# E.A. PONEZHA

Bogolyubov Institute for Theoretical Physics, Nat. Acad. of Sci. of Ukraine (14b, Metrologichna Str., Kyiv 03680, Ukraine)

# RELAXATION TIMES AND CORRELATION FUNCTIONS UNDER THE INFLUENCE OF CROSS-CORRELATED COLORED NOISES FOR THE MODEL OF RESONANT TUNNELING

UDC 519.814, 519.246

The effects of cross-correlated noises on the process of relaxation of fluctuations in the model of resonant tunneling, in which the noise sources due to incident flow intensity fluctuations and frequency fluctuations are assumed to exist, are considered. To characterize the dynamical behavior of the system, the normalized correlation functions and the associated relaxation times are calculated with the help of a projection-operator technique with regard for memory effects. The influence of noise intensities, their correlation times, and the strength of correlation between the noises on these functions has been analyzed. It is found that the strength of a cross-correlation between two noises can facilitate the intensity fluctuation decay. The behavior of the relaxation time with respect to that of the strength of noises can be characterized as a stochastic resonance phenomenon. It is shown that an enhancement of the self-correlation time of the intensity fluctuations accelerates the transition from the unstable state, while the growth of the self-correlation time of frequency fluctuations results in the retardation of the transition, thereby stabilizing the system.

Keywords: resonant tunneling, intensity correlation function, associated relaxation time, strength of cross-correlation, external colored noises.

# 1. Introduction

One of the fundamental problems in studies of the dynamics of nonequilibrium systems is the investigation of their behavior under the influence of noises of different nature. It is reasonable to suggest that there can be more than one source of noise in physical systems, for instance, internal thermal fluctuations and external random perturbations. The former are represented usually as an additive term in the Langevin equation and the latter are related to fluctuations of the external parameter as a multiplicative term. In particular, such treatment was justified for laser models [1–4].

In Ref. [5], it was supposed that fluctuations of some parameters can lead to noises of the additive, as well as multiplicative, character, which are

ISSN 0372-400Х. Укр. фіз. журн. 2015. Т. 60, № 7

not independent. They can have the common origin and, therefore, correlate with one another [5–7]. At the present time, the effects of correlation between noises of different nature are widely investigated within different models [8–11]. In the study of a behavior of unstable systems under the influence of noises, a much attention is paid to the investigation of noises with finite correlation times (colored noises), which allows one to construct, in some cases, a more realistic model for describing the system [4, 10–13].

An important aspect of the dynamical behavior of stochastic systems is the fluctuation decay depending on the system parameters and noises. As the dynamical characteristic of fluctuations in stochastic unstable systems, the correlation functions in a nonequilibrium steady state and the associated relaxation times are often used [4, 8, 10, 12–15].

<sup>©</sup> E.A. PONEZHA, 2015

In the present paper, we study the influence of colored noises of different nature on the process of fluctuation decay in a model system describing the resonant tunneling of electrons in double-barrier nanostructures. It is known that such structures can strongly enhance the transmission coefficient under the resonance condition, which gives opportunity to fabricate resonant tunneling diodes on their base, which have great prospects for their use in various electronic devices [16]. That is why studying the noise effects in such structures in regions of instability is of importance.

In our model of tunneling process [17], the instability takes place during the transition from the state with low tunneling effectiveness to the state with high one. In Refs. [18, 19, 21], we have considered the influence of white noise on the dynamics of this system. In Ref. [20], the influence of colored noise on a mean first passage time close to an instability point has been investigated. In Ref. [21], the problem of mutual influence of the amplitude and phase noises in the incident electron flow on the correlation function of the outgoing flow intensity has been considered by means of the linearization of the evolution equations. In this work, we investigate how the considered tunneling system is affected by the noise caused by fluctuations in the incident flow intensity and by the noise originated from random violations of the resonant frequency detuning. We assume that both noises have finite correlation times and can correlate with each other. The dynamical behavior of the system under the influence of these noises will be described with the help of the correlation function of the outgoing flow intensity and the associated relaxation time.

In Sec. 2, the general expressions used in the calculation of the correlation functions and the associated relaxation times are given. The stochastic model under consideration is represented in Sec. 3. Sec. 4 contains the calculations of the correlation functions and the relaxation times. Conclusions are given in Sec. 5.

# 2. Correlation Function and Associated Relaxation Time

The general Langevin equation with two noise sources can be written as

$$\dot{x} = f(x) + g_1(x)p(t) + g_2(x)q(t), \tag{1}$$

where f(x) is the deterministic term of a nonlinear process, and p(t) and q(t) are Gaussian colored noises

with zero mean  $\langle p(t) \rangle = \langle q(t) \rangle = 0$ , and the correlation functions

$$\langle p(t)p(t')\rangle = \frac{D}{\tau_1} \exp(|t - t'|/\tau_1),$$

$$\langle q(t)q(t')\rangle = \frac{Q}{\tau_2} \exp(|t - t'|/\tau_2),$$

$$\langle p(t)q(t')\rangle = \langle q(t)p(t')\rangle \frac{\lambda\sqrt{DQ}}{\tau_0} \exp(|t - t'|/\tau_0).$$

Here, D and Q are noise intensities,  $g_1(x)$  and  $g_2(x)$ are, in the general case, nonlinear functions of x,  $\tau_1$ and  $\tau_2$  are self-correlation times of two noises, respectively,  $\tau_0$  is the time of their mutual cross-correlation, and  $0 \leq \lambda \leq 1$  defines the correlation strength between these noises.

The approximate Fokker–Planck equation corresponding to Eq. (1) with the use of Novikov's theorem [22] and Fox's approximation [2] is given by the expression [7, 11]

$$\frac{\partial P(x,t)}{\partial t} = -\frac{\partial}{\partial x}A(x)P(x,t) + \frac{\partial^2}{\partial x^2}B(x)P(x,t), \quad (2)$$

where

$$A(x) = f(x) + G'(x)G(x)$$
 and  $B(x) = [G(x)]^2$ .

G(x) with regard for the self-correlation times of noise components and their cross-correlation time for small correlation times can be written as [6, 10, 11, 23]

$$G(x) = \left[\frac{D}{1+a\tau_1}[q_1(x)]^2 + \frac{2\lambda\sqrt{DQ}}{1+a\tau_0}g_1(x)g_2(x) + \frac{Q}{1+a\tau_2}[q_2(x)]^2\right]^{1/2}$$

with a = -f'(x) (index ' stands for the derivative with respect to x). The stationary probability density is given by

$$P_{\rm st} = \frac{N}{G(x)} \exp\left[\int^x \frac{A(x')}{G(x')} dx'\right].$$
(3)

Here, N is the normalization coefficient,  $N^{-1} = \int_0^\infty P_{\rm st}(x) dx$ . The stationary normalized correlation function of the variable x is written as [12]

$$C(s) = \frac{\langle \delta x(t+s)\delta x(t) \rangle_{\rm st}}{\langle (\delta x)^2 \rangle_{\rm st}},\tag{4}$$

ISSN 0372-400X. Укр. фіз. журн. 2015. Т. 60, № 7

where  $\delta x(t) = x(t) - \langle x(t) \rangle$ , and C(s) is a measure of the correlation between fluctuations at times t and t + s.

Performing the Laplace transformation of the function C(s), we obtain

$$\tilde{C}(w) = \int_{0}^{\infty} \exp(-ws)C(s)ds.$$
(5)

The associated relaxation time is generally defined by the expression

$$T_C = \int_0^\infty \frac{C(s)}{\langle (\delta x)^2 \rangle_{\rm st}} ds.$$
(6)

The use of the projection-operation method leads to a continued-fraction expansion for the quantity  $\tilde{C}(w)$ [12]. In the zeroth approximation where the memory effects are completely neglected, C(s) relaxes exponentially,  $C(s) = \exp(-\gamma_0 s)$ , with a relaxation time

$$T_C = \gamma_0^{-1} = \frac{\langle (\delta x)^2 \rangle_{\text{st}}}{\langle G(x) \rangle_{\text{st}}}.$$
(7)

By the truncation of the continued-fraction expansion in the first order, the following formula for  $T_C$  was obtained [12]:

$$T_C = \left[\gamma_0 + \frac{\eta_1}{\gamma_1}\right]^{-1}.$$
(8)

Here,

$$\eta_1 = \frac{\langle G(x)A'(x)\rangle}{\langle (\delta x)^2 \rangle_{\rm st}} + \gamma_0^2,$$
  
$$\gamma_1 = -\frac{\langle G(x)[A'(x)]^2 \rangle}{\eta_1 \langle (\delta x)^2 \rangle_{\rm st}} + \frac{\gamma_0^3}{\eta_1} - 2\gamma_0,$$

where A'(x) is the derivative of A with respect to x.

The first-order approximation for  $\hat{C}(w)$  is determined by the formula [12]

$$\tilde{C} = \frac{w + \gamma_1}{(w + \gamma_0)(w + \gamma_1) + \eta_1}.$$
(9)

Performing the inverse Laplace transformation for  $\tilde{C}(w)$ , it was shown [12] that the correlation function can be written as a sum of two exponentials

$$C(s) = (1 - \Delta) \exp(-\Gamma_1 s) + \Delta \exp(-\Gamma_2 s), \qquad (10)$$

ISSN 0372-400Х. Укр. фіз. журн. 2015. Т. 60, № 7

with

$$\Delta = \frac{\gamma_1 - \Gamma_2}{\Gamma_1 - \Gamma_2},$$

and

$$\Gamma_{1,2} = \frac{1}{2} [\gamma_0 + \gamma_1 \pm \sqrt{(\gamma_1 - \gamma_0)^2 - 4\eta}]$$

The parameter  $\Delta$  defines the importance of the memory effects. If the value  $\Delta$  is close to unity, then we can use the zeroth order.

# 3. Stochastic Model for Resonant Tunneling Process

We consider the model of the tunneling system at a resonance described in Ref. [17]. The model considers the open coherent double-barrier tunneling system, where an electron flow described by a wave packet is injected from the left and passes through the tunneling system only once. In Ref. [17], the following differential equation for the complex amplitude D of the wave packet outgoing of the tunneling system was obtained:

$$\frac{D}{dt} = (iz - 1)D - i|D|^2D + D_0,$$
(11)

where  $D_0$  is the dimensionless amplitude of the wave function in the incoming flow,  $z = L(k-k_r)$  is the parameter proportional to the detuning of the electron wave vector k from the resonance value  $k_r$ , L is the inverse half-width of the resonance level in the k-space, which is connected with a half-width in the energy space by the relation  $\delta E_{1/2} = 2h\nu = \hbar k_r/(m^*L)$ ,  $m^*$ is the effective electron mass, and  $t = \nu t'$  is the dimensionless time.

Going to the intensities  $D_0^2 = I_0$ ,  $D^2 = I$ , we obtain the following relation between the intensities of the incident  $I_0$  and outgoing I electron flows in the stationary case:

$$I_0 = I[1 + (z - I)^2].$$
(12)

Assuming that the phase of the wave function is not changed in the process of tunneling in a vicinity of the stationary state, the following approximate differential equation can be obtained [19]:

$$\frac{dI}{dt} = -I + \frac{I_0}{1 + (z - I)^2}.$$
(13)



Fig. 1. Relaxation time  $T_C$  as a function of the mean value of the incident flow intensity  $I_0$  for the various values of noise strength D and the parameters  $\lambda = 0.1$ , Q = 0.1,  $\tau_1 = \tau_2 =$  $= \tau_0 = 0.1$ 



Fig. 2. Dependence of the relaxation time  $T_C$  on the noise strength D for the various values of incident flow intensity  $I_0$  with the parameters  $\lambda = 0.8$ , Q = 0.1,  $\tau_1 = \tau_2 = \tau_0 = 0.1$ 

The system exhibits the bistability as a function of the parameter  $I_0$ , if the parameter z satisfies the inequality  $z > \sqrt{3}$  [17].

We will consider the intensity of the incident flow  $I_0$  as a stochastic quantity,  $I_0 = \langle I_0 \rangle + p(t)$ , where  $\langle I_0 \rangle$  is the value of incident flow intensity in the absence of noise, and p(t) are fluctuations of the intensity described by a Gaussian noise with zero mean and the correlation function  $\langle p^*(t)p(t) \rangle = \frac{D}{\tau_1} \exp\left(-\frac{|t-t'|}{\tau_1}\right)$ , where D is the noise strength, and  $\tau_1$  is its self-correlation time.

We suppose that there are also fluctuations of the detuning parameter  $z = \langle z \rangle + q(t)$ , where q(t) is a noise term with zero mean and the correlation function  $\langle q^*(t)q(t) \rangle = \frac{Q}{\tau_2} \exp\left(-\frac{|t-t'|}{\tau_2}\right)$ , with Q being the noise intensity, and  $\tau_2$  the time of its self-correlation.

Then we come to the Langevin equation

$$\frac{dI}{dt} = -I + \frac{I_0}{1 + (z - I)^2} + g_1(I)p(t) + g_2(I)q(t) \quad (14)$$

with  $g_1 = \frac{1}{1+(z-I)^2}$ ,  $g_2 = \frac{2I(I-z)}{1+(z-I)^2}$  obtained by expanding the nonlinear terms to the first order in I.

In order to consider the self-correlation times of noises and their mutual cross-correlation time, we have to go to the Fokker–Planck equation given by expression (2). In the numerical calculations of the intensity correlation functions C(s) and the associated relaxation times  $T_C$ , we use expressions (10) and (8), respectively.

### 4. Calculation of the Intensity Correlation Functions and Relaxation Times

We now analyze the behavior of the relaxation time  $T_C$  as a function of the parameters of the process and noises. The relaxation time gives the information about the time scale of the fluctuation evolution.

In Fig. 1, the dependence of the relaxation time  $T_C$  on the mean value of the incoming intensity  $I_0$  for various values of noise strength D is represented.

It can be seen that, for the small noise strength (D = 0.1), the maximum of  $T_C$  (that implies the slowing down of the process) takes place close to the point of the transition from the state with low tunneling effectiveness to the state with high effectiveness  $(I_0 = 5.2)$ . As the noise strength D increases, the maximum shifts to larger values of  $I_0$ , thereby increasing the stability of the system.

In Fig. 2, the relaxation time  $T_C$  as a function of the noise strength D is plotted for various mean intensities. We see that if the noise is absent (D = 0),  $T_C$ has a maximum close to the deterministic transition point (which occurs at  $I_0 = 5.0$  for this set of parameters). Increasing D results in a shift of the transition point to larger values of  $I_0$  similar to Fig. 1. In this region,  $T_C$  as a function of D grows with D, passes through the maximum, and decreases for larger D. Such a behavior is the evidence of a stochastic resonance phenomenon.

ISSN 0372-400X. Укр. фіз. журн. 2015. Т. 60, № 7



Fig. 3. Relaxation time  $T_C$  as a function of the crosscorrelation strength  $\lambda$  for the various values of mean incident flow intensity  $I_0$  with the parameters D = 1, Q = 0.1,  $\tau_1 = \tau_2 = \tau_0 = 0.1$ 

The relaxation time  $T_C$  as a function of the correlation strength  $\lambda$  between two noises is depicted in Fig. 3 for various values of the mean incident flow intensity  $I_0$ .

It can be seen that  $T_C$  monotonically decreases, as  $\lambda$  increases. For a fixed value of  $\lambda$ ,  $T_C$  has the largest value near the threshold ( $I_0 = 6$ ) and the smallest values in the regions below ( $I_0 = 5.2$ ) and above the threshold ( $I_0 = 7$ ).

Now, we analyze how the memory effects influence the relaxation time.  $T_C$  as a function of the selfcorrelation time  $\tau_1$  of the intensity fluctuations p(t)for various values of the mean incident flow intensity  $I_0$  is shown in Fig. 4. For the parameters used, the transition from the low to high state occurred near the point  $I_0 = 5.6$ .

It follows from Fig. 4 that the dependence of  $T_C$  on  $\tau_1$  has the most pronounced character with the maximum at the definite value of  $\tau_1$  only in a vicinity of the transition point. As  $\tau_1$  increases, the maximum of  $T_C$  can be observed at smaller values of the incident intensity. Therefore, we can conclude that the transition is facilitated with the growth of the correlation time of noise p(t).

In Fig. 5, we can see how this dependence for  $I_0 = 5.7$  is changed with the cross-correlation strength  $\lambda$ .

It can be noted that the maximum of  $T_C$  shifts with respect to the cross-correlation strength  $\lambda$  in a different way. For the moderate values of cross-correlation intensities  $\lambda$  ( $\lambda = 0.5$ –0.7), the maximum takes place at the larger values of  $\tau_1$  than for small ones ( $\lambda = 0.3$ ), as well as for large ( $\lambda = 0.8$ ) ones.

ISSN 0372-400Х. Укр. фіз. журн. 2015. Т. 60, № 7



Fig. 4. Dependence of  $T_C$  on the self-correlation time  $\tau_1$  of p(t) noise for the various values of mean incident flow intensity  $I_0$  with the parameters  $\lambda = 0.5$ , D = 0.5, Q = 0.1,  $\tau_2 = \tau_0 = 0.1$ 



Fig. 5. Dependence of  $T_C$  on the self-correlation time  $\tau_1$  of p(t) noise close to the transition point  $(I_0=5.7)$  for the various cross-correlation strengths  $\lambda$  and with the parameters D = 1, Q = 0.1,  $\tau_1 = \tau_2 = \tau_0 = 0.1$ 



Fig. 6. Relaxation time  $T_C$  as a function of the mean incident intensity for the various correlation times of noises with the parameters  $\lambda = 0.5$ , D = 0.5, Q = 0.1



Fig. 7. Correlation functions C(s) for the various mean values of incident intensity  $I_0$  when parameters values were  $\lambda = 0.1$ ,  $D = 0.5, Q = 0.1, \tau_1 = \tau_2 = \tau_0 = 0.1$ 



Fig. 8. Correlation function C(s) in the vicinity of the transition point  $(I_0 = 5.8)$  for the various values of cross-correlation strength  $\lambda$  with the parameters D = 0.5, Q = 0.1,  $\tau_1 = \tau_2 = \tau_0 = 0.1$ 



**Fig. 9.** Correlation function C(s) in a vicinity of the transition point  $(I_0 = 5.8)$  for the various noises strengths Q of the noise q(t) with the parameters D = 0.5,  $\lambda = 0.5$ ,  $\tau_1 = \tau_2 = \tau_0 = 0.1$ 

662



Fig. 10. Correlation functions for the various values of noise p(t) correlations times  $\tau_1$  for the process being before the threshold) with the parameters D = 0.5, Q = 0.1,  $\lambda = 0.5$ ,  $\tau_2 = \tau_0 = 0.01$ 



Fig. 11. Correlation functions for the various values of noise p(t) correlations times  $\tau_1$  for the process being after the threshold with the parameters D = 0.5, Q = 0.1,  $\lambda = 0.5$ ,  $\tau_2 = \tau_0 = 0.01$ 

In Fig. 6, we plot the function  $T_C$  vs  $I_0$  for the various values of self-correlation times  $\tau_1$  and  $\tau_2$  of noises p(t) and q(t), respectively, and of their crosscorrelation time  $\tau_0$ . It can be observed that an increase in the correlation times of noises leads to a shift of the maximum of the relaxation time  $T_C$ . When the correlation times of noises have minimum values  $(\tau_1 = \tau_2 = \tau_0 = 0.01)$ , the maximum of  $T_C$  is situated at  $I_0 = 5.8$  (see the solid curve). The growth of the correlation time  $\tau_1$  of fluctuations of the intensity to the value  $\tau_1 = 0.5$  results in the shift of the maximum to the left to the value of  $I_0 = 5.6$ . The increase of  $\tau_2$  (self-correlation time of frequency fluctuations) to 0.5 causes the shift of the maximum to the right to the value of  $I_0 = 6.1$ . On the other hand, the growth

ISSN 0372-400Х. Укр. фіз. журн. 2015. Т. 60, № 7

of the mutual correlation time  $\tau_0$  to the value of 0.5 does not result in a shift of the curve.

The behavior of the normalized intensity correlation function C(s) for the various parameters of the process and noises is represented in Figs. 7–12. In Fig. 7, the correlation functions of the process are plotted in the cases where the value of mean incident intensity for the given parameters was below the transition point  $(I_0 = 5)$  in its vicinity  $(I_0 = 5.5)$  and above it  $(I_0 = 5.8, 6.0)$ .

It can be seen in Fig. 7 that C(s) decays most rapidly above the transition  $(I_0 = 6)$  and below it  $(I_0 = 5)$ . But, close to the transition  $(I_0 = 5 \div 5.8)$ , its decay slows. Such a behavior indicates that the fluctuations increase as the process approaches the instability point resulting in an increase of the area under the C(s) curve, which defines the relaxation time.

In Fig. 8, the dependence of C(s) on the degree of the correlation between the noises  $\lambda$  is shown. We found that the C(s) decay is slowing, as  $\lambda$  increases from the value 0.2 to 0.4 (the area under the curve increases), and then accelerates for larger values of  $\lambda$ (the area under the curve contracts). Thus, the correlation strength  $\lambda$  between two noises can accelerate the fluctuation decay.

Let us see how the intensity Q of the frequency noise q(t) affects the behavior of the intensity correlation function C(s). This is demonstrated in Fig. 9, where it can be observed that the smallest decay of the correlation function (an increase of fluctuations is a characteristic of the transition) takes place at the definite value of the noise intensity Q. This means the existence of a stochastic resonance phenomena for this noise similar to the p(t) noise.

The influence of the self-correlation times of noises and their mutual cross-correlation time on the behavior of the intensity correlation function depends strongly on the acting point of the process with respect to the point of transition. It is illustrated in Figs. 10–11, where the correlation functions for the various values of p(t) noise correlation times  $\tau_1$  are depicted for the process being below the threshold (Fig. 10) and above it (Fig. 11).

Below the threshold, we can see that the rate of correlation function decay becomes slower, as the noise correlation time  $\tau_1$  increases. On the contrary, above the threshold (Fig. 11), the correlation function decay increases with  $\tau_1$ . Therefore, in the pre-threshold

ISSN 0372-400Х. Укр. фіз. журн. 2015. Т. 60, № 7

region, the growth of the noise correlation time increases fluctuations; while, in the after-threshold region, it results in their decrease.

We have found that the dependences of the correlation function on the self-correlation time  $\tau_2$  of the frequency noise and the cross-correlation time  $\tau_0$  have a similar character.

#### 5. Conclusion

By means of numerical calculations, we have analyzed the influence of colored cross-correlated noises on the outgoing flow intensity correlation function C(s) and the associated relaxation time  $T_C$  within a model of resonant tunneling. It has been found that the stability of the system increases with the strength Dof noise that models fluctuations of the incident flow intensity  $I_0$ . There exists the dependence of the relaxation time on the intensity of the noises that is typical of the stochastic resonance phenomenon. We have shown that  $T_C$  monotonically decreases, as the correlation strength  $\lambda$  between two noises increases. It has been shown also that the cross-correlation strength  $\lambda$  can facilitate the fluctuation decay. We have obtained that the growth of the self-correlation time  $\tau_1$ of the intensity fluctuation noise  $\tau_1$  results in a shift of the threshold of the transition to the high state to lower values of the incident intensity  $I_0$  with respect to the deterministic case, while the increase of the correlation time  $\tau_2$  of frequency fluctuations shifts the threshold to the larger values of  $I_0$ . Thus, the time  $\tau_1$ growth accelerates the transition; on the other hand, the stability of the system increases, when  $\tau_2$  grows.

- S. Zhu, A.W. Yu, and R. Roy, Phys. Rev. A 34, 4333 (1986).
- 2. R.F. Fox and R. Roy, Phys. Rev. A 35, 1838 (1987).
- M. Aguado and M. San Miguel, Phys. Rev. A 37, 450 (1988).
- M. Aguado, E. Hernandez-Garcia, and M. San Miguel, Phys. Rev. A 38, 5670 (1988).
- 5. A. Fulinski and T. Telejko, Phys. Lett. A 152, 11 (1991).
- Wu Da-jin, Cao Li, and Ke Sheng-zhi, Phys. Rev. E 50, 2496 (1994).
- 7. Ya Jia and Jia-rong Li, Phys. Rev. Lett. 78, 994 (1997).
- D. Mei, C. Xie, and Li Zhang, Phys. Rev. E 68, 051102 (2003).
- Han Li-Bo, Cao Li, Wu Da-Jin, and Wang Jun, Commun. Theor. Phys. (Beijing, China) 42, 59 (2004).
- 10. Ping Zhu, Eur. Phys. J. B 55, 447 (2007).
- C-J. Wang, Q. Wei, and B-C. Mei, Phys. Lett. A **372**, 2176 (2008).

- A. Hernandez-Machado, M. San Miguel, and J.M. Sancho, Phys. Rev. A 29, 3388 (1984).
- A. Hernandez-Machado, J. Casademunt, M.A. Rodriguez, L. Pesquera, and M. Noriega, Phys. Rev. A 43, 1744 (1991).
- M. Noriega, L. Pesquera, and M.A. Rodriguez, Phys. Rev. A 43, 4008 (1991).
- M. Noriega, L. Pesquera, and M.A. Rodriguez, J. Casademunt, Phys. Rev. A 44, 2094 (1991).
- V.V. Mitin, V.A. Kochelap, and M.A. Stroscio, *Introduction to Nanoelectronics. Science, Nanotechnology, Engineering, and Applications* (Cambridge Univ. Press, Cambridge, 2008).
- A.S. Davydov and V.N. Ermakov, Physica D 28, 168 (1987).
- V.N. Ermakov and E.A. Ponezha, Metallofiz. Nov. Techn. 30, 585 (2008).
- 19. E.A. Ponezha, Ukr. J. Phys. 55, 244 (2010).
- V.N. Ermakov and E.A. Ponezha, Metallofiz. Nov. Techn. 33, 45 (2011).
- 21. E.A. Ponezha, Metallofiz. Nov. Techn. 36, 713 (2014).
- 22. E.A. Novikov, JETP 47, 1919 (1964).
- 23. P. Jung and P. Hänggi, J. Opt. Soc. Am. B 5, 979 (1988).

Received 05.12.14

#### О.О. Понежа

## ЧАСИ РЕЛАКСАЦІЇ ТА КОРЕЛЯЦІЙНІ ФУНКЦІЇ ПІД ВПЛИВОМ КРОС-КОРЕЛЬОВАНИХ ШУМІВ У МОДЕЛІ РЕЗОНАНСНОГО ТУНЕЛЮВАННЯ

#### Резюме

Розглянуто вплив крос-корельованих шумів на процес релаксації флуктуацій у моделі резонансного тунелювання, в якій передбачалась наявність джерел шумів, пов'язаних з флуктуаціями інтенсивності та частоти в падаючому потоці. Для характеристики динамічної поведінки системи розраховувалися нормовані функції кореляції і асоційовані часи релаксації за допомогою методу проекційного оператора з урахуванням ефектів пам'яті. Проаналізовано вплив на ці функції інтенсивностей шумів, їх часів кореляції, а також сили взаємної кореляції і часу крос-кореляції. Було знайдено, що сила кореляції між двома шумами може прискорити розпад флуктуацій інтенсивності. Залежність часу релаксації від інтенсивності шумів може бути охарактеризована як стохастичний резонанс. Було показано, що збільшення часу кореляції флуктуацій інтенсивності прискорює перехід з нестабільного стану, у той час як ріст часу кореляції флуктуацій частоти затримує перехід, тим самим збільшуючи стабільність системи.

#### Е.А. Понежа

### ВРЕМЕНА РЕЛАКСАЦИИ И КОРРЕЛЯЦИОННЫЕ ФУНКЦИИ ПОД ВЛИЯНИЕМ КРОСС-КОРРЕЛИРОВАННЫХ ШУМОВ В МОДЕЛИ РЕЗОНАНСНОГО ТУННЕЛИРОВАНИЯ

Резюме

Рассмотрено влияние кросс-коррелированных шумов на процесс релаксации флуктуаций в модели резонансного туннелирования, в которой предполагалось наличие источников шумов, связанных с флуктуациями интенсивности и частоты в падающем потоке. Для характеристики динамического поведения системы рассчитывались нормированные функции корреляции и ассоциированные времена релаксации с помощью метода проекционного оператора с учетом эффектов памяти. Проанализировано влияние на эти функции интенсивностей шумов, их времен корреляции, а также силы взаимной корреляции и времени кросскорреляции. Было найдено, что сила корреляции между двумя шумами может ускорить распад флуктуаций интенсивности. Зависимость времени релаксации от интенсивности шумов может быть охарактеризована как стохастический резонанс. Было показано, что увеличение времени корреляции флуктуаций интенсивности ускоряет переход из нестабильного состояния, в то время как рост времени корреляции флуктуаций частоты задерживает переход, тем самым увеличивая стабильность системы.