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A CORRIGENDUM TO ”HEREDITARY PROPERTIES BETWEEN A RING AND ITS MAXIMAL SUBRINGS”

ПОПРАВКА ДО РОБОТИ „СПАДКОВІ ВЛАСТИВОСТІ МІЖ КІЛЬЦЕМ ТА ЙОГО МАКСИМАЛЬНИМИ ПІДКІЛЬЦЯМИ”

Let R be a commutative ring with identity. In [2] (Proposition 3.1), Azarang proved that if R is an integral domain and S is a maximal subring of R, and is integrally closed in R, then $\dim(S) = 1$ implies that $\dim(R) = 1$ if and only if $(S : R) = 0$. An example is given which shows the above mentioned proposition is not correct.

Нехай R — комутативне кільце з одиницею. В роботі [2] (твердження 3.1) Азаранг довів, що у випадку, коли R — інтегральна множина, а S — максимальне підкільце R, інтегрально замкнене в R, із рівності $\dim(S) = 1$ випливає, що $\dim(R) = 1$ тоді і тільки тоді, коли $(S : R) = 0$. Наведено приклад, який показує, що це твердження є неправильним.

1. Introduction. All rings considered throughout are commutative with nonzero identity; all ring extensions, ring homomorphisms, and algebra homomorphisms are unital. Given rings $S \subseteq R$, the conductor $(S\colon R) = \{r \in R\colon rR \subseteq S\}$. Also, dimension(al) refers to Krull dimension. If S is a proper subring of a ring R, then S is a maximal subring of R if there is no ring T such that $S \subset T \subset R$ where $\subset$ denotes proper inclusion.

In [2], Azarang proved in Proposition 3.1 that if R is an integral domain and S is a maximal subring of R, and is integrally closed in R, then $\dim(S) = 1$ implies $\dim(R) = 1$ if and only if $(S\colon R) = 0$. The importance of Proposition 3.1 in [2] is witnessed by the abstract of [2]. We have given an example which shows that the reverse implication of above proposition is not correct.

2. Corrigendum. The following result was proved in [2].

Theorem 2.1 ([2], Proposition 3.1). *Let R be an integral domain and S be a maximal subring of R, and is integrally closed in R. Then the following statements are true*:

(1) *If* $\dim(R) = 1$, *then* $\dim(S) = 1$ *if and only if* $(S\colon R) = 0$.

(2) *If* $\dim(S) = 1$, *then* $\dim(R) = 1$ *if and only if* $(S\colon R) = 0$.

We now present the counter example to show that (2) is not correct. However, (1) is correct.

***Example* 2.1.** Let $R = \mathbb{Q}$ and $S = \mathbb{Z}_{2\mathbb{Z}}$. We assert that S is a maximal subring of R. Suppose there is a ring T such that $S \subset T \subseteq R$. Choose $\dfrac{p}{2^n q} \in T \setminus S$, where p and $2^n q$ are coprime, $n \in \mathbb{N}$. Thus, $(2^{n-1}q)\left(\dfrac{p}{2^n q}\right) \in T$, which gives $1/2 \in T$. Therefore, $T = R$. Hence, S is a maximal subring of R. Since S is a one dimensional valuation domain with quotient field R, S is integrally

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closed in R. Now, suppose $p/q \in (S : R)$, where p and q are coprime. Clearly, q must be odd. Also, $\dfrac{p}{2^n q} \in S$ for all $n \in \mathbb{N}$, implies that $p = 0$. Therefore, $(S : R) = 0$. Clearly, $\dim(S) = 1$ but $\dim(R) = 0$. This counters (2) of above mentioned theorem.

***Remark* 2.1.** Note that under the stated conditions of Theorem 2.1, if $\dim(S) = 1$, then $\dim(R) \leq 1$ by [1] (Proposition 4.1), and $(S : R) = 0$ by [3] (Theorem 7).

References

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