Torsion-induced optical "triaxiality" of $LiNbO_3$ crystals

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Abstract. In the present work we have revealed that three topological defects of optical indicatrix orientation appear whenever torsion stresses are applied to $LiNbO_3$ and a divergent optical beam is incident upon this crystal. The above defects have half-integer strengths. We have found that the places where the defects are localized are characterized by a zero phase difference. These phenomena enable one to consider the three defects as outlets of optic axes in an inhomogeneous, 'optically triaxial' $LiNbO_3$ crystal. After analyzing all of possible scenarios for the interaction among the defects, we have shown that no reaction of their fusion can be expected. Besides, there can appear no defects with intermediate strength values ranging from ½ to 1 and, moreover, no optical vortices with fractional charge are generated by these defects.

Keywords: optical anisotropy, $LiNbO_3$ crystals, torsion stresses, optically triaxial state

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1. Introduction

Optical vortices (OVs) that appear as a result of screw-like phase-front dislocations [1] have been attracting a lot of attention of researchers in the last two decades. This is due to unusual physical behaviour of OVs and promises of their applications in such novel branches of technology as quantum encoding [2], quantum teleportation [3] and microparticles manipulation [4]. Most of these applications are associated with quantum properties of the OVs, because they bear nonzero orbital angular momentum $l\hbar$ (with l being integer) that represents their charge [5].

It is known that one of the methods used to generate the OVs is linked with inhomogeneous anisotropic media, e.g. liquid crystalline plates with embedded structural defects of director orientation in the centre of the plate. This structural defect is nothing but a topological defect of the field of director orientation, which can easily be created by technological manipulations with liquid crystalline cells. When a circularly polarized wave bearing a nonzero spin angular momentum is incident on a cell, one can generate the emergent optical wave that includes a so-called doughnut mode. The amplitude of the emergent wave can be written as [6]

$$E^{out}(\varphi) = E^{in}\cos\frac{\Delta\Gamma}{2}\begin{bmatrix}1\\ \pm i\end{bmatrix} + iE^{in}\sin\frac{\Delta\Gamma}{2}e^{\pm i2q\varphi \pm i2\alpha_0}\begin{bmatrix}1\\ \mp i\end{bmatrix}, \quad (1)$$

where q is the strength of topological defect, $m = \pm 2q$ the OV charge, $\Delta\Gamma$ the phase difference and α_0 the initial orientation of director, whereas E^{in} and $E^{out}(\varphi)$ are the amplitudes of the incident and emergent waves, respectively. The first term in the r. h. s. of Eq. (1) describes a nearly plane wave with the same spin angular momentum as that of the incident wave (i.e., $-\hbar$), while the second term corresponds to the wave with a helical front carrying a nonzero orbital angular momentum.

Note that a liquid-crystalline matrix with the structural defects created technologically represents a structure with a 'stiff' director orientation. As a consequence, the parameters of the OV beam cannot easily be operated by external fields. In view of this shortcoming, we have suggested recently creating of the topological defects of optical indicatrix orientation in solid crystalline materials using inhomogeneous external fields (e.g., torsion and bending stresses [7–9] or conically shaped electric fields [10, 11]). Then the topological defects can generate the OVs in the emergent beam owing to conversion of spin angular momentum into orbital angular one. In particular, the topological defect of optical indicatrix orientation with the strength ½ appears at the axis of torsion, whenever optically uniaxial crystals belonging to certain symmetry groups are twisted around the optic axis direction [8]. In this case one deals with a parallel, expanded incident optical beam. The above topological defect generates a singly charged OV [7].

On the other hand, it is known (see, e.g., Ref. [12]) that a different topological defect of optical indicatrix orientation, which has the unit strength, appears at the beam axis under the condition that a divergent beam propagates along the optic axis in a uniaxial crystal. This produces a doubly charged OV. The following question arises in connexion with the above processes: What will happen with the topological defects of optical indicatrix orientation if the torsion stress is applied to the uniaxial crystal in the same geometry under the condition of divergent incident beams? In other words, how will the initial topological defect (with the unit strength observed in the divergent beam) transform into the defect with the strength equal to ½ under nonzero torque moment? Through some intermediate values, or in some other manner? The present work is aimed at answering these questions with experimental means.

2. Experimental procedures

In our polarimetric experiments, we used a $LiNbO_3$ crystal belonging to the point symmetry group $3m$. Some of its faces were perpendicular to the principal axes *X*, *Y* and *Z* of Fresnel ellipsoid. Here $X \| a$, $Y \| m$ and $Z \| c$, where *a* and *c* are the crystallographic axes, and *m* denotes the mirror symmetry plane. Our sample had an octahedral prism shape. Its sizes were equal to 14.9 mm along the *Z* axis and 6.0 mm along the *X* and *Y* axes. To obtain spatial distributions of the phase difference and the optical indicatrix orientation, a circularly polarized optical beam of He–Ne laser (the optical wavelength $\lambda = 632.8\text{ nm}$) was focused with an optical lens onto entrance sample face and propagated along the *Z* axis (see Fig. 1). Then the light was analyzed with a linear rotating polarizer and, finally, fell upon a CCD camera.

As a result, one can obtain experimental angular distributions of the phase difference $\Delta\Gamma(\theta_X, \theta_Y) = 2\pi\Delta n(\theta_X, \theta_Y)d / \lambda$ (with $\Delta n(\theta_X, \theta_Y)$ denoting the angular distribution of optical birefringence, λ the wavelength and d the sample thickness along the direction defined by the angles (θ_X, θ_Y)). The orientation angle of optical indicatrix is given by the relation $\tan 2\zeta_Z(\theta_X, \theta_Y) = B_{ij} / (B_{ii} - B_{jj})$, where B_{ij} are the components of optical-frequency impermeability tensor.

The dependence of the intensity I on the analyzer azimuth α reduces to

$$I = \frac{I_0}{2}\left\{1 + \sin\Delta\Gamma(\theta_X, \theta_Y)\sin\left[2\left(\alpha - \varphi(\theta_X, \theta_Y)\right)\right]\right\}, \tag{2}$$

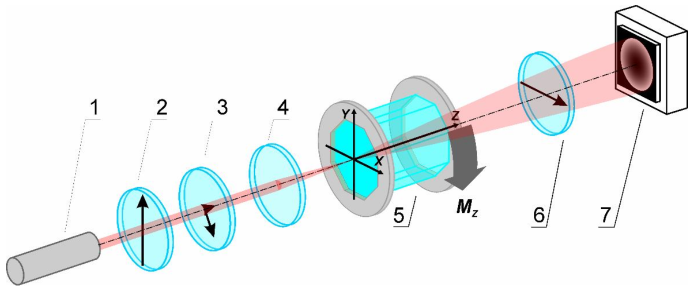


Fig. 1. Experimental setups used for determining angular distributions of the phase difference and the optical indicatrix orientation: 1 – He–Ne laser, 2 – linear polarizer, 3 – quarter-wave plate (QWP), 4 – lens, 5 – sample under torque moment M_Z, 6 – linear analyzer, and 7 – CCD camera.

where $\varphi(\theta_X, \theta_Y)$ is the angle of orientation of the principal axis of polarization ellipse behind the sample. After recording and filtering CCD images, azimuthal dependences of the intensity I are fitted by the sine function for each pixel of the image:

$$I = C_1 + C_2 \sin\left[2\left(\alpha - C_3\right)\right], \quad (3)$$

where C_1, C_2 and C_3 are fitting coefficients. Using the relation for the optical retardation $\Delta\Gamma(\theta_X, \theta_Y)$ and Eq. (3), one can write out the fitting coefficients as

$$C_1 = \frac{I_0}{2}, \quad C_2 = \frac{I_0}{2} \sin \Delta\Gamma(\theta_X, \theta_Y), \quad C_3 = \varphi(\theta_X, \theta_Y). \quad (4)$$

Then the optical retardation can be determined directly by the fitting coefficients C_1 and C_2:

$$\sin \Delta\Gamma(\theta_X, \theta_Y) = \frac{C_2}{C_1}, \quad (5)$$

while the angular orientation of the intensity minimum depends upon the orientation $\varphi(\theta_X, \theta_Y)$ of the principal axis of polarization ellipse and the coefficient C_3. Thus, fitting the dependences of light intensity for each pixel of the sample image behind the analyzer on the azimuth enables one to construct 2D maps of optical anisotropy parameters of the sample under test, namely the optical retardation and the orientation of the principal axis of optical indicatrix.

3. Results and discussion

When the light suffers acute focusing (the focal length of the lens is equal to f = 15 mm) and the torque moment is zero, one can detect a zero phase difference in the centre of the map (see Fig. 2a). This corresponds to outlet of the optic axis in our uniaxial $LiNbO_3$ crystal. The topological defect of optical indicatrix orientation, which corresponds to this axis, manifests the strength equal to unity, i.e. the optical indicatrix is rotated by 2π rad whenever the tracing angle is changed by the same value.

When a nonzero torque moment (M_Z = 0.127 N×m) is applied to crystalline rod, two topological defects with the strengths equal to 1/2 appear, instead of a single defect with the unit strength. This implies the topological reaction 1 = ½ + ½ (see Fig. 3a). The both defects occur in those places of the azimuthal map where the phase difference is equal to zero (see Fig. 3b). Increasing torque moment leads to further increase in the angular distance between the defects. It is obvious that the above defects correspond to outlets of the optic axes, so that our inhomogeneous crystal seems to become optically biaxial. The plane where the optic axes lie is almost parallel to bisector of the ZX and ZY planes. Change in the sign of the torque moment 'switches' the plane of the optic axes by 90 deg around the Z axis.

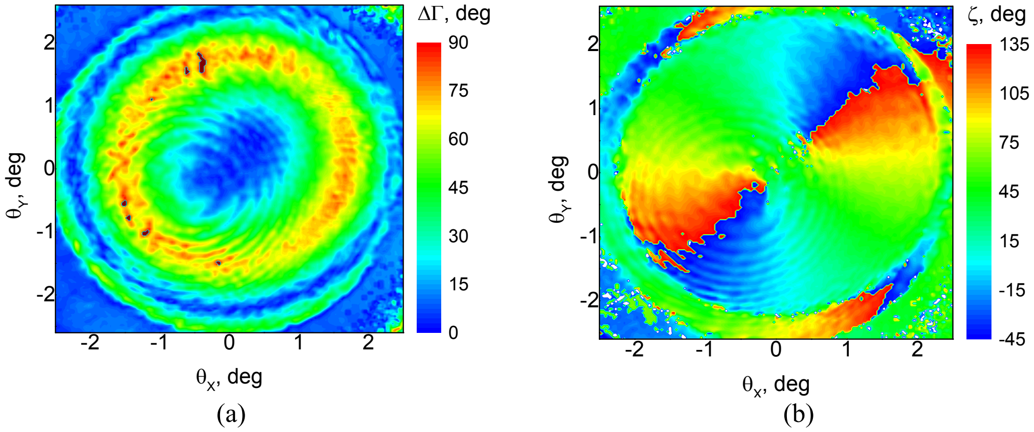


Fig. 2. Phase difference (a) and angle of optical indicatrix orientation (b) detected at $f = 15$ mm and zero momentum M_Z.

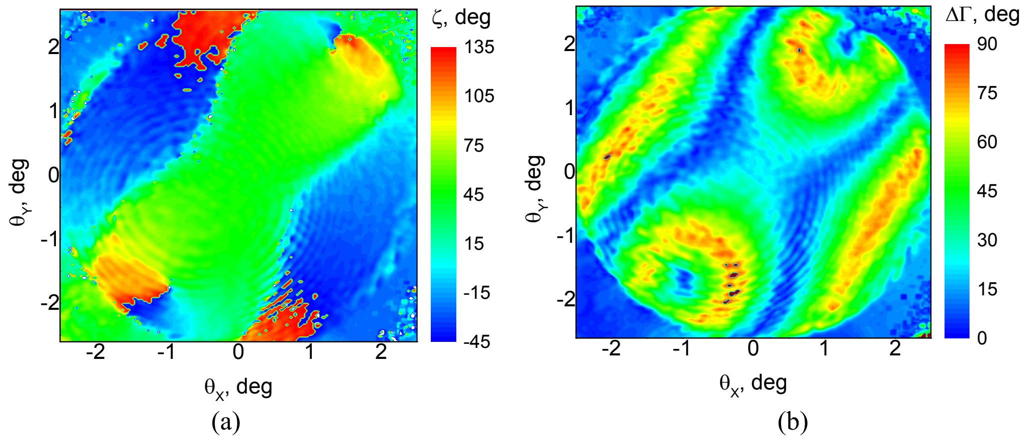
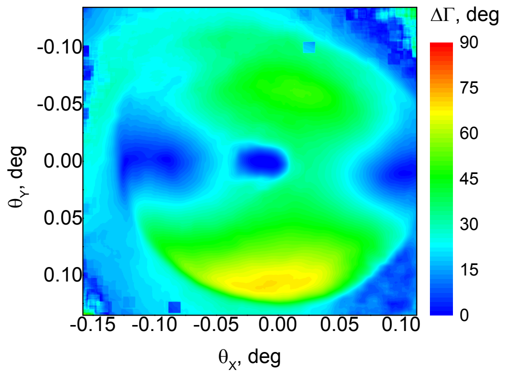


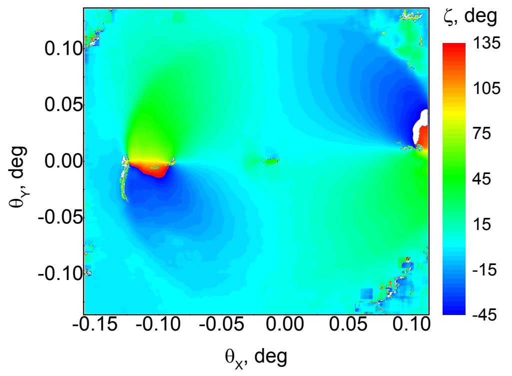
Fig. 3. Angle of optical indicatrix orientation (a) and phase difference (b) detected at $f = 15$ mm and $M_Z = 0.127$ N×m.

As a consequence of the above experimental data, the following question appears: Why do we not observe the topological defect of optical indicatrix orientation located in the beam centre, which has earlier been observed under conditions of parallel optical beam and crystal torsion [7]? It is evident from Fig. 3a that this defect has not been detected in case of the focal length equal to $f = 15$ mm. The defect with the strength ½ observed in the parallel beams propagating through the sample has to be located somewhere in a narrow central part of the map, where the elementary beams are close to parallel. Under acute focusing, this part of map is too small to make detecting this defect possible. Hence, one has to magnify the central part of map, or use a lens with weaker focusing.

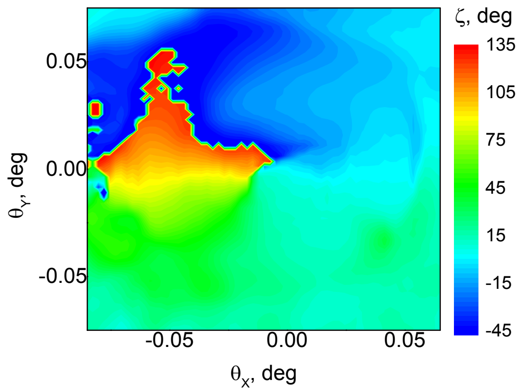
As seen from Fig. 4a, three places with a zero phase difference are observed on the map when the entrance lens with the focal length $f = 300$ mm is used and the torque moment $M_Z = 0.036$ N×m is applied. This phenomenon has to imply that an 'optically triaxial' state appears in our crystal. One must realize that, as a matter of fact, we deal with optically inhomogeneous medium which can, in principle, manifest such an exotic optical state. One of the places with zero phase difference occurs in the very centre of the map. Obviously, it should correspond to a torsion-induced topological defect, which we have not detected in our previous experiments. The two other defects are shifted along the $\pm X$ directions (see Fig. 4a). These lateral defects are clearly seen in Fig. 4b at the edge of the map, whereas the central one is almost invisible under these conditions. It is interesting that the two lateral defects appear in the *XZ* plane rather than in the bisector plane, as it has been observed for higher torque moments and acute focusing. This phenomenon will be scrutinized in our forthcoming studies.



(a)



(b)



(c)

Fig. 4. Phase difference (a) and angle of optical indicatrix orientation (b) detected at $f = 300$ mm and $M_Z = 0.036$ N×m; (c) angle of optical indicatrix orientation detected at $f = 500$ mm and $M_Z = 0.036$ N×m.

In order to detect the central defect, we have magnified the central part using the entrance lens with the focal length $f = 500$ mm, which makes the incident beam closer to parallel. As seen from Fig. 4c, the torsion-induced topological defect with the strength ½ is present in the centre of the map. Hence, we have successfully detected all of the three defects that exist simultaneously in our crystal. Then $LiNbO_3$ can surely be described by the ‘optically triaxial’ state. Notice also that the defects are clearly separated spatially. To implement the fusion reaction with the three defects and obtain the resultant defect with the strength 3/2, one must have needed to decrease the torque moment or, alternatively, decrease the divergence angle of the optical beam. However, the central defect has to disappear when the torque moment decreases down to zero value (only then the two lateral defects would reach the centre of the map). On the other hand, the lateral defects have to disappear when the divergence angle decreases to zero. Thus, no reaction of fusion among the three defects can be expected. As a result, one concludes that neither defects with intermediate strengths from the range ½÷1 can exist in the crystal. The same holds true of the OVs with fractional charges generated by the three defects under test.

4. Conclusion

In this work we have studied the topological defects of optical indicatrix orientation, which are induced by the torsion mechanical stresses in the $LiNbO_3$ crystals. We have found that three different topological defects appear under external torque moment, provided that the incident optical beam is divergent. One of these defects, the central one, has the strength equal to ½. It is located along the torsion axis that coincides with the optic axis in the uniaxial $LiNbO_3$ crystals. The angular position of this defect is stable and does not depend on the torque moment. The other two topological defects are lateral and also have the strength ½. They appear due to torsion-induced splitting of the central defect with the unit strength. The phase difference is equal to zero in the place where the central defect is located. Concerning the two lateral defects, the phase difference can acquire integrated zero values, being nonzero inside the sample. The phenomena observed by us enable considering the three defects as the outlets of optic axes in spatially inhomogeneous 'optically triaxial' crystal.

The two following scenarios of interaction among the defects are possible: (i) the two lateral defects reach the centre of the map and the central defect disappears when the torque moment decreases down to a zero value, and (ii) the lateral defects disappear and only the central defect remains when the divergence angle tends to zero. Therefore no reaction of fusion among the three defects can be expected. As a result, we conclude that no defects with intermediate strengths in the range from ½ to 1 can exist. Moreover, no OVs with fractional charges generated by these defects can appear.

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Анотація. *Встановлено, що за умов падіння на кристал* $LiNbO_3$ *розбіжного пучка світла та дії торсійних напружень у кристалі з'являються три топологічні дефекти орієнтації оптичної індикатриси. Ці дефекти мають напівцілу силу, а місця їхнього розташування характеризуються нульовою різницею фаз. Виявлений ефект дає змогу розглядати згадані вище три дефекти як виходи оптичних осей в неоднорідному "оптично тривісному" кристалі. На підставі аналізу можливих сценаріїв взаємодії між дефектами показано, що реакцію злиття трьох дефектів не слід очікувати. Неможливими є й проміжні між ½ і 1 значення сили дефектів, а також генерація оптичних вихорів із дробовими зарядами.*