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## Polarisation singularities of optical beam propagating in imperfect quartz crystals

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**Abstract.** We have revealed experimentally polarisation singularities in the optical beam propagating through quartz crystals along the direction of optic axis and discussed their origin. It has been supposed that the singularities are caused by a screw dislocation of crystalline structure.

**Keywords:** crystalline structure, dislocations, polarisation singularities, optical vortices

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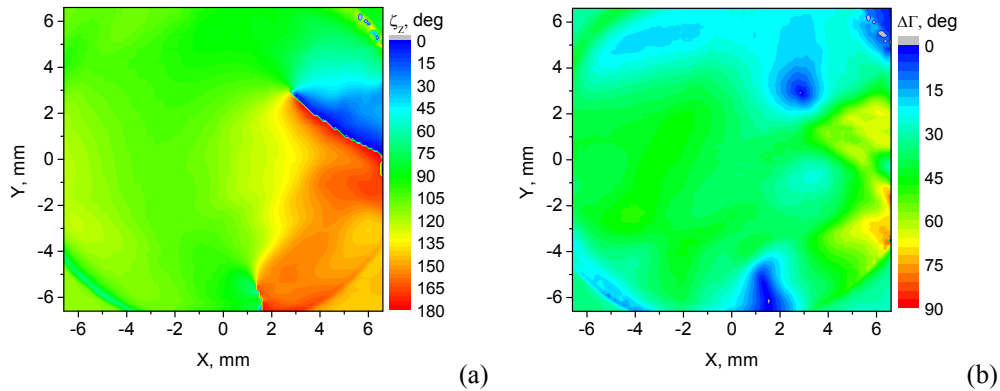
**UDC:** 535.5, 548.4

Recently we have developed an accurate method for measuring piezooptic coefficients, which is based on creation of 2D spatially inhomogeneous mechanically stressed state in crystals via torsion [1–4]. Using LiNbO<sub>3</sub> and  $\alpha$ -BaB<sub>2</sub>O<sub>4</sub> crystals as examples, we have determined the coefficients  $\pi_{14}$  with very low relative errors not exceeding a few percent [2]. We have also demonstrated that the piezooptic coefficient  $\pi_{14}$  can be quite simply determined for the point symmetry groups 3m and  $\bar{3}m$  of a trigonal crystallographic system. These are just the groups that characterise LiNbO<sub>3</sub> and  $\alpha$ -BaB<sub>2</sub>O<sub>4</sub> crystals, respectively.

Notice that quartz crystals belonging to the point symmetry group 32 have a piezooptic tensor of the same form as that for the groups 3m and  $\bar{3}m$ . As a consequence, the initial goal of the present study has been determining the piezooptic coefficient  $\pi_{14}$  for the quartz crystals. However, we have found out that, up to the torsion moment of  $M_Z = 0.15$  Nm applied around the principal Z axis, no optical birefringence changes are detected for the light propagating along the Z direction which coincides with the optic axis. Of course, this could have been caused by the fact that the piezooptic coefficient  $\pi_{14}$  for the quartz crystals is comparatively small (according to the data [5], it is equal to  $-0.11 \times 10^{-12}$  m<sup>2</sup>/N only). Nonetheless, we have revealed a following interesting phenomenon. When a wide parallel beam propagates along the optic axis of a SiO<sub>2</sub> crystal, which has not been subjected to torsion stresses, two polarisation singularities exist in different parts of a sample (see Fig. 1). A corresponding experimental set up and a method for measuring the optical phase difference  $\Delta\Gamma$  and the angle of optical indicatrix rotation  $\zeta_Z$  have been described in detail in our recent work [6]. The aim of the present work to describe and attempt to interpret the singularities mentioned above.

The singularities found by us are characterised by a zero value of a so-called residual linear optical birefringence ( $\Delta n_{XY} = \Delta\Gamma\lambda / 2\pi d_Z$ , with  $\lambda = 632.8$  nm being the light wavelength and  $d_Z = 15.85$  mm the sample thickness along the Z direction) and by the optical indicatrix rotation of

180 deg occurring when the tracing angle changes by 360 deg. Hence, these singularities reveal the topological defect strength equal to  $\pm 1/2$ . Notice also that the two singularities differ by their signs of optical indicatrix rotation. Namely, the indicatrix of the upper part of the sample rotates anticlockwise (the topological defect strength is equal to  $-1/2$ ), whereas the clockwise rotation is peculiar for the lower part (the topological defect strength of  $1/2$ ).



**Fig. 1.** Spatial maps of optical indicatrix rotation angle (a) and optical phase difference (b) measured for the  $\text{SiO}_2$  crystals.

Now let us analyse possible origins of the appearance of polarisation singularities mentioned above. A first reason can be a residual optical birefringence induced by residual mechanical stresses. Then the crystal should become optical biaxial and the nuclei of the singularities should correspond to the outlets of optic axes. However, in our case the singularity nuclei are not characterised by a zero phase difference, otherwise the optic axes must have been inclined by some small angle with respect to the direction of beam propagation. Moreover, the directions of optical indicatrix rotations should have been the same in the case of optical biaxiality (cf., e.g., with the appearance of optical vortex at conical refraction [7]). A second possible reason can be induction of a stress field of special configuration by some ordered structural imperfections. Linear defects of crystalline structure (e.g., dislocations) can be such imperfections [8]. For example, a line of screw dislocations with the Burgers vector  $b$  parallel to  $[001]$  direction can produce the stress tensor components

$$\sigma_4 = -\frac{G_1 b}{2\pi} \frac{X}{X^2 + Y^2}, \quad \sigma_5 = -\frac{G_1 b}{2\pi} \frac{Y}{X^2 + Y^2}, \quad \sigma_1 = \sigma_2 = \sigma_3 = \sigma_6 = 0, \quad (1)$$

where  $|b| = c/2$ ,  $c$  denotes the lattice parameter,  $G_1 = 1/S_{44}$  the shear module,  $S_{44}$  the elastic compliance tensor component, and  $X$  and  $Y$  the coordinates.

Notice that the screw dislocations with the mentioned Burgers vector are indeed peculiar for the crystalline quartz [9]. Hence, if the screw dislocation appears in  $\text{SiO}_2$  crystals, with the  $b$  vector parallel to the  $[001]$  direction, the relations for the increments of optical impermeability tensor components  $\Delta B_i$  in  $XY$  plane may be written as

$$\Delta B_1 = \pi_{14} \sigma_4, \quad \Delta B_2 = -\pi_{14} \sigma_4, \quad \Delta B_6 = \pi_{14} \sigma_5. \quad (2)$$

Then the equations for the optical birefringence and the angle of optical indicatrix rotation acquire the forms

$$\Delta n_{XY} = n_o^3 \pi_{14} \sqrt{\sigma_4^2 + \sigma_5^2} = \frac{n_o^3 \pi_{14} G_1 b}{2\pi \sqrt{(X^2 + Y^2)}} = \frac{n_o^3 \pi_{14} G_1 b}{2\pi \rho}, \quad (3)$$

$$\tan 2\zeta_Z = \frac{2B_6}{B_1 - B_2} = \frac{\sigma_5}{\sigma_4} = \frac{Y}{X} = \tan \varphi, \quad \zeta_Z = \varphi / 2, \quad (4)$$

where  $n_o$  is the ordinary refractive index,  $X = \rho \cos \varphi$  and  $Y = \rho \sin \varphi$ .

A comparison of experimental data with the theoretical relations testifies that the rotation of optical indicatrix around the polarisation singularity cores is well described by Eq. (4). Then the screw dislocations observed by us should be characterised with the Burgers vectors of the opposite signs. However, the experimental spatial distribution of the phase difference not too well conforms to Eq. (3). Of course, the maximum residual birefringence is not so high, being approximately equal to  $7 \times 10^{-6}$ . Nevertheless, this birefringence value is reached at the distance of few millimetres from a dislocation core. At the same time, the corresponding distance dealt with in the study [9] has been equal to tens of micrometers only. The latter peculiarity may be related to the fact that Eqs. (1) describing the coordinate dependence of the stress tensor component represent only a rough approximation [10]. More strictly, they have been derived exclusively on the basis of linear elasticity and so do not account for possible influences of a finite size and a nature of the dislocation core

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*Анотація.* Експериментально виявлено поляризаційні сингулярності в промені, що поширюється вздовж оптичної осі в кристалах кварцу, та обговорено природу цих сингулярностей. Припускається, що вони спричинені гвинтовими дислокаціями кристалічної структури.