Topological defects of optical indicatrix orientation in optically biaxial crystals. The case of light propagation in the directions close to the optic axes

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Abstract. We have developed analytical approach to determine the orientations of cross sections of optical indicatrix (OI) around the optic axes (OAs) in biaxial crystals. It has been found that the angular distribution of cross sections of the OI by the planes perpendicular to the directions close to the OA reveals a topological defect of OI orientations with the strength equal to ½. When a conical circularly polarized wave with cone's axis coinciding with the OA in a biaxial crystal propagates through a sample, a singly charged optical vortex is generated. We have shown that splitting of a single OA in optically uniaxial crystals into two OAs due to electrooptic effect is accompanied by the topological reaction that involves dividing a single defect with the unit strength into two defects with the strengths equal to ½. We have experimentally discovered topological dipoles that consist of topological defects with the strengths of each defect within the pair equal to +½ and –½.

Keywords: topological defects, optical indicatrix of biaxial crystals, optical vortices, topological reactions

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1. Introduction

Optical vortices bearing orbital angular momentum can be used in different branches of technologies, e. g. quantum information processing [1], microparticles manipulation [2] and beam focusing below the diffraction limit [3]. Following from the principles of singular optics [4], novel possibilities arise if one uses both spin and orbital angular momentums in an optical beam, which represent the entities closely related tp quantum computing, cryptography, and quantum teleportation [5–7]. Since the spin angular momentum can acquire only two different values ($s = +1$ or $s = -1$ in the units of Planck constant $\hbar$ [8]) and the optical angular momentum can, in principle, be infinite ($m = 0, \pm 1, \pm 2, ...$) [9], the information can be encoded by multiplying a number of distinguishable states. In this sense a photon can carry arbitrarily large amount of information distributed over its spin and orbital quantum states [10]. Hence, studies of the basic principles of generation, propagation and interaction of optical vortex are very important for the optical technologies.

Optical vortices are generated with the aid of various techniques, which in fact can be divided into two different groups. The methods based on optically isotropic and inhomogeneous media (e.g., an inhomogeneous distribution of material density) belong to the first group. When propagating through these media, an initially nearly plane wave undergoes a perturbation of its phase front, with appearance of scalar-field singularities or optical vortices. These experimental techniques deal, e. g., with computer-generated holograms and optical fibres [4, 11–13]. The second group embraces the methods that lead to appearance of singularities of vector fields. Such singularities can be induced by different inhomogeneous fields [14–19] under propagation of nearly parallel beams through crystals or glasses. The effect can be observed if conical beams propagate in optically uniaxial crystals along the optic axis (OA) direction [20–25]. The same is true for the case of optically biaxial crystals, provided that the beam propagates along one of the OAs under the conditions of conical refraction [26–28].

In the case of uniaxial crystals, distribution of orientations of the cross sections of optical indicatrix (OI) around the OA reveals a topological defect in its centre, with the strength equal to unity. This leads to appearance of a doubly charged optical vortex in one of the components of emergent circularly polarized wave [23, 29]. This fact can be explained by the relation suggested in the work [30] for liquid crystalline q-plates having a topological defect of director orientation in its centre. The electric field of the emergent light can be written as [30]

$$E^{out}(\varphi) = E_A \cos\frac{\Delta\Gamma}{2}\begin{bmatrix} 1 \\ \pm i \end{bmatrix} + iE_A \sin\frac{\Delta\Gamma}{2} e^{\pm i2q\varphi \pm i2\alpha_0}\begin{bmatrix} 1 \\ \mp i \end{bmatrix}, \quad (1)$$

where q is the strength of topological defect, $m = \pm 2q$ the vortex charge, $\Delta\Gamma$ the phase difference, α_0 the initial orientation of director in the q-plate, and E_A and $E^{out}(\varphi)$ are respectively the incident and emergent wave amplitudes.

The first term in Eq. (1) describes the plane wave with the same spin angular momentum as in the incident wave (i.e., $-\hbar$), while the second one corresponds to the wave with the helical front, which carries a nonzero orbital angular momentum. When the crystal symmetry lowers from uniaxial to biaxial, the orbital vortices with unit charges can be generated [31] whenever conical circularly polarized light beams propagate along each of the OAs. This process seems to be explained as a result of conservation of the optical angular momentum taking place under splitting of OAs, i.e. we have $2\hbar = \hbar + \hbar$. However, the law of conservation of the orbital angular momentum is applicable only in the case of axial symmetry, which does not hold in the case described above.

The following question appears in connection with the conical refraction: can a singly charged optical vortex be generated when the light propagates along one of the OAs in biaxial crystals under the condition that the angle of conical refraction is negligibly small (e. g., in case of small birefringence)? It has been shown in a number of works [31–33] that a polarization singularity (a so-called C-point) appears in the emergent light propagating along the OAs in biaxial crystals. The distribution of polarization states around this polarization singularity testifies that this C-point represents a topological defect with the strength equal to ½. It seems to be evident that a specific distribution of polarization states that includes the topological defect in the centre of beam should be caused by the same distribution of the cross sections of OI. Nonetheless, to our best knowledge, this effect has not been proved analytically.

In the present work we derive phenomenological relations allowing one to conclude that the OAs in optically biaxial crystals represent topological defects of OI orientation with the strength equal to ½.

2. Analytical method and results

The OI equation in the coordinate system XYZ can be written as

$$\frac{x^2}{n_x^2} + \frac{y^2}{n_y^2} + \frac{z^2}{n_z^2} = 1. \quad (2)$$

Let us assume that the inequalities $n_X < n_Y < n_Z$ hold true. Then the transformation of the coordinate system XYZ under successive rotations by the angle φ around the Z axis and the angle θ around the Y' axis (see Fig. 1) can be represented by the relations

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \mathbf{R}_z(\varphi)\cdot\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix}\cdot\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}, \tag{3}$$

$$\begin{bmatrix} X'' \\ Y'' \\ Z'' \end{bmatrix} = \mathbf{R}_Y(\theta)\cdot\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \mathbf{R}_Y(\theta)\cdot\mathbf{R}_Z(\varphi)\cdot\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \mathbf{R}_\mathrm{t}(\varphi,\theta)\cdot\begin{bmatrix} x \\ y \\ z \end{bmatrix}, \tag{4}$$

where $\mathbf{R}_Z(\varphi)$ and $\mathbf{R}_Y(\theta)$ denote the matrices of rotations around the Z and Y' axes, respectively.

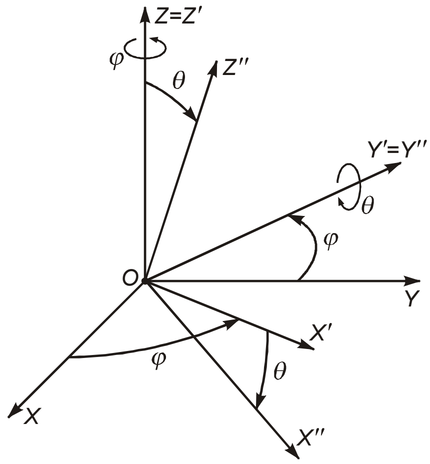

Fig. 1. Transformation of coordinate system $XYZ\to X'Y'Z'\to X''Y''Z''$ occurring under successive rotations by the angle φ around the Z axis and the angle θ around the Y' axis.

The resulting transformation matrix $\mathbf{R}_\mathrm{t}(\varphi,\theta)$ reads as

$$\mathbf{R}_\mathrm{t}(\theta,\varphi) = \mathbf{R}_Y(\theta)\cdot\mathbf{R}_Z(\varphi) = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}\cdot\begin{bmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} \cos\theta\cos\varphi & \cos\theta\sin\varphi & -\sin\theta \\ -\sin\varphi & \cos\varphi & 0 \\ \sin\theta\cos\varphi & \sin\theta\sin\varphi & \cos\theta \end{bmatrix}. \tag{5}$$

The coordinates in the doubly primed coordinate system and the coordinates in the initial system are linked by the relations

$$\begin{aligned} X'' &= X\cos\theta\cos\varphi + Y\cos\theta\sin\varphi - Z\sin\theta \\ Y'' &= -X\sin\varphi + Y\cos\varphi \\ Z'' &= X\sin\theta\cos\varphi + Y\sin\theta\sin\varphi + Z\cos\theta \end{aligned} . \tag{6}$$

The matrix $\mathbf{R}_t^{-1}(\theta,\varphi)$ of the inverse coordinate transformation $X''Y''Z''\to XYZ$ can be written as

$$\mathbf{R}_\mathrm{t}^{-1}(\theta,\varphi) = \mathbf{R}_Z(-\varphi)\cdot\mathbf{R}_Y(-\theta) = \begin{bmatrix} \cos\theta\cos\varphi & -\sin\varphi & \sin\theta\cos\varphi \\ \cos\theta\sin\varphi & \cos\varphi & \sin\theta\sin\varphi \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}. \tag{7}$$

Quite naturally, the inverse-transformation matrix represents a transposed directtransformation matrix, i.e. $\mathbf{R}_t^{-1}(\theta,\varphi)=\mathbf{R}_t^{T}(\theta,\varphi)$ (see Eqs. (5) and (7)). Now one can express the coordinates in the initial coordinate system in terms of the coordinates in the doubly primed system:

$$\begin{aligned} X &= X''\cos\theta\cos\varphi - Y''\sin\varphi + Z''\sin\theta\cos\varphi \\ Y &= X''\cos\theta\sin\varphi + Y''\cos\varphi + Z''\sin\theta\sin\varphi \\ Z &= -X''\sin\theta + Z''\cos\theta \end{aligned} \quad . \tag{8}$$

Substituting Eqs. (8) into Eq. (2), one obtains the OI equation in the doubly primed coordinate system:

$$\begin{aligned} &X''^2\left[\cos^2\theta\left(n_X^{-2}\cos^2\varphi+n_Y^{-2}\sin^2\varphi\right)+n_Z^{-2}\sin^2\theta\right]+Y''^2\left(n_X^{-2}\sin^2\varphi+n_Y^{-2}\cos^2\varphi\right)+ \\ &Z''^2\left[\sin^2\theta\left(n_X^{-2}\cos^2\varphi+n_Y^{-2}\sin^2\varphi\right)+n_Z^{-2}\cos^2\theta\right]-X''Y''\cos\theta\sin2\varphi\left(n_X^{-2}-n_y^{-2}\right)+ \\ &X''Z''\sin2\theta\left[\left(n_X^{-2}\cos^2\varphi+n_Y^{-2}\sin^2\varphi\right)-n_Z^{-2}\right]-Y''Z''\sin\theta\sin2\varphi\left(n_X^{-2}-n_Y^{-2}\right)=1. \end{aligned} \tag{9}$$

If the wave vector of the incident light is directed along the Z'' axis, the cross section of OI by the plane $X''OY''$ represents an ellipse. Its principal axes define the orientations and the refractive indices of the eigenwaves that propagate along a given direction. The equation of this ellipse can easily be obtained from Eq. (9) under the condition $Z''=0$:

$$A\,X''^2+B\,X''Y''+C\,Y''^2=1\,, \tag{10}$$

where

$$\left.\begin{aligned} A &= \cos^2\theta\left(n_X^{-2}\cos^2\varphi+n_Y^{-2}\sin^2\varphi\right)+n_Z^{-2}\sin^2\theta \\ B &= -\cos\theta\sin2\varphi\left(n_X^{-2}-n_Y^{-2}\right) \\ C &= n_X^{-2}\sin^2\varphi+n_Y^{-2}\cos^2\varphi \end{aligned}\right\}. \tag{11}$$

If we have $B \neq 0$, the principal axes of the ellipse are rotated about the coordinate axes X'' and Y'' by the angle

$$\psi = \arctan\left(\frac{C-A-\sqrt{(A-C)^2+B^2}}{B}\right). \tag{12}$$

Then the refractive indices n_I and n_{II} of the eigenwaves propagating along the direction Z'' are as follows:

$$n_{I,II}=\sqrt{\frac{2\left(A+C\pm\sqrt{(A-C)^2+B^2}\right)}{4AC-B^2}}\,, \tag{13}$$

where the signs «+» and «–» correspond to n_I and n_{II}, respectively. After the angle ψ for the light propagation directions close to the OA in biaxial crystals is known, one can determine the orientations of elliptical cross sections of the OI for the above directions.

To be specific, we consider as example a biaxial crystal of tellurite (TeO_2), which belongs to the symmetry group *mmm* [34]. Its refractive indices are equal to $n_X = 2.00$, $n_Y = 2.18$ and $n_Z = 2.35$ [34]. Fig. 2 displays the maps of angular distributions of the optical birefringence and the orientation of slow principal OI axis, which are calculated using the theoretical relations derived above and the optical parameters of TeO_2.

One can see from Fig. 2 that the orientations of the cross sections of OI around the OA correspond to a spatial distribution that involves a topological defect in its centre. The strength of this defect is equal to ½. Now it becomes clear why the conical circularly polarized incident beam propagating along the OAs in biaxial crystals induces a singly charged vortex in the orthogonal circular polarizations of the emergent beam (see the second term in the r. h. s. of Eq. (1)). We have also to notice that lowering of the crystal symmetry from uniaxial to biaxial (e. g., under some external field applied to a sample) has to be accompanied by a topological reaction. The latter would imply division of the topological defect with the strength 1, which is peculiar for the uniaxial crystals, into two defects with the strengths ½, which are peculiar for the biaxial crystals. Below we demonstrate this fact experimentally.

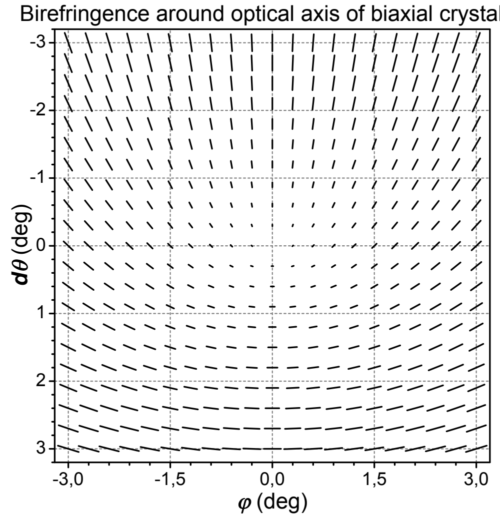

Fig. 2. Distribution of optical birefringence calculated in the vicinity of OA in TeO_2 crystals. The length of segments is proportional to the birefringence value $\Delta n = n_I - n_{II}$, while their orientations indicate polarization directions of the slow wave (i.e., that with the refractive index n_I).

In our polarimetric experiment, we have used a $LiNbO_3$ crystal of which faces are perpendicular to the principal axes *X*, *Y* and *Z* of the Fresnel ellipsoid. In other words, we have $X \| \mathrm{a}$, $Y \| \mathrm{m}$ and $Z \| \mathrm{c}$, where *a* and *c* denote the crystallographic axes, and *m* is the mirror symmetry plane. The sample has a parallelepiped shape with the sizes equal to 25.0 along the *Z* axis and 9.2 mm along the *X* and *Y* axes. The electric DC voltage 500 V has been applied along the *X* axis.

A circularly polarized optical beam propagating along the *Z* axis has been focused by an optical lens at the entrance surface of our sample. Then the beam is analyzed by a linear rotating polarizer and finally falls at the CCD camera. As a result, one obtains some angular distribution of the phase difference and the orientation angle of OI. The relevant experimental results are presented in Fig. 3. Note that the method used for determining these parameters has been described in detail in our work [14].

As seen from Fig. 3a and Fig. 3b, nine topological defects (TD_i) of OI orientation exist in the crystal under zero voltage. These are the central defect TD_0 and the four pairs of defects (TD_1 and TD_2, TD_3 and TD_4, TD_5 and TD_6, and TD_7 and TD_8), which in fact represent topological dipoles. The central defect TD_0 has the init strength. The lateral defects have the strength modules equal to ½ and the opposite signs of the strength within each pair. Moreover, one can detect some additional dipoles of topological defects at still larger angles of beam divergence. The phase difference in the places where all the mentioned defects are located is equal to $2\pi n$, where *n* is an integer and therefore it looks as if it is equal to zero. (see Fig. 3a). The existence of these defects agrees well with the simulation results presented in Ref. [35]. However, we have not observed a topological defect between the two defects in the dipoles, which might have led to appearance of

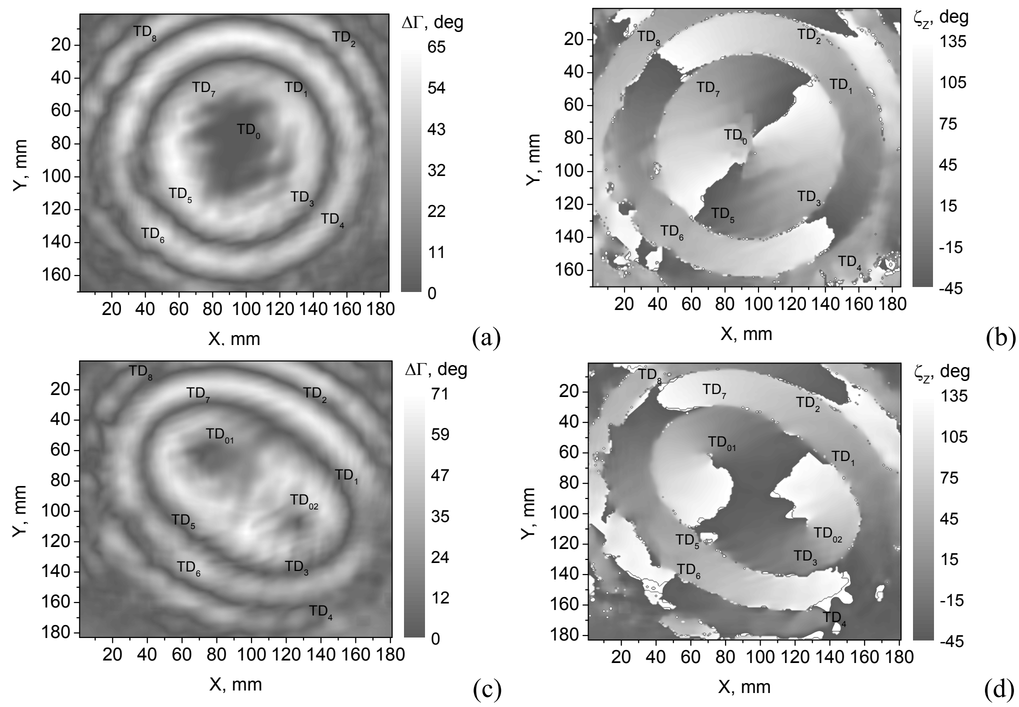

Fig. 3. The distributions of phase difference (panels (a) and (c)) and orientation angle of OI orientation (panels (b) and (d)) in $LiNbO_3$ crystals within the angular beam aperture. Panels (a), (b) and (c), (d) correspond to zero voltage and V = 0.5 kV.

singly charged optical vortex, as predicted in the work [35]. Application of the voltage 0.5 kV leads to splitting of the central defect TD_0 into two defects, TD_{01} and TD_{02}, of which strengths are equal to ½. This process can be described as a topological reaction that results in division of the defect with the unit strength into two defects with half-unit strengths. This process is described by the equation $1 = ½ + ½$ in the units of $\hbar$. At the same time, the two diametrically opposite dipoles consisting of topological defects move away from the beam's cone axis, while the other two defect pairs move towards it.

3. Conclusions

In the present work we have developed a simple analytical approach that enables determining of orientation of the cross sections of OI around the OAs in optically biaxial crystals. We have found that the angular distribution of the cross sections of OI by the planes perpendicular to the directions close to the OA reveals a topological defect of OI orientations with the strength equal to ½ . This topological defect corresponds to the outlet of OA in the biaxial crystals. This statement implies that, if the conical circularly polarized incident beam propagates along the OA in biaxial crystals, a singly charged optical vortex is generated in the outgoing beam, with the circular polarizations of the opposite handednesses. Notice that this explanation of vortex generation does not involve the effect of conical refraction.

We have shown experimentally that splitting of a single OA in uniaxial crystals into two OAs is accompanied by the topological reaction that divides a defect with the unit strength into two defects with the strengths ½. At the same time, we have revealed at least four pairs of additional topological defects. These are topological dipoles with the strengths of each defect within the pair equal to +½ and –½.

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Krupych O., Vasylkiv Yu., Kryvyy T., Skab I. and Vlokh R. 2017. Topological defects of optical indicatrix orientation in optically biaxial crystals. The case of light propagation in the directions close to the optic axes. Ukr.J.Phys.Opt. **18**: 131 – 138

Анотація. *У роботі представлено аналітичний підхід до опису орієнтації перетинів оптичних індикатрис за умови просвічуванні оптично двовісного кристала в довільному напрямку. Виявлено, що кутовий розподіл перетинів оптичних індикатрис площинами, перпендикулярними до напрямків, близьких до оптичної осі двовісного кристала, містить топологічний дефект орієнтації оптичних індикатрис із силою ½. При поширенні конічної циркулярно поляризованої хвилі вздовж оптичних осей у двовісних кристалах буде генеруватися вихор одиничного заряду. Показано, що розщеплення єдиної оптичної осі в одновісному кристалі внаслідок електрооптичного ефекту і відповідна поява двох оптичних осей супроводжується топологічною реакцією розпаду дефекту з силою 1 на два дефекти з силою ½. Експериментально виявлено топологічні диполі, які складаються з пари топологічних дефектів із силами +½ і –½.*