m^2/N}$.

3. Conclusion

In conclusion, we have shown that the singularity of optical wave front can be used in practice when determining so-called non-principal piezooptic coefficients. In the present work we have determined the piezooptic coefficient π_{14} for optically active lead germanate crystals. This has been achieved by means of observation of the optical vortex appearing under crystal torsion. It has been found that piezooptic coefficient π_{14} is equal to $\pi_{14} = -(0.35 \pm 0.09) \times 10^{-12}\,\mathrm{m}^2/\mathrm{N}$, while for the coefficient π_{25} we have $|\pi_{25}| \le 0.09 \times 10^{-12}\,\mathrm{m}^2/\mathrm{N}$.

References

1. Skab I, Vasylkiv Yu, Savaryn V and Vlokh R, 2010. Relations for optical indicatrix parameters in the conditions of crystal torsion. Ukr. J. Phys. Opt. **11**: 193–240.
2. Vasylkiv Yu, Savaryn V, Smaga I, Skab I and Vokh R, 2010. Determination of piezooptic coefficient π_{14} of $LiNbO_3$ crystals under torsion loading. Ukr. J. Phys. Opt. **11**: 156–164.
3. Skab I, Smaga I, Savaryn V, Vasylkiv Yu and Vlokh R, 2011. Torsion method for measuring piezooptic coefficients. Cryst. Res. & Techn. **46**: 23–36.
4. Y Vasylkiv, V Savaryn, I Smaga, I Skab and R Vlokh, 2011. On determination of sign of the piezooptic coefficients using torsion method. Appl. Opt. **50** (to be published).
5. Skab I, Vasylkiv Yu, Smaga I, Savaryn V and Vlokh R, 2011. On the method for measuring piezooptic coefficients π_{25} and π_{14} in the crystals belonging to point symmetry groups 3 and $\bar{3}$. Ukr. J. Phys. Opt. **12**: 28–35.
6. Narasimhamurty T S, Photoelastic and electrooptic properties of crystals. New York: Plenum Press (1981).
7. Skab I, Vasylkiv Y, Zapeka B, Savaryn V and Vlokh R, 2011. On the appearance of singularities of optical field under torsion of crystals containing three-fold symmetry axes. J. Opt. Soc. Amer. A (to be published).
8. Konstantinova A F, Grechushnikov B N, Bokut' D V and Valyashko E G, Optical properties of crystals. Minsk: Nauka i Tekhnika (1995).
9. Vlokh O G, Spatial dispersion phenomena in parametric crystal optics. Lviv: Vyshcha Shkola (1984).
10. Vlokh O G and Kobylyanskii B V, 1974. Influence of parameters of gyrotropic crystals on the character of light polarization. Ukr. Fiz. Zhurn. **19**: 1129–1135.
11. Skab I, Vasylkiv Y, Savaryn V and Vlokh R, 2011. Optical anisotropy induced by torsion stresses in $LiNbO_3$ crystals: appearance of an optical vortex. J. Opt. Soc. Amer. A **28**: 633–640.
12. Kay M I, Newnham R E and Wolfe R W, 1975. The crystal structure of the ferroelectric phase of $Pb_5Ge_3O_{11}$. Ferroelectrics. **9**: 1–6.
13. Iwata Y, 1977. Neutron diffraction study of the structure of paraelectric phase of $Pb_5Ge_3O_{11}$. J. Phys. Soc. Japan. **43**: 961–967.
14. Ermakov A S and Duda V M, 2010. Nonstoichiometry and conductivity anisotropy of lead germanate single crystals. Ukr. J. Phys. **55**: 218–221.

15. Sirotin Yu I and Shaskolskaya M P, Fundamentals of crystal physics. Moscow: Nauka (1979).
16. Berry M V, Jeffrey M R and Mansuripur M, 2005. Orbital and spin angular momentum in conical diffraction. J. Opt. A: Pure Appl. Opt. **7**: 685–690.
17. Vlokh R, Volyar A, Mys O and Krupych O, 2003. Appearance of optical vortex at conical refraction. Examples of $NaNO_2$ and $YFeO_3$ crystals. Ukr.J.Phys.Opt.**4**: 90–93.
18. Berry M V, 2004. Conical diffraction asymptotics: fine structure of Poggendorff rings and axial spike. J. Opt. A: Pure Appl. Opt. **6**: 289–300.
19. Dennis M R, 2004. Local phase structure of wave dislocation lines: twist and twirl. J. Opt. A: Pure Appl. Opt. **6**: S202–S208.
20. Beth R A, 1936. Mechanical detection and measurement of the angular momentum of light. Phys. Rev. **50**: 115–125.

Vasylkiv Yu., Skab I. and Vlokh R., 2011. Measurements of piezooptic coefficients π_{14} and π_{25} in $Pb_5Ge_3O_{11}$ crystals using torsion induced optical vortex Ukr.J.Phys.Opt. **12**: 101–108.

Анотація. *В роботі показано, яким чином можна використати сингулярності оптичного поля для вимірювання п'єзооптичних коефіцієнтів при крученні кристалів, за умови, що природна оптична активність створює перешкоди прямим поляриметричним дослідженням. Експериментальне дослідження оптичного вихору, який виникав при крученні кристалів германату свинцю дозволило визначити, так звані, неголовні п'єзооптичні коефіцієнти (π_{14} та π_{25}). Отримано співвідношення між спіновим кутовим моментом пучка і п'єзооптичними коефіцієнтами.*