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Approximative properties of the Weierstrass integrals on the classes $W^r_\beta H^\alpha$

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Abstract. The work focuses on the solution of the one problem of approximation theory. The problem is to investigate approximative properties of the Weierstrass integrals on the classes $W^r_\beta H^\alpha$. We obtain asymptotic equalities for the upper borders of defluxion of functions from the classes $W^r_\beta H^\alpha$ from the Weierstrass integrals.

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1. Introduction

Let L be the space of 2π–periodic summable on the period functions having the norm $\|f\|_L = \|f\|_1 = \int_{-\pi}^{\pi} |f(t)|dt$; L_∞ be the space of 2π–periodic measurable and essentially bounded functions having the norm $\|f\|_\infty = \operatorname{ess\,sup}_t |f(t)|$; C be the space of 2π-periodic functions having the norm $\|f\|_C = \max_t |f(t)|$.

Let $r > 0$ and β be a fixed real number. If the series

$$\sum_{k=1}^{\infty} k^r \left(a_k \cos\left(kx + \frac{\beta\pi}{2}\right) + b_k \sin\left(kx + \frac{\beta\pi}{2}\right)\right) \qquad (1.1)$$

is the Fourier series of some summable function φ, then we can introduce the (r, β)-derivative of the function f in the Weyl–Nagy sense and denote it by f^r_β (see, e.g., [1, p. 130]). By W^r_β we denote the set of all functions satisfying this condition.

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If $f \in W^r_\beta$ and, in addition, $f^r_\beta \in H^\alpha$, that is satisfies the Lipschitz condition of order α:

$$|f^r_\beta(x+h) - f^r_\beta(x)| \le |h|^\alpha, \quad 0 < \alpha \le 1, h \in \mathbb{R},$$

we say that f belongs to the class $W^r_\beta H^\alpha$. In the case where $\alpha = 0$ we set $W^r_\beta H^0 = W^r_{\beta,\infty}$.

If $\beta = r$, $r \in \mathbb{N}$, then the classes $W^r_\beta H^\alpha$ coincide with the famous Sobolev classes $W^r H^\alpha$. Note that W^r_∞ are the classes of functions f such that $\|f^{(r)}\|_\infty \le 1$.

Let $f \in L$. The quantity

$$W(\rho; f; x) = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t+x) \left\{ \frac{1}{2} + \sum_{k=1}^{\infty} \rho^{k^2} \cos kt \right\} dt, \ 0 \le \rho < 1,$$

is called the Weierstrass integral of the function f. Setting $\rho = e^{-\frac{1}{\delta}}$ the Weierstrass integral can be rewritten as follows (see, e.g., [2])

$$W_\delta(f; x) = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t+x) \left\{ \frac{1}{2} + \sum_{k=1}^{\infty} e^{-\frac{k^2}{\delta}} \cos kt \right\} dt, \ \delta > 0.$$

In the present paper we investigate asymptotic behavior of the quantity

$$\mathcal{E}(W^r_\beta H^\alpha; W_\delta)_C = \sup_{f \in W^r_\beta H^\alpha} \|f(\cdot) - W_\delta(f; \cdot)\|_C, \quad \delta \to \infty. \tag{1.2}$$

The problem of establishing the asymptotic equality for the quantity (1.2), according to Stepanets [1, p. 198], is called the Kolmogorov–Nikolsky problem for the Weierstrass integral W_δ on the class $W^r_\beta H^\alpha$ in the uniform metric.

In the uniform metrics the Kolmogorov–Nikolsky problem on the Zygmund classes Z_α, $Z_\alpha := \big\{ f(x) \in C : |f(x+h) - 2f(x) + f(x-h)| \le 2|h|^\alpha$, $0 \le \alpha \le 2$, $|h| \le 2\pi \big\}$, Korovkin [3], Bausov [4], Falaleev [5]; on the Sobolev classes W^r_∞ — in the papers of Bausov [6], Baskakov [7]. Asymptotic equalities for approximation of the Stepanets classes [1] by the Weierstrass integrals were obtained in the papers [2, 8, 9].

2. The approximation by Weierstrass integrals on the classes $W^r_\beta H^\alpha$

Analogously to the paper [10] for the Weierstrass integral for $r > 0$ and $\delta > 0$, we consider the following function $\tau(u)$ continuous on $[0; \infty)$:

$$\tau(u) = \tau_\delta(r, u) = \begin{cases} (1 - e^{-u^2})\delta^{\frac{r}{2}}, & 0 \le u \le \frac{1}{\sqrt{\delta}}, \\ (1 - e^{-u^2})u^{-r}, & u \ge \frac{1}{\sqrt{\delta}}, \end{cases} \tag{2.1}$$

whose Fourier transform

$$\hat{\tau}_\beta(t) = \frac{1}{\pi}\int_0^\infty \tau(u)\cos\left(ut + \frac{\beta\pi}{2}\right)du \tag{2.2}$$

is summable on the whole real axis (this fact is proved in [2]).

In what follows, by K, K_i, $i = 1, 2$, we denote constants whose values may be different in different places.

Theorem 2.1. *For $r > 2$, $0 < \alpha < 1$ and $\delta \to \infty$ the following asymptotic equality is true*

$$\mathcal{E}\left(W^r_\beta H^\alpha; W_\delta\right)_C = \frac{1}{\delta}\sup_{f\in W^r_\beta H^\alpha}\|f''\|_C + O\bigg(\frac{1}{\delta^{\frac{r+\alpha}{2}}} + \frac{1}{\delta^2}\bigg), \tag{2.3}$$

where f'' be the second derivative of function f.

Proof. Similarly, as in the paper [11], let us rewrite the function $\tau(u)$ defined by the relation (2.1) in the form $\tau(u) = \varphi(u) + \mu(u)$, where

$$\varphi(u) = \begin{cases} u^2\delta^{\frac{r}{2}}, & 0 \le u \le \frac{1}{\sqrt{\delta}}, \\ u^{2-r}, & u \ge \frac{1}{\sqrt{\delta}}, \end{cases} \tag{2.4}$$

$$\mu(u) = \begin{cases} \big(1 - e^{-u^2} - u^2\big)\delta^{\frac{r}{2}}, & 0 \le u \le \frac{1}{\sqrt{\delta}}, \\ \big(1 - e^{-u^2} - u^2\big)u^{-r}, & u \ge \frac{1}{\sqrt{\delta}}. \end{cases} \tag{2.5}$$

It is known that Fourier transforms $\widehat{\varphi}(t)$ and $\widehat{\mu}(t)$ of kind (2.2) of functions $\varphi(u)$ and $\mu(u)$ are summable on whole real axis (see, e.g., [2]).

According to the theorem 3 of Bausov [6], if the integrals

$$A(\alpha,\varphi) = \frac{1}{\pi}\int_{-\infty}^\infty |t|^\alpha \left|\int_0^\infty \varphi(u)\cos\left(ut + \frac{\beta\pi}{2}\right)du\right| dt, \tag{2.6}$$

$$A(\alpha,\mu) = \frac{1}{\pi}\int_{-\infty}^\infty |t|^\alpha \left|\int_0^\infty \mu(u)\cos\left(ut + \frac{\beta\pi}{2}\right)du\right| dt, \tag{2.7}$$

are convergent and $A(\alpha,\mu) = o\big(A(\alpha,\varphi)\big)$, then the following asymptotic equality holds

$$\mathcal{E}(W^r_\beta H^\alpha; W_\delta)_C = \frac{1}{\delta^{\frac{r}{2}}}\sup_{f\in W^r_\beta H^\alpha}\|f_\varphi\|_C + O\left(\frac{1}{\delta^{\frac{r+\alpha}{2}}}A(\alpha,\mu)\right), \tag{2.8}$$

where $f_\varphi(x) := \int_{-\infty}^\infty \Big(f^r_\beta\big(x + \frac{t}{\sqrt{\delta}}\big) - f^r_\beta(x)\Big)\widehat{\varphi}(t)dt$.

To prove the convergence of the integral $A(\alpha,\varphi)$, according to theorem 1 of the paper of Bausov [6], let us consider the integrals

$$\int_0^{\frac{1}{2}} u^{1-\alpha}\,\big|d\varphi'(u)\big|,\ \int_{\frac{1}{2}}^{\frac{3}{2}} |u-1|^{1-\alpha}\,\big|d\varphi'(u)\big|,\ \int_{\frac{3}{2}}^{\infty} (u-1)\,\big|d\varphi'(u)\big|, \tag{2.9}$$

$$\Big|\sin\frac{\beta\pi}{2}\Big|\int_0^{\infty}\frac{|\varphi(u)|}{u^{1+\alpha}}du,\ \int_0^1\frac{|\varphi(1-u)-\varphi(1+u)|}{u^{\alpha+1}}du, \tag{2.10}$$

and obtain the upper bounds.

For the first integral from (2.9) we get

$$\int_0^{\frac{1}{2}} u^{1-\alpha}\,\big|d\varphi'(u)\big| = \int_0^{\frac{1}{\sqrt{\delta}}} u^{1-\alpha}\,\big|d\varphi'(u)\big| + \int_{\frac{1}{\sqrt{\delta}}}^{\frac{1}{2}} u^{1-\alpha}\,\big|d\varphi'(u)\big|$$

$$= 2\delta^{\frac{r}{2}}\int_0^{\frac{1}{\sqrt{\delta}}} u^{1-\alpha}du + (2-r)(1-r)\int_{\frac{1}{\sqrt{\delta}}}^{\frac{1}{2}} u^{1-r-\alpha}du = O\bigg(\frac{1}{\delta^{1-\frac{r+\alpha}{2}}}\bigg),\quad r>2. \tag{2.11}$$

In view of the fact that the function $|u-1|^{1-\alpha}|d\varphi'(u)|$ is continuous on the segment $[\frac{1}{2},\frac{3}{2}]$ it is bounded on this segment. Thus

$$\int_{\frac{1}{2}}^{\frac{3}{2}} |u-1|^{1-\alpha}|d\varphi'(u)| = O(1). \tag{2.12}$$

Now we estimate the third integral from (2.9):

$$\int_{\frac{3}{2}}^{\infty}(u-1)|d\varphi'(u)| \le \int_{\frac{3}{2}}^{\infty} u|d\varphi'(u)|$$

$$= (2-r)(1-r)\int_{\frac{3}{2}}^{\infty} u^{1-r}du \le K,\quad r>2. \tag{2.13}$$

For first integral from (2.10) we get

$$\int_0^{\infty}\frac{|\varphi(u)|}{u^{1+\alpha}}du = \delta^{\frac{r}{2}}\int_0^{\frac{1}{\sqrt{\delta}}} u^{1-\alpha}du + \int_{\frac{1}{\sqrt{\delta}}}^{\infty} u^{1-r-\alpha}du$$

$$= O\bigg(\frac{1}{\delta^{1-\frac{r+\alpha}{2}}}\bigg),\quad r>2. \tag{2.14}$$

Let us estimate the second integral from (2.10). Similarly to formula (30) of [12], it can be shown that for the function φ, given by the relation (2.4), we have the equality

$$\int_0^1\frac{|\varphi(1-u)-\varphi(1+u)|}{u^{1+\alpha}}du = \int_0^1\frac{|\lambda(1-u)-\lambda(1+u)|}{u^{1+\alpha}}du + O(H(\alpha,\varphi)), \tag{2.15}$$

where $\lambda(u) = 1 - u^2$ and

$$H(\alpha, \varphi) = \big|\varphi(0)\big| + \big|\varphi(1)\big| + \int_0^{\frac{1}{2}} u^{1-\alpha}\big|d\varphi'(u)\big| + \int_{\frac{3}{2}}^{\infty} (u-1)\big|d\varphi'(u)\big|.$$

Since $\int_0^1 \frac{|\lambda(1-u)-\lambda(1+u)|}{u^{1+\alpha}}du = O(1)$, according to (2.11) and (2.12) from (2.15) we get

$$\int_0^1 \frac{|\varphi(1-u)-\varphi(1+u)|}{u^{1+\alpha}}du = O\Big(\frac{1}{\delta^{\frac{1-r-\alpha}{2}}}\Big), \quad r > 2. \tag{2.16}$$

Applying theorem 1 of [6] and taking into account relations (2.11)–(2.16) we show that the Fourier transform of the function φ of the form (2.2) is summable on the real axis, and the following estimate holds

$$A(\alpha, \varphi) = O\Big(\frac{1}{\delta^{\frac{1-r-\alpha}{2}}}\Big), \quad r > 2. \tag{2.17}$$

We now show the convergence of the integral $A(\alpha, \mu)$. Let us consider the following integrals

$$\int_0^{\frac{1}{2}} u^{1-\alpha}\,\big|d\mu'(u)\big|, \int_{\frac{1}{2}}^{\frac{3}{2}} |u-1|^{1-\alpha}\,\big|d\mu'(u)\big|, \int_{\frac{3}{2}}^{\infty} (u-1)\,\big|d\mu'(u)\big|, \tag{2.18}$$

$$\Big|\sin\frac{\beta\pi}{2}\Big| \int_0^{\infty} \frac{\big|\mu(u)\big|}{u^{1+\alpha}}du, \int_0^1 \frac{|\mu(1-u)-\mu(1+u)|}{u^{\alpha+1}}du, \tag{2.19}$$

and get the corresponding upper bounds.

Let us investigate the first integral from (2.18). Similarly, as in the proof of Lemma 1 from [13], we divide the segment $\big[0, \frac{1}{2}\big]$ into two parts: $\big[0, \frac{1}{\sqrt{\delta}}\big]$, $\big[\frac{1}{\sqrt{\delta}}, \frac{1}{2}\big]$. According to the inequality

$$2u^2 e^{-u^2} - e^{-u^2} + 1 \le 3u^2, \quad u \in \mathbb{R}, \tag{2.20}$$

and the fact that if $u \in \big[0, \frac{1}{\sqrt{\delta}}\big]$ then $\mu''(u) \le 0$, we can write

$$\int_0^{\frac{1}{\sqrt{\delta}}} u^{1-\alpha}|d\mu'(u)| = 2\delta^{\frac{r}{2}} \int_0^{\frac{1}{\sqrt{\delta}}} u^{1-\alpha}(2u^2e^{-u^2} - e^{-u^2} + 1)du$$

$$\le 6\delta^{\frac{r}{2}} \int_0^{\frac{1}{\sqrt{\delta}}} u^{3-\alpha}du \le \frac{K}{\delta^{2-\frac{r+\alpha}{2}}}. \tag{2.21}$$

Let us estimate the integral on the segment $\left[\frac{1}{\sqrt{\delta}}, \frac{1}{2}\right]$. From (2.5) we get

$$\mu''(u) = (1 - e^{-u^2} - u^2)r(r+1)u^{-r-2} + 4(-r)u(e^{-u^2} - 1)u^{-r-1}$$
$$+2u^{-r}(e^{-u^2} - 2u^2e^{-u^2} - 1). \tag{2.22}$$

One can verify that $\mu''(u) \le 0$. Further, according to (2.20) and, in addition, to the inequalities

$$e^{-u^2} + u^2 - 1 \le \frac{u^4}{2}, \quad 1 - e^{-u^2} \le u^2, \quad u \in \mathbb{R}, \tag{2.23}$$

we have

$$\int_{\frac{1}{\sqrt{\delta}}}^{\frac{1}{2}} u^{1-\alpha}|d\mu'(u)| \le r(r+1)\int_{\frac{1}{\sqrt{\delta}}}^{\frac{1}{2}} (u^2 + e^{-u^2} - 1)u^{-1-r-\alpha}du$$
$$+4r\int_{\frac{1}{\sqrt{\delta}}}^{\frac{1}{2}} (1 - e^{-u^2})u^{1-r-\alpha}du + 2\int_{\frac{1}{\sqrt{\delta}}}^{\frac{1}{2}} (2u^2e^{-u^2} - e^{-u^2} + 1)u^{1-r-\alpha}du$$
$$\le K\int_{\frac{1}{\sqrt{\delta}}}^{\frac{1}{2}} u^{3-r-\alpha}du \le K_1 + \frac{K_2}{\delta^{2-\frac{r+\alpha}{2}}}. \tag{2.24}$$

In view of (2.21) and (2.24) we find

$$\int_0^{\frac{1}{2}} u^{1-\alpha}|d\mu'(u)| = O\Big(1 + \frac{1}{\delta^{2-\frac{r+\alpha}{2}}}\Big). \tag{2.25}$$

By the same way as to estimate the second integral from (2.18), we get

$$\int_{\frac{1}{2}}^{\frac{3}{2}} |u-1|^{1-\alpha}|d\mu'(u)| = O(1). \tag{2.26}$$

To estimate the last integral from (2.18) we use (2.22) and the following inequalities

$$e^{-u^2} \le 1, \quad 1 - e^{-u^2} \le 1, \quad u^2e^{-u^2} \le 1, \quad u \in \mathbb{R}. \tag{2.27}$$

We obtain

$$\int_{\frac{3}{2}}^{\infty} (u-1)|d\mu'(u)| \le \int_{\frac{3}{2}}^{\infty} u|d\mu'(u)| \le K\int_{\frac{3}{2}}^{\infty} u^{-r+1}du \le K_1, \quad r > 2. \tag{2.28}$$

Analogously to the estimation (36) of [14], to estimate the first integral with (2.19) we divide the interval $[0, \infty)$ into three parts: $\left[0, \frac{1}{\sqrt{\delta}}\right]$,

$\left[\frac{1}{\sqrt{\delta}},1\right]$, $\left[1,\infty\right]$. According to the first inequalities from (2.23) and (2.27) we get

$$\int_0^{\infty}\frac{|\mu(u)|}{u^{1+\alpha}}du=\delta^{\frac{r}{2}}\int_0^{\frac{1}{\sqrt{\delta}}}\frac{e^{-u^2}+u^2-1}{u^{1+\alpha}}du$$

$$+\left(\int_{\frac{1}{\sqrt{\delta}}}^{1}+\int_1^{\infty}\right)(e^{-u^2}+u^2-1)u^{-1-r-\alpha}du\leq\frac{1}{2}\delta^{\frac{r}{2}}\int_0^{\frac{1}{\sqrt{\delta}}}u^{3-\alpha}du$$

$$+\frac{1}{2}\int_{\frac{1}{\sqrt{\delta}}}^{1}u^{3-r-\alpha}du+\int_1^{\infty}u^{1-r-\alpha}du\leq K_1+\frac{K_2}{\delta^{2-\frac{r+\alpha}{2}}}. \tag{2.29}$$

Let us estimate the second integral from (2.19). Similarly, as for function φ, the following relation holds

$$\int_0^1\frac{|\mu(1-u)-\mu(1+u)|}{u^{1+\alpha}}du=\int_0^1\frac{|\lambda(1-u)-\lambda(1+u)|}{u^{1+\alpha}}du+O(H(\alpha,\mu)), \tag{2.30}$$

where $\lambda(u)=e^{-u^2}+u^2$ and

$$H(\alpha,\mu)=\left|\mu(0)\right|+\left|\mu(1)\right|+\int_0^{\frac{1}{2}}u^{1-\alpha}\left|d\mu'(u)\right|+\int_{\frac{3}{2}}^{\infty}(u-1)\left|d\mu'(u)\right|.$$

Since $\int_0^1\frac{|\lambda(1-u)-\lambda(1+u)|}{u^{1+\alpha}}du=O(1)$, according to (2.25) and (2.28) from (2.30) we get

$$\int_0^1\frac{|\mu(1-u)-\mu(1+u)|}{u^{1+\alpha}}du=O\left(1+\frac{1}{\delta^{2-\frac{r+\alpha}{2}}}\right),\quad r>2. \tag{2.31}$$

Applying theorem 1 from the paper of Bausov [6] and taking into account estimates (2.29) and (2.31), we have

$$A(\alpha,\mu)=O\left(1+\frac{1}{\delta^{2-\frac{r+\alpha}{2}}}\right),\quad r>2. \tag{2.32}$$

Thus, we showed the convergence of integrals $A(\alpha,\varphi)$, $A(\alpha,\mu)$ and validity of the relation $A(\alpha,\mu)=o\left(A(\alpha,\varphi)\right)$. So equality (2.8) holds. Whereas an estimation (2.32) from (2.8) we receive

$$\mathcal{E}(W^r_\beta H^\alpha;W_\delta)_C=\frac{1}{\delta^{\frac{r}{2}}}\sup_{f\in W^r_\beta H^\alpha}\|f_\varphi\|_C+O\left(\frac{1}{\delta^{\frac{r+\alpha}{2}}}+\frac{1}{\delta^2}\right). \tag{2.33}$$

It is well known that Fourier series of the function $f_\varphi(x)$ (see, e.g., [15]) is of the form:

$$S[f_\varphi(x)]=\sum_{k=1}^{\infty}\varphi\left(\frac{k}{\sqrt{\delta}}\right)k^r(a_k\cos kx+b_k\sin kx).$$

From here, considering formulas (2.4) and (1.1) we have that

$$S[f_\varphi(x)] = \sum_{k=1}^{\infty} \frac{k^2}{\delta^{1-\frac{r}{2}}}(a_k \cos kx + b_k \sin kx)$$

$$= \frac{1}{\delta^{1-\frac{r}{2}}} f_0^{(2)}(x) = -\frac{1}{\delta^{1-\frac{r}{2}}} f''(x), \tag{2.34}$$

where f'' be the second derivative of function f.

Substituting (2.34) into (2.33), we obtain (2.3). Theorem is proved. □

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