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CAN GLOBULAR CLUSTERS IN THE GALAXY BE CLASSIFIED BY THE VELOCITY ANISOTROPY PARAMETER?

Classification problems of globular star clusters in the Galaxy have been briefly discussed. The values of the velocity anisotropy parameter are calculated for 36 globular clusters, by using relatively new data for their apparent surface density. Those values are grouped into three classes. The correlation of the examined parameter with other parameters of globular clusters is analyzed.

Keywords: globular star clusters, globular cluster classification, velocity anisotropy parameter.

Till now, 157 globular star clusters (GSCs) were revealed in the Galaxy, and there is a large bank of observation data for them, which have been collected for a long time (see, e.g., catalog [1] and references therein). A lot of works are devoted to theoretical aspects of the origin and evolution of GSCs. Nevertheless, the problem of GSC classification still remains open, although some of well-known authors tried to solve it.

In order to understand the current state of the problem of GSC classification, it is necessary at first to present here, at least briefly, the main results obtained in this area. First of all, the proposition made by Shapley and Sawyer [2] should be mentioned. The cited authors were first who proposed to classify GSCs according to the degree of apparent star concentration toward the cluster center. They introduced 12 classes of concentration toward the center and denoted them using Roman numerals. For this purpose, they firstly created an almost uniform series of photos for 103 GSCs. Class I included clusters with the highest concentration, and class XII is characterized by the lowest concentration toward the center. Note that the cited authors did not give real values for the degree of physical star concentration, so that it is difficult to apply this classification in practice. Moreover, there is no correlation between the degree of star concentration toward the cluster center and other known physical characteristics of GSCs.

Further researches in the domain of GSC classification were described rather completely in the work by Kukarkin [3], so that we will not dwell on them. In work [3], the author proposed, for the first time, the concept of the "GSC index of richness" and denoted this quantity as IR. It is evident that this concept also means, to some extent, the concentration classification. In work [3], the values of IR index varied from 0 to 1. As a result, it was found that the index of richness correlates well with the absolute value of the cluster and is weakly sensitive to the gradient of the surface star density. Thus, Kukarkin managed to find a linear equation describing the relation between the IR index and the integrated absolute value of the cluster. However, this index has not been used in GSC catalogs till now.

A purely mathematical approach to the problem of classification in work [4] should also be mentioned. The authors of this paper applied a cluster data analysis and proposed some variants of the GSC classification in the space of various astrophysical parameters. In work [4], the distributions of 100 GSCs in our Galaxy and 167 clusters in the M31 galaxy were considered in the three-dimensional coordinate space $[(B-V)_0, (U-B)_0, M_V]$. Following this way, the cited authors managed to identify certain clusters that are not isolated, but are continuously transforming into one another. As one can see, this approach is also difficult to be applied in practice and does not give a solution of the GSC classification problem.

This circumstance forces the researchers to analyze some other GSC parameters. The velocity anisotropy parameter can be one of them, in principle. Various authors define it differently: as a simple ratio between the average values of the kinetic energy component in the radial and transverse motions, $2\langle T_r\rangle / \langle T_\perp\rangle$, or as the quantity $1-\sigma_t^2/\sigma_r^2$, where σ_r^2 and σ_t^2 are the radial and transverse, respectively, components of the velocity dispersion. Below, as the velocity anisotropy parameter, we take the quantity [5, 6]

$$A = \frac{2\,\overline{\Pi^2} - \overline{T^2}}{\overline{\Pi^2}}, \quad (1)$$

where $\overline{\Pi^2}$ and $\overline{T^2}$ are the radial and transverse, respectively, components of the velocity dispersion. To begin with, we assume that the parameter A is constant for the whole system, although, actually, this quantity can be a function of the distance from the cluster center. This parameter is of interest, first of all, from the viewpoint of its possible dependences on other physical GSC characteristics, although it is of interest *per se.* The anisotropy parameter is convenient, when analyzing the stages of the cluster evolution, because every stage is characterized by a definite structure, as well as by the density and velocity distributions. At the transition from the previous stage to the next one, the structure becomes smoother and tends to a more isotropic velocity distribution (unlike a radially elongated distribution at the early stages). The value of the velocity anisotropy parameter $A = 0$ corresponds to the spherical velocity distribution (mainly at the late evolution stage). The value $A = 2$ corresponds to a radially elongated distribution (at a rather early stage of evolution).

In order to determine the velocity anisotropy parameter for a GSC from the observation data obtained for the apparent surface density, let us consider a stationary model with the spherical symmetry. It is easy to derive the equation

$$\frac{d}{dr}\left[\frac{r^2}{D}\frac{d}{dr}\left(D\overline{\Pi^2}\right) + \left(2\overline{\Pi^2} - \overline{T^2}\right) r\right] = -4\pi G D r^2, \quad (2)$$

by excluding the potential from the hydrodynamic equation with the help of the Poisson equation. In Eq. (2), the dimensionless density D was introduced. If we introduce a new variable $x = 4\pi G r/\overline{\Pi^2}$, then Eq. (2) takes a relatively simple form

$$\frac{d}{dx}\left(\frac{x^2}{D}\frac{dD}{dx}\right) = -Dx^2 - A. \quad (3)$$

By solving Eq. (3) numerically, we obtain the dependences of D and $D' = dD/dx$ on x, with the dimensionless density D being an explicit function of the anisotropy parameter. Equation (3) was solved for various A-values from the interval $[0, 2]$. Knowing the D- and D'-values, we can change to the surface density, i.e. to the projection of a spatial density onto the tangent plane. Then we calculate the surface density by the formula

$$F(r) = -A \int\limits_{r/\alpha}^{\infty} \sqrt{x^2 - \left(\frac{r}{\alpha}\right)^2} D'(x)\, dx. \quad (4)$$

After having calculated the surface density $F(r)$ for the given anisotropy parameter values, we compare the obtained F-values with the observed surface densities for specific systems and choose the optimal A-value. The scaling factors are determined by minimizing the difference between the observed and theoretical densities. On the basis of the physical nature of this problem, it is convenient to apply a rather reliable symplectic method for the minimization of the function

$$f = \sum_{i=1}^{N}\left[-A \int\limits_{r_i/\alpha}^{\infty} \sqrt{x^2 - \left(\frac{r}{\alpha}\right)^2} D'(x)\, dx - F_0\right]^2 \quad (5)$$

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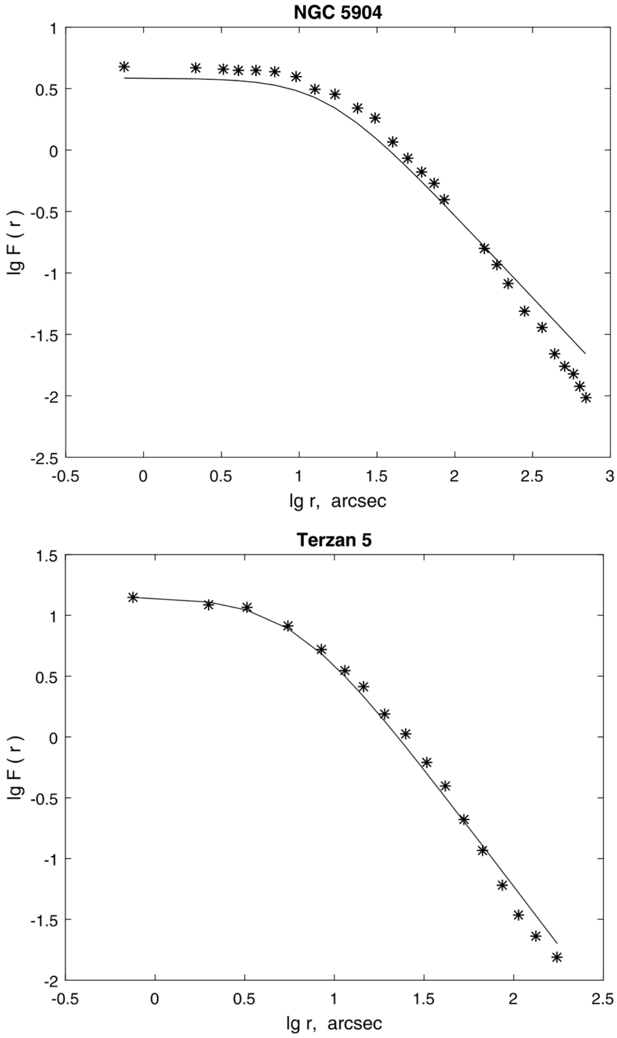
with respect to the parameters α, A, and F_0 (see also work [5]). Here, N is the number of circular zones counted. Then the value of the anisotropy parameter that gives a minimum f-value is adopted as the most probable value of A. The calculation procedure described above was implemented numerically for the first time and with a good accuracy by Ashurov [6]. In order to verify our algorithm and the calculation method, we preliminary repeated calculations of work [5] for 10 GSCs and got a good confirmation of the results. The algorithm was used to calculate the value of the parameter A for 36 GSCs. The calculation results are quoted in Table 1.

Table 1. **Calculation results for the anisotropy parameter A**

No.	GSC name	R_{Gal}, kpc	[Fe/H]	M_V	A
1	NGC 104	7.4	–0.72	–9.42	0.20
2	NGC 288	12.0	–1.32	–6.75	0.28
3	NGC 1261	18.1	–1.27	–7.80	1.80
4	AM 1	124.6	–1.70	–4.73	1.80
5	Eridanus	95.0	–1.43	–5.13	1.72
6	NGC 1851	16.6	–1.18	–8.33	0.04
7	NGC 1904	18.8	–1.60	–7.86	0.72
8	NGC 2419	89.9	–2.15	–9.42	0.96
9	NGC 2808	11.1	–1.14	–9.39	1.84
10	Pal 3	95.7	–1.63	–5.69	1.64
11	Pal 4	111.2	–1.41	–3.11	1.88
12	NGC 4147	21.4	–1.80	–6.17	1.80
13	NGC 4590	10.2	–2.23	–7.37	0.00
14	NGC 5024	18.4	–2.10	–8.71	0.50
15	NGC 5272	12.0	–1.50	–8.88	0.04
16	NGC 5466	16.3	–1.98	–6.98	0.00
17	NGC 5824	25.9	–1.91	–8.85	0.00
18	NGC 5904	6.2	–1.29	–8.81	0.60
19	Pal 14	71.6	–1.62	–4.80	1.64
20	NGC 6121	5.9	–1.16	–7.19	0.00
21	NGC 6205	8.4	–1.53	–8.55	0.88
22	NGC 6229	29.8	–1.47	–8.06	0.62
23	NGC 6218	4.5	–1.37	–7.31	0.04
24	NGC 6254	4.6	–1.56	–7.48	0.60
25	NGC 6266	1.7	–1.18	–9.18	0.80
26	NGC 6341	9.6	–2.31	–8.21	1.51
27	Terzan 5	1.2	–0.23	–7.42	0.20
28	NGC 6626	2.7	–1.32	–8.16	0.00
29	NGC 6715	18.9	–1.49	–9.98	0.08
30	NGC 6723	2.6	–1.10	–7.83	0.08
31	NGC 6809	3.9	–1.94	–7.57	0.12
32	NGC 6864	14.7	–1.29	–8.57	0.64
33	NGC 6934	12.8	–1.47	–7.45	0.04
34	NGC 6981	12.9	–1.42	–7.04	1.80
35	NGC 7089	10.4	–1.65	–9.03	0.20
36	NGC 7492	25.3	–1.78	–5.81	1.80



Some examples of the comparison between the observed surface density (symbols) and the corresponding theoretical curve for indicated GSCs

When calculating the velocity anisotropy parameter, we mainly used relatively new data taken from the work by Miocchi *et al.* [7], which were obtained from the results of space- and ground-based observations. In addition, we partially used the results of observations from some articles of well-known authors (e.g., Peikov and Rusev [8, 9]). From Table 1, one can see that, for the majority of clusters, the velocity anisotropy parameter is less than 1, i.e. they have an almost spherical distribution of star velocities and are at a relatively late evolution stage. Note that it is at the calculated values of the anisotropy parameter that the corresponding theoretical results agree well with the observed surface density functions (see Figure).

For 11 clusters, the values of the velocity anisotropy parameter fall within an interval of 1.50–1.85. Therefore, the velocity distributions in those clusters are close to the radially extended one, which may probably be a result of their nonstationary state in a regular field.

Finally, let us return to the main question: Can the GSCs be classified according to the velocity anisotropy parameter? To answer it, let us compare the values the GSC anisotropy parameter. Then, it is evident that they can be reliably grouped into three classes (see Table 2). Note that there are no GSCs in the interval $1.0 < A < 1.5$. This fact requires that additional data should be collected in the future concerning the apparent density of GSCs. Nevertheless, it is of interest to compare the average values of basic physical parameters for three GSC classes.

Table 3 exhibits the average values of some characteristics for the mentioned GSC classes: the average distance $\langle R\rangle$ from the class members to the galactic center, the average metallicity $\langle$Fe/H$\rangle$, the average age $\langle \lg t\rangle$, the average GSC size $\langle D_g\rangle$, and the average index of richness $\langle$IR$\rangle$. The GSCs of class I are, on average. twice as closer to the galaxy center as the GSCs of class II, and the GSCs of class III are located in the external galaxy regions. The metallicity and the age of globular star clusters decrease with the distance growth. The average size of clusters in classes I and III are evidently larger than in class II. It is of interest that the index of richness for clusters in class II is larger than in other two classes.

Table 2. **GSC classification by the anisotropy parameter**

Class	A-interval	Number of GSCs
I	$0.0 \le A < 0.5$	16
II	$0.5 \le A < 1.0$	9
III	$1.5 \le A < 2.0$	11

Table 3. **Average values of main parameters for three GSC classes**

Group	$\langle R\rangle$, kpc	[Fe/H]	$\langle \lg t\rangle$	$\langle [D_g]\rangle$, pc	$\langle$IR$\rangle$
I	10.21	–1.41	10.18	7.10	0.55
II	21.39	–1.57	10.16	6.35	0.66
III	54.23	–1.59	10.13	7.41	0.44

We also note the results of our calculations for the correlation coefficient of the anisotropy parameter A with well-known data for other physical parameters. In principle, they can serve as a basis for the creation of one-dimensional classifications. Unfortunately, the values of those coefficients are small and do not allow a two-dimensional classification to be made.

In the future, we intend to search for observation data on the apparent density in order to continue the calculation of the velocity anisotropy parameter for other GSCs. We will include them into the general statistics in order to find new empirical dependences of the parameter A on separate physical parameters of GSCs.

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ЧИ МОЖНА КЛАСИФІКУВАТИ КУЛЬОВІ СКУПЧЕННЯ ГАЛАКТИКИ ПО ЇХ ПАРАМЕТРУ АНІЗОТРОПІЇ ШВИДКОСТЕЙ?

Р е з ю м е

Коротко обговорюються проблеми класифікації кульових скупчень нашої Галактики. Обчислені значення параметра анізотропії швидкостей для 36 кульових скупчень, використовуючи порівняно нові дані для видимої поверхневої густини. За значеннями параметра анізотропії кульові скупчення зірок розділені на три класи. Вивчено кореляцію даного параметра з іншими характеристиками кульових скупчень.