# IMPROVED CRITERION OF NONEQUILIBRIUM IN ELECTRIC ARC PLASMA INDUCED BY RADIATION TRANSFER

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More accurate calculations, which allowed the principal role of radiation emission processes in the formation of a local thermodynamic equilibrium (LTE) state in plasma to be estimated, are carried out. The solution of the problem is obtained in the framework of the LTE approximation with regard for the radiation emission and radiation losses in plasma. The results of numerical simulation testify to a deviation of the population at the excited levels of a copper atom from the equilibrium distribution, which is induced by the resonance radiation absorption onto both the ground and metastable levels.

## 1. Introduction

Electric arc plasma is usually studied under the assumption that it is in a local thermodynamic equilibrium (LTE) state. This assumption corresponds to an equilibrium distribution of excitation and ionization states in plasma, irrespective of the absence of equilibrium radiation emission according to the Kirchhoff law. A general condition for the LTE state to be established in plasma is its high density. Specifically, it must be high enough for the collision processes that are responsible for the population of excited levels and the ionization of plasma-forming gas to prevail over the competing processes associated with radiation transfer in plasma or radiation escape beyond the plasma boundaries.

Estimations testify that, for the majority of plasma-forming gases, the plasma density is not high enough for the radiation losses from the arc volume, which are considered as a nonequilibrium factor in plasma, to be neglected even under the conditions that exist in an atmospheric electric arc characterized by a substantial current. The most illustrative in this regard is the intensity of resonance line emission, which, as a rule, considerably exceeds the intensity of other spectral lines. To provide a balance between the processes of excitation and de-excitation in an optically thin plasma of hydrogen-like particles, a considerable electron concentration of about $10^{18}$ cm$^{-3}$ is needed in the majority of practical cases. However, the characteristic path length of resonance radiation is, as a rule, much shorter than the typical radius of an electric arc obtained at atmospheric pressure. The existing radiation self-absorption effectively reduces the role of the radiation-induced de-excitation of energy levels and, accordingly, diminishes the electron concentration threshold, at which the LTE in an optically dense uniform plasma is attained.

An essentially new effect can be expected, if the radiation absorption occurs under the condition of temperature-induced plasma inhomogeneity. In this case, the radiation that arrives from hotter plasma regions can not only compensate radiative energy losses, but can also invoke the inverse effect, the repopulation of the resonance level.

A nonequilibrium state in the plasma of a free-burning electric arc between fusible copper electrodes in the atmospheric environment, considered as a result of the transfer of resonance emission by copper atoms, was studied experimentally for the first time in works [1–3] by V.A. Zhovtyansky *et al.* at the Kyiv University. Later, a simple model was proposed for nonequilibrium plasma. It consisted in that the population of the resonance level of copper atoms along the arc radius corresponded to a temperature characteristic of the axial arc region [4]. This assumption, as was shown in work [4], may result in the enlightenment effect for the electric arc channel, i.e., in a reduction of the channel resistance to the electric current flow [4, 5]. The physical origin of this effect consists in an effective reduction of the ionization potential for plasma-forming atoms at the arc periphery owing to the repopulation of resonance levels of atoms in this region.

In works [6, 7], we predicted theoretically that the state of plasma of an electric arc free-burning between fusible electrodes in the atmosphere deviates from the equilibrium one. Those works were based on the application of a criterion approach, which allowed the problem of radiation transfer to be solved under the assumption that the equilibrium conditions in plasma took place. However, even in this simplified variant, the problem remains rather difficult from the mathematical viewpoint, because the influence of radiation that arrives from other plasma regions must be taken into account for every physically small volume. In this connection, we adopted certain simplifications in our detailed work [7]. They allowed us to obtain a final solution of the problem and to show that the transfer of the resonance radiation emitted by copper atoms at the arc periphery owing to their absorption of the resonance radiation arriving from the region near the arc axis, which is characterized by the highest temperature, does take place.

However, as a result of the adopted simplifications, the nonequilibrium criterion in work [7] turned out insensitive to a deviation from the equilibrium population of the metastable levels of a copper atom. This result does not correspond to experimental ones obtained in work [1]. In this connection, we carried out an additional analysis of the radiation transfer problem in this work and derived an advanced nonequilibrium criterion, which is more sensitive to the metastable level population.

## 2. Statement of the Problem and Solution Technique

The crucial role of the resonance transition onto an equilibrium plasma state can explain the popularity of the so-called two-level model of atom with two energy states (levels): basic (1) and excited (2). In the framework of this model, the following relation is valid for $n_2(r,t)$ [8, 9]:

$$\frac{dn_2(r,t)}{dr} = -n_2(r,t)A_{21} - n_2(r,t)\omega_{21} + n_1(r,t)\omega_{12} + \int\limits_V n_2(r',t)A_{21}K(|r-r'|)dV', \quad (1)$$

where

$$K(\rho) = (4\pi\rho^2)^{-1}\int \varepsilon_\nu k_\nu \exp\left(-k_\nu \rho\right) d\nu; \quad (2)$$

$n_k$ is the population of the $k$-th level, $\omega_{kl}$ the frequency of the atomic excitation and de-excitation between levels $k$ and $l$, $A_{kl}$ the probability of the corresponding resonance radiative transition, $\varepsilon_\nu$ the frequency distribution of photons determined by the line shape and normalized to 1, and $k_\nu$ the spectral absorption factor. The integral term in expression (1) corresponds to the radiation transfer. The kernel $K(|r-r'|)$ in the case of a stationary temperature distribution is equals to the probability that a photon emitted at the point $r'$ is absorbed in a physically small volume characterized by the coordinate $r$.

In the stationary case, it is convenient to rewrite Eq. (1) for the reduced population of the excited state, $y_2(r) = n_2(r)/n_2^0(r)$, where $n_2^0$ is the Boltzmann population number at the temperature of electrons. Let it also be independent of coordinates. Then we have

$$y_2(r) = (1-\beta)^{-1}\int\limits_V y_2(r')K(|r-r'|)dV' + \beta/(1+\beta). \quad (3)$$

Here, the notation $\beta = \omega_{21}/A_{21}$ was introduced. We also used the following relation between $\omega_{12}$ and $\omega_{21}$, which follows from the detailed balancing principle [8]:

$$n_k\omega_{kl} = n_l\omega_{lk}. \quad (4)$$

We can use once more the advantage of the criterion description, which allows the populations of the ground and excited states to be described in the equilibrium approximation, i.e. by the Boltzmann distribution. Accordingly, Eq. (3) can be rewritten in the following form [5]:

$$y_2(r) = \frac{1}{1+\beta(r)}\int\limits_V \frac{n_1^0(r')}{n_1^0(r)}\exp\left[\frac{h\nu_0}{k}\left(\frac{1}{T(r)} - \frac{1}{T(r')}\right)\right] \times$$

$$\times y_2(r')K(r,r')dV + \frac{\beta}{(1+\beta(r))}, \quad (5)$$

where $\nu_0$ is the frequency at the line center; $h$ and $k$ are the Planck and Boltzmann constants, respectively;

$$K(r',r) = \frac{1}{4\pi}\int\limits_0^\infty \frac{\varepsilon_\nu(r')k_\nu(r')}{|r-r'|^2}\exp\left[-\int\limits_r^{r'} k_\nu(r'')d\mathbf{l}\right]d\nu, \quad (6)$$



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and $\int_r^{r'} k_\nu(r'')d\mathbf{l}$ is the contour integral along the ray connecting the points with coordinates $r$ and $r'$. Let us also suppose that plasma is concentrated in a long enough cylindrical volume of radius $R$. In contrast to the criterion proposed in work [7], the integrand in expression (5) involves not only the temperature dependence of the absorption factor, but also the dependence of the reduced local population of the level in the equilibrium state on the temperature distribution in the arc.

Searching for an analytical solution of the equation describing the radiation-induced excitation is a task connected with substantial difficulties. In the cases where the analytical solution was obtained, the corresponding result turned out rather cumbersome. Therefore, the application of numerical methods for the solution of such problems is of special importance.

Note also that, even if the dependence $n_2(r)$ is strong, the quantity $y_2(r)$, which characterizes the local deviation from equilibrium, can change rather weakly. This circumstance gives us ground to factor the function $y_2(r')$ outside the integral sign [9]. Then, after simple transformations, we obtain the relation

$$y_2(r) = \frac{\beta(r)}{\theta(r)+\beta(r)}, \tag{7}$$

where

$$\theta(r) = 1 - \frac{1}{4\pi}\iiint\limits_V\int\limits_0^\infty \frac{n_1^0(r')}{n_1^0(r)}\exp\left[\frac{h\nu_0}{k}\left(\frac{1}{T(r)}-\frac{1}{T(r')}\right)\right]\times$$

$$\times\frac{\varepsilon_\nu(r')k_\nu(r')}{|r-r'|^2}\exp\left[-\int\limits_{\mathbf{r}}^{\mathbf{r}'}k_\nu(\mathbf{r}'')d\mathbf{l}\right]d\nu dV'. \tag{8}$$

In this form, Eq. (8) becomes convenient for the physical interpretation. Now, the function $\theta(r)$ cannot be regarded as a probability for the photon to escape beyond plasma boundaries, as was true in the case of a uniform temperature distribution over the arc. Now, this parameter takes the influence of two probabilistic processes into account, which, however, oppositely affect the population at the energy level of a copper atom in a local plasma volume. These are a reduction of the level population owing to losses of radiation emitted in this volume and the population growth associated with the absorption of radiation emitted in other regions, where plasma has a high temperature. Therefore, the function $\theta(r)$ can acquire both positive and negative values. However, the combination $A_{21}\theta(r)$ still produces an approximate value for the divergence of a photon flux at the point $r$.

If $\theta(r)$ is negative at the point $r$, the reduced population at this point exceeds the local equilibrium value, i.e. the intense radiation fluxes coming from hot plasma regions are absorbed in cooler ones. A similar effect not only compensates the loss of excitation connected with the radiation escape, but induces the inverse effect, so that the population at the excited state can exceed the local equilibrium value.

To calculate the integral in expression (8) numerically, it is convenient to change to the local spherical coordinate system moving with the observation point $r$. Then, changing the integration order over the spatial coordinates and the frequency, we arrive at the relation

$$I = \frac{1}{4\pi}\int\limits_0^\infty\iiint\limits_V \frac{n_1^0(r')}{n_1^0(r)}\exp\left[\frac{h\nu_0}{k}\left(\frac{1}{T(r)}-\frac{1}{T(r')}\right)\right]\times$$

$$\times\frac{\varepsilon_\nu(r')k_\nu(r')}{|r-r'|^2}\exp\left[-\int\limits_{\mathbf{r}}^{\mathbf{r}'}k_\nu(\mathbf{r}'')d\mathbf{l}\right]dV'd\nu =$$

$$= \frac{1}{4\pi}\int\limits_0^\infty\int\limits_0^{2\pi}\int\limits_0^\pi\int\limits_0^{R(\varphi)} \frac{n_1^0(r')}{n_1^0(r)}\exp\left[\frac{h\nu_0}{k}\left(\frac{1}{T(r)}-\frac{1}{T(r')}\right)\right]\times$$

$$\times\frac{\varepsilon_\nu(\rho)k_\nu(\rho)}{\rho^2}\exp\left[-\int\limits_0^\rho k_\nu(t))dt\right]\rho^2\sin(\theta)d\rho d\theta d\varphi d\nu, \tag{9}$$

where $\rho = |\mathbf{r}-\mathbf{r}'|$, and $0 \le \rho < \infty$ in view of the choice of the local coordinate system origin at the point $r$. Basing on the symmetry of the problem, changing to the integration variable $\omega = (\nu-\nu_0)/\triangle\nu(r')$, and expanding the integration limits over the frequency, we obtain the expression

$$I = \frac{1}{\pi}\int\limits_{-\infty}^\infty\int\limits_0^{2\pi}\int\limits_0^\pi\int\limits_0^{R(\varphi)} \frac{n_1^0(r')}{n_1^0(r)}\exp\left[\frac{h\nu_0}{k}\left(\frac{1}{T(r)}-\frac{1}{T(r')}\right)\right]\times$$

$$\times\varepsilon_\omega(\rho)k_\omega(\rho)\exp\left[-\int\limits_0^\rho k_\omega(t))dt\right]\sin(\theta)d\rho d\theta d\varphi d\omega. \tag{10}$$

Carrying out the sequentially change of variables – $\rho = \rho'/\sin(\theta)$ and $\rho' = \rho''/\sin(\theta)$ – we transform expression (10) to the form

$$I = \frac{1}{\pi}\int\limits_{-\infty}^\infty\int\limits_0^{\pi}\int\limits_0^{\pi/2}\int\limits_0^{r_0(\varphi)} \frac{n_1^0(r')}{n_1^0(r)}\exp\left[\frac{h\nu_0}{k}\left(\frac{1}{T(r)}-\frac{1}{T(r')}\right)\right]\times$$

$$\times\varepsilon_\omega(\rho)k_\omega(\rho)\frac{\exp\left[-\int\limits_0^{\rho''}k_\omega(t))dt/\sin^2(\theta)\right]}{\sin(\theta)}d\rho''d\theta d\varphi d\omega, \tag{11}$$

where $r_0(\varphi)$ satisfies the equation $R^2 = r^2 + r_0^2(\varphi) - 2rr_0(\varphi)\cos(\pi-\varphi)$, and $R$ is the arc radius. In essence, $r_0(\varphi)$ is a projection of the radius-vector that starts from the point $r$ and scans the internal surface of the cylinder onto the polar plane $\theta = \pi/2$. Bearing in mind the representation for the modified second-kind Bessel function (the Macdonald function)

$$\int\limits_0^{\pi/2}\frac{\exp\left[-z/\sin^2(\theta)\right]}{\sin(\theta)}d\theta = \frac{1}{2}\exp\left(-\frac{z}{2}\right)K_0\left(\frac{z}{2}\right),$$

let us integrate in Eq. (11) over the azimuthal angle $\theta$. We obtain

$$I = \frac{1}{2\pi}\int\limits_{-\infty}^{\infty}\int\limits_0^{\pi}\int\limits_0^{r_0(\varphi)}\frac{n_1^0(r')}{n_1^0(r)}\exp\left[\frac{h\nu_0}{k}\left(\frac{1}{T(r)}-\frac{1}{T(r')}\right)\right]\times$$

$$\times\varepsilon_\omega(\rho)k_\omega(\rho)\exp\left(-\frac{Z}{2}\right)K_0\left(\frac{Z}{2}\right)d\rho''d\theta d\varphi d\omega, \tag{12}$$

where

$$Z = \int\limits_0^{\rho''}k_\omega(t)dt. \tag{13}$$

Since the Macdonald function diverges at a vicinity of zero, let us consider its asymptotic representation

$$K_0(t) \approx -\ln\left(\frac{t}{2}\right) - \frac{\gamma}{2}, t\in(0,1),$$

where $\gamma$ is the Euler constant, and write down an auxiliary relation for the internal integral in Eq. (12),

$$J = \int\limits_0^{r_0(\varphi)}\frac{n_1^0(r')}{n_1^0(r)}\exp\left[\frac{h\nu_0}{k}\left(\frac{1}{T(r)}-\frac{1}{T(r')}\right)\right]\varepsilon_\omega(\rho)k_\omega(\rho)\times$$

$$\times\exp\left(-\frac{Z}{2}\right)\left(K_0\left(\frac{Z}{2}\right)+\ln\left(-\frac{Z}{2}\right)+\frac{\gamma}{2}\right)d\rho''. \tag{14}$$

Integral (14) is finite in a vicinity of zero, because its integrand tends to zero as $Z\to 0$. In view of the fact that the contour integral (13) is calculated along the ray, which connects the points $r$ and $r'$ and coincides with the direction of integration over $\rho''$, let us pass to the integration over the variable $Z$ in expression (14),

$$J = \int\limits_0^{Z_R}\frac{n_1^0(r')}{n_1^0(r)}\exp\left[\frac{h\nu_0}{k}\left(\frac{1}{T(r)}-\frac{1}{T(r')}\right)\right]\varepsilon_\omega(\rho)\times$$

$$\times\exp\left(-\frac{Z}{2}\right)\left(K_0\left(\frac{Z}{2}\right)+\ln\left(-\frac{Z}{2}\right)+\frac{\gamma}{2}\right)dZ, \tag{15}$$

where $Z_R = \int_0^{r_0(\varphi)}k_\omega(t)dt$, and apply the mean-value theorem to result (15). We note that the application of this theorem to the functions depending on $\rho$, $r$, and $r'$ would not bring about a substantial loss of accuracy, provided that the step of integration over $Z$ is small enough,

$$J_k \approx \frac{n_1^0(r')}{n_1^0(r_k)}\exp\left[\frac{h\nu_0}{k}\left(\frac{1}{T(r)}-\frac{1}{T(r'_k)}\right)\right]\varepsilon_\omega(\rho_k)\times$$

$$\times\int\limits_{Z_{k-1}}^{Z_k}\exp\left(-\frac{Z}{2}\right)\left(K_0\left(\frac{Z}{2}\right)+\ln\left(-\frac{Z}{2}\right)+\frac{\gamma}{2}\right)dZ, \tag{16}$$

where $r'_k(t)$ and $\rho_k(t)$ stand for the values of corresponding quantities at a certain point within the interval $[t_{k-1}, t_k]$. In the framework of the assumptions made, the integral of the product of the exponential function and the sum of a logarithmic function and a constant in expression (16) can be calculated analytically,

$$I_1 = \int\limits_{Z_{k-1}}^{Z_k}\exp\left(-\frac{Z}{2}\right)\left(\ln\left(-\frac{Z}{2}\right)+\frac{\gamma}{2}\right)dZ =$$

$$= 2\left(\exp\left(-\frac{Z_{k-1}}{2}\right)D_{k-1} - \exp\left(-\frac{Z_k}{2}\right)D_k\right), \tag{17}$$

where $D_i = \left(\gamma - \exp\left(\frac{Z_i}{2}\right)Ei\left(-\frac{Z_i}{2}\right)+\ln\left(\frac{Z_i}{2}\right)\right)$, and $Ei(z) = -\int_{-z}^{\infty}\frac{\exp(-t)}{t}dt$ is the exponential integral. Then, the internal integral in expression (12) reads

$$I \approx \frac{1}{2\pi}\int\limits_{-\infty}^{\infty}\int\limits_0^{\pi}\sum_k\frac{n_1^0(r'_k)}{n_1^0(r)}\exp\left[\frac{h\nu_0}{k}\left(\frac{1}{T(r)}-\frac{1}{T(r'_k)}\right)\right]\times$$

$$\times\varepsilon_\omega(\rho_k)\,(I_k - I_1)\,d\varphi d\omega, \qquad (18)$$

where

$$I_i = \int\limits_{Z_{i-1}}^{Z_i} \exp\left(-\frac{Z}{2}\right)\left(K_0\left(\frac{Z}{2}\right) + \ln\left(-\frac{Z}{2}\right) + \frac{\gamma}{2}\right) dZ,$$

the quantity $I_1$ is determined from expression (17), $k = 0 \ldots N$, and $Z_N = Z_R$. All terms in expression (18) are finite at zero; therefore, the last one can be integrated with a high enough accuracy, even if a "loose integration mesh" is applied. For testing the accuracy of the proposed "semianalytical" procedure, we calculated the integral of $K_0(t)$ over the interval $[0, 1]$. The value obtained coincides with the tabulated one to the sixth digit after the decimal point. Integration in formula (18) over the reduced radius, the polar angle, and the reduced frequency was carried out by the trapezium method.

In Table, a list of some spectral lines emitted by a copper atom from the resonance levels $E_u = 3.79$ and 3.82 eV is quoted. They were used as a reference for studying the role of radiation emission effects. One of them has the ground (nonexcited) level and the other the metastable one, $E_l = 1.39$ eV, as the lower level of the corresponding spectral transition. The table also quotes the statistical weights of levels $g$ (the subscripts $u$ and $l$ denote the upper and lower, respectively, transition levels), the oscillator strengths $f$, which are proportional to the transition probability [10], and the Stark broadening parameters $\triangle\lambda_s$, which correspond to the concentration of charged particles $n_e = n_i = 10^{17}$ cm$^{-3}$ [11]. They are necessary for the effects of spectral line emission and radiation absorption to be taken into account properly [12]. The broadening of the 327.3-nm spectral line is associated with the Doppler effect. The natural and collision-induced broadening of spectral lines was also taken into account, and the resulting broadening was presented by the Voigt contour [13]. The issues concerning the account of spectral plasma characteristics were discussed in more details in our recent work [7].

The concentrations of charged particles were determined from the Saha and Boltzmann equations with regard for the Dalton equation

$$[N_a + (1 + \chi_{\mathrm{Cu}})N_i]kT = \chi_{\mathrm{Cu}}p,$$

**T a b l e. Spectral parameters of the resonance lines of copper atom**

| Line, nm | $E_k$, eV | $g_k$ | $E_i$, eV | $g_i$ | $\triangle\lambda_s$, nm* | $f$ |
|---|---|---|---|---|---|---|
| 510.5 | 3.82 | 4 | 1.39 | 6 | 0.021 | 0.0051 |
| 327.3 | 3.79 | 2 | 0 | 2 | | 0.220 |

where $p$ is the atmospheric pressure, $\chi_{\mathrm{Cu}}$ the content of copper vapor in the plasma-forming mixture, and $N_a$ and $N_i$ are the concentrations of copper atoms and ions, respectively.

## 3. Results of Calculations

For calculations, we used a temperature profile obtained as a solution of the energy balance equation for a wall-stabilized cylindrical arc in the one-dimensional approximation (the Elenbaas–Heller equation) [6, 7],

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dS}{dr}\right) + \sigma E^2 = 0, \quad S = \int\limits_0^T \lambda(T)dT, \qquad (19)$$

where $r$ is the radial coordinate, $\sigma(S)$ the electrical conductivity, $E$ the electric field strength, $S$ the thermal potential, and $\lambda(T)$ the thermal conductivity. The corresponding boundary conditions are

$$\left.\frac{dS}{dr}\right|_{r=0} = 0, \quad S|_{r=r_w} = S_w, \qquad (20)$$

where the parameter $S_w$ corresponds to the temperature $T_w = 1000$ °C of some effective cooling wall (a quasiwall) of the radius $R_w = 3$ mm.

To find the numerical solution of the nonlinear boundary-value problem (19) and (20), we applied the method of solution continuation in a parameter [14]. At every parameter step, the linearized differential equation of the second order was solved by reducing the boundary-value problem to a sequence of Cauchy problems, the latter being integrated with the use of the Dormand–Prince fifth-order method [15]. The electric current and the absorbing quasiwall radius were taken as fixed parameters. As the first approximation, we selected the analytical solution obtained in the framework of the quasichannel model [4].

To integrate the internal integral in the contour integral (13), as well as in the case of the Voigt profile, the trapezium method was applied. The functional dependence of the temperature on the radius at every integration point was found with the use of a linear interpolation of the temperature profile determined by solving problem (19), (20). Figures 1 to 3 exhibit some results of our numerical calculations obtained for an open electric arc in copper vapor at a discharge current of 30 A. The fraction of copper vapor was equal to 0.1, 1, and 10%.

Under the adopted assumptions, the temperature distributions are identical for various concentrations of copper vapor in the plasma volume (Fig. 1). In view of the

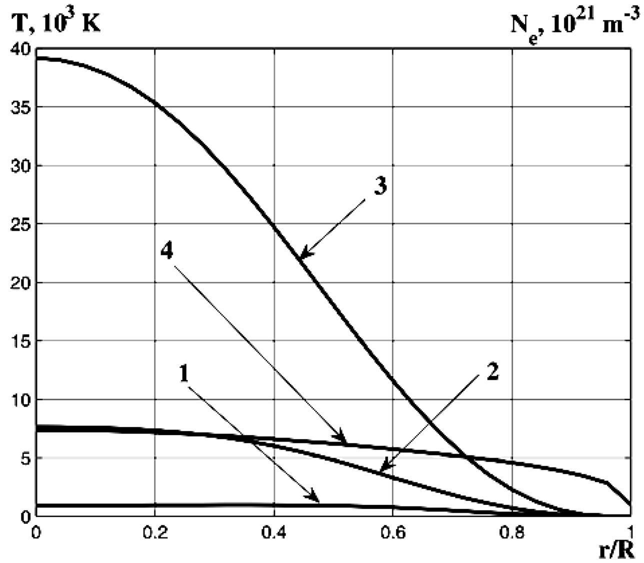


Fig. 1. Radial distributions of the electron concentration in an arc with a copper content of 0.1 (*1*), 1 (*2*), and 10% (*3*). Curve *4* describes the temperature distribution in all three cases

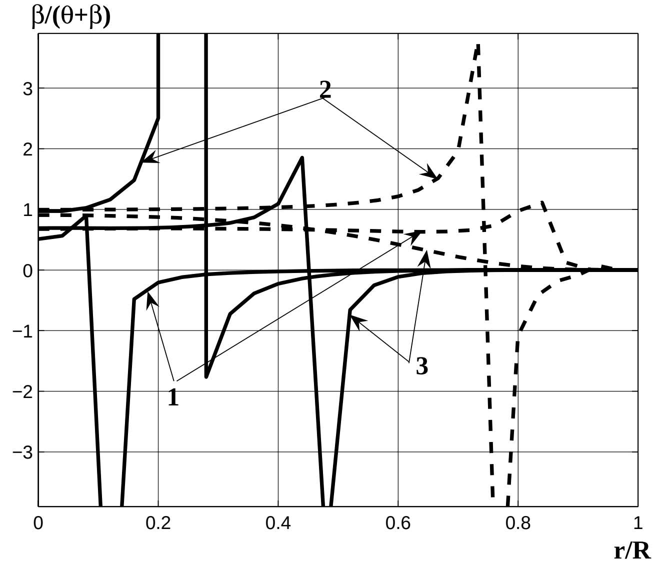


Fig. 3. Improved (solid curves) and approximate [7] (dashed curves) criteria for the Voigt profile of the 327.3-nm line in arcs with copper contents of 0.1 (*1*), 1 (*2*), and 10% (*3*)

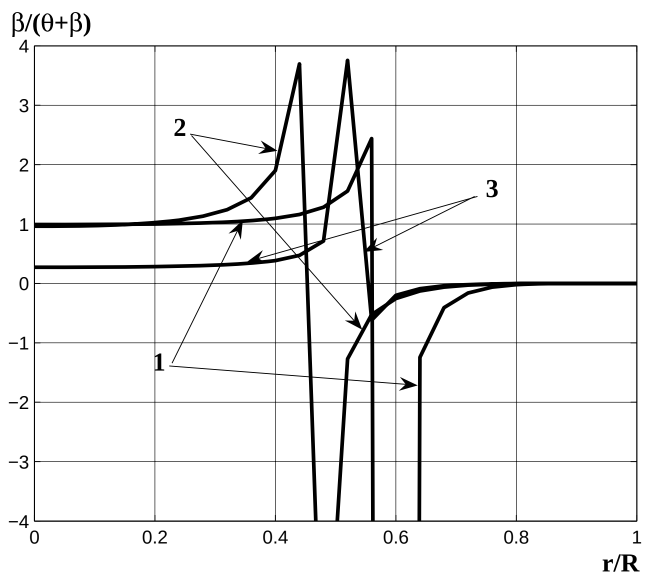


Fig. 2. Improved criterion for the Voigt profile of the 510.5-nm line in arcs with copper contents of 0.1 (*1*), 1 (*2*), and 10% (*3*)

fact that copper is the dominating plasma-forming component in an air–copper vapor mixture, different concentrations of copper vapor give rise to different distributions of the electron concentration (curves *1* to *3* in Fig. 1).

Figure 2 shows the results of demonstration calculations for the 510.5-nm spectral line, for which the lower level of the spectral transition corresponds to the metastable level in the electron structure of a copper atom. The Lorentz contour is typical of this line in the dense plasma region, whereas the Gaussian profile, in accordance with the mechanism of Doppler broadening, prevails in a vicinity of the quasiwall, where the concentration of electrons is insignificant. Therefore, while calculating the influence of radiation effects on a deviation of the population distribution from the equilibrium one in the "metastable level – resonance level"system, the Voigt profile, which takes both mechanisms of line broadening into account, was applied. Hence, the nonequilibrium criterion proposed in this work turns out much more sensitive than the approximate criterion [7], which is insensitive to the nonequilibrium population of a metastable level.

Figure 3 demonstrates similar results obtained for the 327.3-nm resonance line, the lower transition level for which is the ground atomic one. For resonance lines, which lie deeply in the energy structure of the atom, the mechanism of Doppler broadening dominates, because the influence of the Stark effect is minimal owing to a substantial screening of spectral transition levels from the action of external electromagnetic fields. When comparing the curves in Fig. 3 with the results obtained in work [7] (the dashed curves), it should be noted that the nonequilibrium zone, in which the parameter $\beta(r)/(\theta(r)+\beta(r))$ is negative in comparison with the effective lifetime criterion in the cited work, becomes shifted toward the axial region, which considerably expands the region of a plasma nonequilibrium state. This fact testifies that the factor of radiation influence becomes prevailing in this region. It also means that one should expect a substantial absorption of resonance spectral lines here, which ultimately gives rise to a deviation from the LTE state. This scenario is completely in agreement with the results of experimental researches [1].

A special attention is attracted by the transient region in Figs. 2 and 3, which is located between the axial and peripheral plasma zones. When the radial coordinate increases, the equilibrium plasma region firstly gradually transforms into the nonequilibrium zone, where the contribution of radiation absorption to the resonance level population starts to grow. Since this contribution, which is described by the term $\theta(r)$, is a compensatory, the ratio $\beta(r)/(\theta(r)+\beta(r))$ grows with the radial coordinate to the values, when the terms in the denominator mutually compensate each other; then, the ratio changes its sign and gradually approaches zero, when $\theta(r) \gg \beta(r)$.

However, it should be noted that those results were obtained under the assumption of a weak dependence of the quantity $y_2$, which characterizes, according to the results of work [9], a local deviation from the equilibrium state, on the radial component. Owing to this assumption, the last multiplier was factored outside the integral sign, and, hence, the integral equation (5) for $y_2(r)$ was transformed into the algebraic one (7). For such reasons, it is the ratio $\beta(r)/(\theta(r)+\beta(r))$ that is reckoned along the ordinate axis in Figs. 2 and 3.

The results obtained open the prospects for detailed calculations of equilibrium plasma parameters with regard for the radiation transfer and including the kinetics of metastable and resonance level populations into the model.

## 4. Conclusions

When the processes of radiation transfer are taken into account, there arise the substantial difficulties associated with a drastic dependence of the photon path length on the frequency. Therefore, the appreciable mutual influence of elementary plasma volumes located rather far from one another and characterized by strongly different temperatures is observed. For the mathematical description of the radiation transfer processes to be adequate, it becomes necessary to use integral relations, which make allowance for the mutual influence of such processes over the whole plasma volume.

In this work, the more accurate calculations were carried out, which allowed the principal role of radiation emission processes in the formation of an equilibrium plasma state to be estimated. Since the general problem of radiation transfer in a nonequilibrium medium is rather difficult, its solution was obtained in the framework of the LTE model with regard for the processes of radiation emission and losses in plasma. This formulation made the problem concerned somewhat simpler and allowed us to confine the consideration by assuming the equilibrium plasma state.

The results of numerical simulation evidence a deviation from the equilibrium population of the energy levels of a copper atom, which takes place owing to the absorption of resonance radiation on the ground and metastable levels.

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## УДОСКОНАЛЕНИЙ КРИТЕРІЙ НЕРІВНОВАЖНОСТІ ПЛАЗМИ ЕЛЕКТРИЧНОЇ ДУГИ, ЗУМОВЛЕНОЇ ПЕРЕНЕСЕННЯМ ВИПРОМІНЮВАННЯ

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Р е з ю м е

У роботі виконано уточнені розрахунки, що дозволяють принципово оцінити роль процесів випромінювання у формуванні рівноважного стану плазми. Розв'язок задачі отримано у варіанті критерію застосовності припущення локальної термодинамічної рівноваги (ЛТР) з урахуванням ролі процесів випромінювання у плазмі та його втрат. Дані числового моделювання доводять наявність ефектів порушення рівноважної заселеності енергетичних рівнів атома міді, зумовлену поглинанням резонансного випромінювання як на основний, так і на метастабільний рівень.