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DIELECTRIC MODEL OF ENERGY LOSSES BY A MASSIVE CHARGED PARTICLE MOVING THROUGH COLD MAGNETIZED PLASMA

Energy losses by a charged particle moving in infinite magnetized plasma have been calculated in the framework of the dielectric model and with the use of the correspondence principle. This principle enabled us not to use a phenomenological cutoff parameter for matching with the theory of binary collisions. Analytical expressions for energy losses were derived for the motions of a particle directed along and perpendicularly to the magnetic field. They were confirmed by numerical calculations for a charged particle moving in a magnetic field of an arbitrary strength and at an arbitrary angle to the field direction. The results obtained are compared with those obtained in quantum field theory.

Keywords: dielectric model, cutoff parameter, energy losses, principle of correspondence, binary collisions, magnetized plasma.

1. Introduction

Interaction of beams of charged particles with matter has been a subject of active scientific researches during the whole century. Owing to the development of accelerators and detectors of charged particles, a large practical contribution has been made not only to the fundamental physics, but also to medical radiology, materials science, thermonuclear physics, and so forth. In all those cases, a comprehensive and adequate understanding of processes that take place in the course of interaction between charged particles and various substances is required.

Recently, a large attention has been paid to the processes, in which strong magnetic fields (so strong that the cyclotron radius of electrons is much smaller than the linear scales of the examined processes, and the cyclotron period is much shorter than the runtime of those processes) play a crucial role. Such fields are used in installations for the magnetic confinement of thermonuclear plasma [1, 2], for holding electrons in electron coolers in storage rings [1–12]

As for the interaction between a heavy charged particle and magnetized plasma, the project FAIR (Facility for Antiproton and Ion Research, Darmstadt), where an electron cooler will be used to accumulate antiprotons, is of interest. In the PANDA (Antiproton Annihilation at Darmstadt) experiment on the High Energy Storage Ring (HESR), owing to the combination of two ways (the electron cooling and the stochastic one) to reduce the phase volume of experimental charged-particle beams, the expected spread of antiproton momenta should be $\frac{\Delta p}{p} \sim 10^{-5}$ [2]. The accumulation of antiprotons is a very complicated process. On the one hand, the number of antiprotons is small, since the coefficient of proton–antiproton conversion in nuclear reactions on the target does not exceed 10^{-7}. On the other hand, antiprotons that are created from the initial proton beam fly out within a wide solid angle and have a large energy spread. For this reason, the process of cooling in storage rings is desirable for protons and vitally important for antiprotons [9]. In addition, protons (antiprotons) will be cooled down by relativistic electron beams with the relativistic parameter $\gamma = 9.5$.

The interaction between an incident charged particle and the substance, for example, when considering the slowing-down (cooling) of heavy ions in electron plasma, can be described in the framework of two theoretical models that supplement each other. These are the dielectric (plasma) model, where the deceleration of a charged particle is associated with the excitation of electron plasma oscillations by the particle itself [13], and the theory of binary collisions, in which the particle loses its energy at consecutive binary collisions at small impact parameters. The plasma approach demands that small impact parameters should be cut off, because perturbation theory works properly only if the energy transferred by the particle to an electron at their interaction is much lower than the particle's energy itself.

Collisions at small impact parameters can be taken into account in the framework of the binary collision theory. By cutting off the large values of impact parameters in the theory of binary collisions, we consider, in such a manner, the process of charge screening by plasma electrons. The total energy losses by a particle at its motion in electron plasma become equal to the sum of results obtained by both methods after the matching procedure [13].

The energy losses by a charged particle in magnetized plasma at the electron cooling can be calculated with the use of the numerical simulation PIC (particle-in-cell) [7], the Monte Carlo method (classical trajectories), or the software package BetaCool [14]. The latter is based on formulas obtained by Budker [4, 10], Parkhomchuk, or Derbenev–Skrinskii [14] in the framework of binary collision theory. However, as was shown by Alfvén and Spitzer [15, 16], the contribution of collective scattering processes at small angles to total energy losses is of the same order as that of binary collisions in plasma with a medium concentration, and by an order of magnitude larger in rarefied plasma.

Therefore, when considering the scattering of an ion at electrons, it is important to consider the contribution made by collective processes. For the cases of strong magnetic fields and the absence of a magnetic field, the analysis of energy losses in the framework of the plasma model was done in works [1, 13] and, in the case of weak fields, in work [7]. The case of the motion along the magnetic field was analyzed in works [17–21].

In this work, we develop the results of work [9], where the polarization losses of a charged particle in electron plasma were found in the framework of the dielectric model for the case of the motion of a particle at an arbitrary angle with respect to the external magnetic field with an arbitrary strength. In work [9], the method of matching the far- and near-range asymptotics was used to find total losses, which is a conventional practice in such problems. A shortcoming of this method consists in that the introduction of an intermediate phenomenological cutting parameter is required, which gives rise to a necessity of engaging the results obtained in the framework of another theory, the theory of binary collisions [1, 7, 9, 13]. In this work, we propose a different approach to avoid this shortcoming. The results obtained are verified on the basis of the correspondence principle, according to which the results obtained in a more complicated case should not contradict to a simpler theory if the passage to the corresponding limit is made.

The case with a zero magnetic field is considered in Section 3. Analytical expressions for the energy losses by a heavy charged particle at its motion along and across the magnetic field are obtained in Sections 4 and 5, respectively. We show that, in accordance with the correspondence principle, when the external magnetic field is "switched-off", we arrive at a simpler case without magnetic field [1, 13]. In Section 6, the contribution of collective effects is numerically calculated and analyzed in the framework of the dielectric model for a test particle that moves at an arbitrary angle ($0 < \alpha < \pi/2$) in external uniform magnetic fields of various strengths. The results obtained are compared with those obtained in work [13] in the framework of a more general quantum field theory.

2. Formulation of the Problem

For the sake of completeness, we repeat some calculations from the previous work [9].

Let a charged particle (a proton, an antiproton) of mass M and charge q move at a constant relative velocity $\mathbf{V}_0 = \mathbf{v}_i - \mathbf{v}_e$ in infinite uniform electron plasma ($\lambda_\mathrm{D} \ll l$) at the angle α with respect to an external uniform magnetic field $\mathbf{H}$. The following assumptions are made:

1. The linear dimensions of electron plasma, l, considerably exceed the Debye radius λ_D, so the boundary conditions can be neglected.

2. The initial electron concentration n_e is spatially uniform.

3. The perturbation of plasma induced by the moving ion is insignificant, i.e. the perturbed electron concentration is relatively small in comparison with the initial concentration, $\delta n_e \ll n_{0e}$;

4. The external magnetic field is uniform, with its strength and direction being constant.

5. Since the ion is a heavy particle, its velocity can be taken as constant on the scale of the ion-electron interaction.

6. The quasineutrality of electron plasma is provided by external fields.

The initial system of equations includes the following components:

1. The Vlasov–Boltzmann equation for collisionless electron plasma,

$$\frac{df_e}{dt}=\frac{\partial f_e}{\partial t}+\mathbf{v}\frac{\partial f_e}{\partial \mathbf{r}}-\frac{e}{m}\left(\mathbf{E}+\frac{1}{c}[\mathbf{v}\times\mathbf{B}]\right)\frac{\partial f_e}{\partial \mathbf{v}}=0, \tag{2.1}$$

where $f_e = f_{0e} + \delta f_e$ is the distribution function, f_{0e} and δf_e are its equilibrium and perturbed components, and $\left(\mathbf{E}+\frac{1}{c}[\mathbf{v}\times\mathbf{B}]\right)$ is the Lorentz force that acts on plasma electrons;

2. The Poisson equation,

$$\Delta\varphi=-4\pi q\delta(\mathbf{r}-\mathbf{V}_0t)-4\pi e n_e(\mathbf{r},t), \tag{2.2}$$

where $q\delta(\mathbf{r}-\mathbf{V}_0t)$ is the charge of the incoming particle, $\mathbf{V}_0$ its velocity, $en_e(\mathbf{r},t)$ the electron charge, and $\delta(x)$ the delta-function;

3. The equation of motion for electron plasma,

$$\frac{d\mathbf{V}_e}{dt}=-\frac{e}{m}\mathbf{E}-\frac{e}{m}[\mathbf{V}_e\mathbf{H}_0]; \tag{2.3}$$

4. The continuity equation for electron plasma,

$$\frac{\partial n_e}{\partial t}+\mathrm{div}(n_e\mathbf{V}_e)=0, \tag{2.4}$$

where $n = n_{0e} + \delta n_e$ is the electron concentration, n_{0e} is its equilibrium component, and δn_e is the concentration of perturbed electrons.

On the basis of the initial system of equations (2.1)–(2.4) and the assumptions made above, the energy losses by a charged particle in electron plasma can be written down in the tensor form as follows [25]:

$$-\frac{d\mathcal{E}}{dt}=\frac{iq^2}{2\pi^2}\int\limits_{-\infty}^{\infty}\frac{\mathbf{k}\mathbf{V}_0}{\varepsilon_{\alpha\beta}k_\alpha k_\beta}\,d^3k. \tag{2.5}$$

For making the further calculations more convenient, let us change to dimensionless counterparts of the quantities in the integrand. The dimensionless wave vector $\mathbf{w}$ equals

$$\mathbf{k}=\mathbf{w}\frac{\omega_\mathrm{P}}{V_0}, \tag{2.6}$$

where ω_P is the plasma frequency of the electron component, and V_0 the ion velocity. The magnetic field parameter is

$$h=\frac{\omega_H}{\omega_\mathrm{P}}. \tag{2.7}$$

Using Eq. (2.6), we obtain

$$d^3k=d^3w\left(\frac{\omega_\mathrm{P}}{V_0}\right)^3 \tag{2.8}$$

and

$$\mathbf{k}\mathbf{V}_0=\omega_\mathrm{P}\mathbf{w}\mathbf{n}_0 \quad\Rightarrow\quad \mathbf{w}\mathbf{n}_0=\frac{w}{\omega_\mathrm{P}}, \tag{2.9}$$

where $\mathbf{n}_0=\mathbf{V}_0/V_0$.

The dielectric permittivity tensor $\varepsilon_{\alpha\beta}$ for a cold magnetoactive electron plasma looks like [13]

$$\varepsilon_{\alpha\beta}=\begin{pmatrix}\varepsilon_\perp & ig & 0\\ -ig & \varepsilon_\perp & 0\\ 0 & 0 & \varepsilon_\parallel\end{pmatrix}, \tag{2.10}$$

where, with regard for Eqs. (2.6) and (2.7), we obtain the following expressions for the components of the dielectric permittivity tensor (2.10):

$$\begin{aligned}\varepsilon_\perp&=1-\frac{1}{(\mathbf{w}\mathbf{n}_0)^2-h^2},\\ \varepsilon_\parallel&=1-\frac{1}{(\mathbf{w}\mathbf{n}_0)^2},\\ g&=-\frac{h}{(\mathbf{w}\mathbf{n}_0)\left((\mathbf{w}\mathbf{n}_0)-h^2\right)}.\end{aligned} \tag{2.11}$$

Let us introduce the notation

$$-\frac{d\mathcal{E}}{dt}=\frac{q^2\omega_\mathrm{P}}{V_0}\frac{1}{2\pi^2}\Im\int\limits_{-\infty}^{\infty}\frac{w}{\varepsilon_{\alpha\beta}w_\alpha w_\beta}\,d^3w=\frac{q^2\omega_\mathrm{P}}{V_0}\tilde{S}, \tag{2.12}$$

where the polarization energy losses $\tilde{S}$ normalized to $q^2\omega_\mathrm{P}/V_0$ read

$$\tilde{S}=-\frac{i}{2\pi^2}\int\limits_{-\infty}^{\infty}\frac{w}{\varepsilon_{\alpha\beta}w_\alpha w_\beta}\,d^3w. \tag{2.13}$$

3. Energy Losses by a Particle in Plasma without External Magnetic Field

Consider a case where the external magnetic field is absent, i.e. $h = 0$. Then the components of the dielectric permittivity tensor look like

$$\varepsilon_{11}=\varepsilon_{22}=\varepsilon_{33}=\varepsilon=1-\frac{\omega_\mathrm{P}^2}{\omega^2}=\frac{(\mathbf{w}\mathbf{n}_0)^2-1}{(\mathbf{w}\mathbf{n}_0)^2}. \tag{3.1}$$

Let us select a coordinate system so that $\mathbf{e}_z \upuparrows \mathbf{n}_0$. Then

$$\varepsilon = \frac{w_z^2 - 1}{w_z^2}, \tag{3.2}$$

and the polarization energy losses $\tilde{S}$ equal [19, 25]

$$\tilde{S} = \int_0^\infty \frac{dw}{w}. \tag{3.3}$$

Using the relation $\omega = \mathbf{kV}_0 = kV_0 \cos\theta$, equality (3.2) can be written down in the form $\varepsilon = 1 - (k\lambda_a \cos\theta)^{-2}$, which means that if the ion velocity considerably exceeds the velocity of electron thermal motion, $\mathbf{v}_i \gg \mathbf{v}_e$, the field around the ion becomes insensible at distances of an order of the adiabatic cut-off parameter $\lambda_a = V_0/\omega_p$ [1]. This result corresponds to the binary collision theory. Hence, the parameter

$$\tilde{S} = \int_1^\infty \frac{dw}{w} = \ln w_{\max} = \ln \frac{k_{\max} V_0}{\omega_{\mathrm{P}}} \tag{3.4}$$

is a logarithm of the ratio between the maximum and minimum impact parameters, which looks like [13]

$$w_{\max} = \frac{m_e M V_0^3}{(M + m_e) q^2 \omega_{\mathrm{P}}}. \tag{3.5}$$

Below, to avoid the necessity of using the procedure of matching for the determination of $w_{\max}$ and, accordingly, engaging the theory of binary collisions, we use the method of correspondence. This means that if the magnetic field is switched off, the results obtained should correspond to the case where the magnetic field is absent, and $w_{\max}$ will be determined from Eq. (3.5).

4. Particle Motion along the Magnetic Field, $\mathbf{V_0} \upuparrows \mathbf{B}$ ($\boldsymbol{\alpha = 0}$)

If the external longitudinal magnetic field $\mathbf{B}$ is introduced into a system of an interacting heavy charged particle and electron plasma–e.g., to hold electrons in the cooling section–and this magnetic field is so strong that $\omega_H > \omega_{\mathrm{P}}$, the experiment shows that the cooling scenario substantially changes [4].

In the theory of binary collisions, two limiting cases are distinguished [11, 23]. These are rapid collisions that occur without the participation of an external magnetic field ($\omega_H^{-1} = \infty$) and slow ones, for which $\mathbf{B}$ is considered as infinitely large ($\omega_H^{-1} = 0$). This classification is incorrect, especially for slow collisions that are characteristic of slow ions and occur in rather a narrow interval of the impact parameters (the Coulomb logarithm $L_{\mathrm{C}} \approx 2$) [23]. Moreover, a drastic difference between the impact parameters may give rise to mystical corrections to the friction force. For non-zero magnetic fields, there also arises an issue concerning the screening of electric fields [1].

All these difficulties bring about a necessity to use the dielectric model for the description of the electron cooling in an external magnetic field. Let us select a coordinate system so that $\mathbf{wn}_0 = w_z$, i.e. let the particle move in parallel to the force line of the external uniform magnetic field. Then the components of the dielectric permittivity tensor have the form

$$\varepsilon_\parallel = \frac{w_z^2 - 1}{w_z^2}, \quad \varepsilon_\perp = \frac{w_z^2 - 1 - h^2}{w_z^2 - h^2}. \tag{4.1}$$

In the cylindrical coordinates, the normalized energy losses (2.13) look like

$$\tilde{S} = -\frac{i}{2\pi^2} \int_0^{2\pi} d\varphi \int_0^\infty dw\, w^2 \int_0^\pi \sin\theta d\theta \frac{w\cos\theta}{g(w,\theta)}, \tag{4.2}$$

where

$$g(w,\theta) = \frac{w^2 \left(w^2 \cos^2\theta - 1 - h^2 + \frac{h^2}{w^2}\right)}{w^2\cos^2\theta - h^2}. \tag{4.3}$$

Integration over φ gives

$$\tilde{S} = -\frac{i}{\pi} \int_0^\infty \frac{dw}{w} \int_0^\pi d\theta \frac{\sin\theta w \cos\theta \left(w^2\cos^2\theta - h^2\right)}{w^2 \left(w^2\cos^2\theta - 1 - h^2 + \frac{h^2}{w^2}\right)}. \tag{4.4}$$

Changing the variables,

$$t = w\cos\theta; \quad dt = -w\sin\theta d\theta, \tag{4.5}$$

we obtain

$$\tilde{S} = -\frac{i}{\pi} \int_0^\infty \frac{dw}{w} \int_{-w}^{w} \frac{t\left(t^2 - h^2\right)}{t^2 - t_0^2}, \tag{4.6}$$

where $t_0^2 = 1 + h^2 - h^2/w^2$. The variable t depends on the variable w. The analysis of the integration region shows that the additional conditions are imposed on Eq. (4.6) after the integration over t: normalized losses are real-valued in the regions, where

$$0 \le t_0^2 \le w^2. \tag{4.7}$$

Figure 1 illustrates the dependence of t_0^2 on w^2. Contributions are given by those regions, in which the curve passes under the line $t^2 = w^2$. With the use of the dependence $t_0^2(w_0^2) = 0$, we obtain

$$w_0^2 = h^2/(1 + h^2). \tag{4.8}$$

The points of intersection between the t_0^2-curve and the line $t^2 = w^2$ are determined from the condition $t_0^2 = w^2$. If $h > 1$,

$$\begin{aligned} &w_1^2 = 1; \\ &w_2^2 = h^2. \end{aligned} \tag{4.9}$$

From condition (4.7), it follows that the integration interval should be broken into two sections. Taking advantage of the theory of residues, let us write down the normalized losses as a sum

$$\tilde{S} = \int\limits_{w_2}^{\infty} \frac{dw}{w}\left(1 - \frac{h^2}{w^2}\right) - \int\limits_{w_0}^{w_1} \frac{dw}{w}\left(1 - \frac{h^2}{w^2}\right). \tag{4.10}$$

The ultimate expression for the normalized energy losses is obtained by integrating Eq. (4.10) over w:

$$\tilde{S} = \ln w_{\max} - \ln\sqrt{1 + h^2}. \tag{4.11}$$

Having comparing expressions (3.3) and (4.11) for the energy losses, we can write down

$$-\frac{d\mathcal{E}}{dt} = \frac{q^2\omega_{\mathrm{P}}^2}{V_0}\left(\ln w_{\max} - f(h)\right), \tag{4.12}$$

where

$$f(h) = \ln\sqrt{1 + h^2}. \tag{4.13}$$

The result obtained for the case of the longitudinal motion of a charged particle with respect to the magnetic field in electron plasma corresponds to those obtained in the framework of quantum electrodynamics [13].

If we put $h = 0$ (no external magnetic field), then $f(h) = 0$, and Eq. (4.12), in accordance with the correspondence principle, transforms into Eq. (3.3), with the phenomenological parameter $w_{\max}$ being determined by equality (3.5).

5. Particle Motion across the Magnetic Field, $\mathbf{V_0} \perp \mathbf{B}$ $(\alpha = \pi/2)$

Now consider the case of a heavy charged particle moving with velocity $\mathbf{V}_0$ in the direction perpendicular to the external longitudinal magnetic field $\mathbf{H}$ in a magnetized electron gas. The properties of the latter are described by the dielectric permittivity tensor $\varepsilon_{\alpha\beta}$, the explicit expressions for which in the case of cold magnetized electron plasma are given by formulas (2.10) and (2.11). Let us select the coordinate system so that the axis z coincides with the direction of the external magnetic field $\mathbf{H}$, and the axis x with the direction of the charged particle motion, $\mathbf{wn}_0 = w_x$. Then

$$\varepsilon_{\alpha\beta} w_\alpha w_\beta = \varepsilon_\perp \left(w_x^2 + w_y^2\right) + \varepsilon_\parallel w_z^2. \tag{5.1}$$

For further calculations, it is convenient to rotate the system until $w_y \upuparrows \mathbf{B}$. Then

$$\varepsilon_{\alpha\beta} w_\alpha w_\beta = w^2 - w_\perp^2\left(\frac{1}{w_z^2 - h^2}\right) - w_\parallel^2 \frac{1}{w_z^2}, \tag{5.2}$$

where $w_\perp^2 = w_z^2 + w_x^2$ and $w_\parallel^2 = w_y^2$. Now, we should change to the spherical coordinate system,

$$\tilde{S} = -\frac{i}{2\pi^2}\int\limits_0^{2\pi} d\varphi \int\limits_0^{\infty} dw \int\limits_0^{\pi} w^2 \sin\theta d\theta \frac{w\cos\theta}{\varepsilon}, \tag{5.3}$$

where

$$\varepsilon = \frac{w^2 w_z^4 - w^2 w_z^2 h^2 - w_z^2 w_x^2 - w_z^4 - w_y^2 w_z^2 + w_y^2 h^2}{w_z^2\left(w_z^2 - h^2\right)}. \tag{5.4}$$

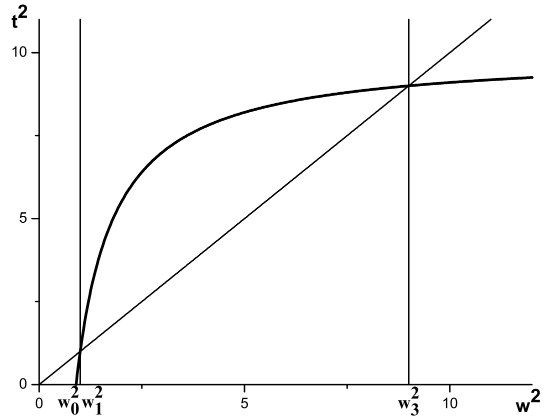


Fig. 1. Dependence $t_0^2(w^2)$. $t_0^2(w_0^2) = 0$. At $w^2 = w_1^2$ and $w^2 = w_2^2$, the curve $t_0^2(w^2)$ intersects the straight line $t^2 = w^2$

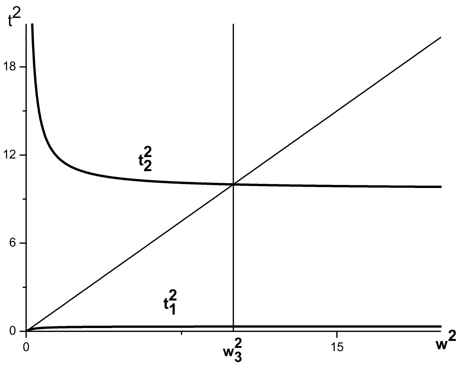


Fig. 2. Dependences $t_1^2(w^2)$ and $t_2^2(w^2)$. At $w^2 = w_3^2$. the curve $t_2^2(w^2)$ intersects the straight line $t^2 = w^2$

Changing the variables according to formulas (4.5), we rewrite Eq. (5.3) the new coordinates as follows:

$$\tilde{S} = -\frac{i}{2\pi^2}\int\limits_0^{2\pi} d\varphi \int\limits_0^{\infty}\frac{dw}{w}\int\limits_{-w}^{w}\frac{t^3\left(t^2-h^2\right)dt}{g(t,w,\varphi)}, \qquad (5.5)$$

where

$$g(t,w,\varphi) = t^4 - t^2\left(h^2+1\right) + h^2\left(1 - t^2/w^2\right)\sin^2\varphi. \qquad (5.6)$$

The roots of the function $g(t,w,\varphi)$ are

$$t_{1,2}^2 = \frac{1}{2}\left(1 + h^2 + \frac{h^2\sin^2\varphi}{w^2}\right) \mp$$
$$\mp\frac{1}{2}\sqrt{\left(1 + h^2 + \frac{h^2\sin^2\varphi}{w^2}\right)^2 - 4h^2\sin^2\varphi}. \qquad (5.7)$$

The variable t depends on w. The analysis of the integration region shows that two cases are possible for the integration over t (see Fig. 2).

1. In the interval $\left[0, w_3^2\right]$, the integral equals the residue at t_1 $\left(t_1 < w < t_2\right)$. At w_3^2, the curve t_2^2 intersects the line $t^2 = w^2$.

$$\int\limits_0^w dt\, f(t) = \int \pi i \operatorname{res}[f(t), t_1]; \qquad (5.8)$$

2. In the interval from w_3 to ∞, the both zeros of the function $g(t,w,\varphi)$ $(t_1 < t_2 < w)$ give a contribution,

$$\int\limits_0^w dt\, f(t) = \pi i \sum_{n=1}^{2}\operatorname{res}[f(t), t_n]. \qquad (5.9)$$

Actually, we may write down

$$\tilde{S} = -\frac{i}{2\pi^2}\int\limits_0^{2\pi} d\varphi\left(\int\limits_0^{w_3} + \int\limits_{w_3}^{\infty}\right)\frac{dw}{w}\int\limits_{-w}^{w}\frac{t^3\left(t^2-h^2\right)dt}{\left(t^2-t_1^2\right)\left(t^2-t_2^2\right)}, \qquad (5.10)$$

where, with regard for the condition $t_2^2 = w^2$, $w_3^2 = 1 + h^2$.

1. If $t_1 < w < t_2$, the integral equals

$$\int\limits_0^w dt\, f(t) = \pi i \operatorname{res}[f(t), t_1] = \pi i\frac{t_1^2\left(t_1^2 - h^2\right)}{t_1^2 - t_2^2}. \qquad (5.11)$$

2. At $t_1 < t_2 < w$, substituting the values of residues at t_1 and t_2 into Eq. (5.9), we obtain

$$\int\limits_0^w dt\, f(t) = \pi i \sum_{n=1}^{2}\operatorname{res}[f(t), t_n] = \pi i\left(t_2^2 + t_1^2 - h^2\right). \qquad (5.12)$$

Consider two limiting cases: weak and strong magnetic fields. If $h \gg 1$, the final integral looks like

$$-\frac{d\mathcal{E}}{dt} = \frac{q^2\omega_p}{V_0}\left(\ln|w_{\max}| - \frac{1}{4} - \frac{1}{2}\ln\frac{h}{2}\right). \qquad (5.13)$$

In the case $h \ll 1$, we obtain

$$-\frac{d\mathcal{E}}{dt} = \frac{q^2\omega_p}{V_0}\left(\ln|w_{\max}| - \frac{h^2}{4} - \frac{h^4}{8}\ln\frac{h}{2}\right). \qquad (5.14)$$

From formula (5.14), one can see that, putting $h = 0$, we obtain–in accordance with the correspondence principle–results without magnetic field (3.3), where $w_{\max}$ is determined by equality (3.5).

6. General Case

Now, let us consider the case of the motion of a particle at an arbitrary angle with respect to the magnetic field $(\mathbf{V}_0\mathbf{H} = = |V_0||H|\cos\alpha)$. As was done above, the coordinate system is so selected that $\mathbf{H} \upuparrows \mathbf{z}$.

The energy losses by an ion in the cold magnetized electron plasma are given by expression (2.5). Let us rotate the coordinate frame around the axis $0y$ until $\mathbf{V_0} \upuparrows z$; then $\mathbf{wn}_{\mathbf{V}_0} = w_z$, equation

$$\begin{aligned} w_x &= w_x'\cos\alpha + w_z'\sin\alpha, \\ w_z &= -w_x'\sin\alpha + w_z'\cos\alpha, \\ w_y &= w_y'. \end{aligned} \qquad (6.1)$$

With the use of Eq. (2.10), let us rewrite the expression for the energy losses in the new coordinates (for convenience, the primes at the variables are omitted),

$$-\frac{d\mathcal{E}}{dt}=\frac{q^2\omega_{\mathrm{P}}^2}{V_0}\frac{1}{2\pi^2}\Im\iiint\frac{w_z\,d^3w}{g_0(w_x,w_y,w_z)}=\frac{q^2\omega_{\mathrm{P}}^2}{V_0}\tilde{S},\tag{6.2}$$

where the denominator $g_0(w_x,w_y,w_z)$ in the integrand, in view of Eq. (6.1), looks like

$$g_0(w_x,w_y,w_z)=\varepsilon_\perp\left(w_y^2-w_{y0}^2\right),\tag{6.3}$$

where

$$w_{y0}^2=-\frac{\varepsilon_\perp\cos^2\alpha+\varepsilon_\parallel\sin^2\alpha}{\varepsilon_\perp}\,(w_x-w_{x1})\,(w_x-w_{x2}),\tag{6.4}$$

and

$$w_{x1,2}=-w_z\frac{\cos\alpha\sin\alpha\left(\varepsilon_\perp-\varepsilon_\parallel\right)}{\varepsilon_\perp\cos^2\alpha+\varepsilon_\parallel\sin^2\alpha}\mp$$
$$\mp w_z\sqrt{\left(\frac{\cos\alpha\sin\alpha\left(\varepsilon_\perp-\varepsilon_\parallel\right)}{\varepsilon_\perp\cos^2\alpha+\varepsilon_\parallel\sin^2\alpha}\right)^2-\frac{\varepsilon_\parallel\cos^2\alpha+\varepsilon_\perp\sin^2\alpha}{\varepsilon_\perp\cos^2\alpha+\varepsilon_\parallel\sin^2\alpha}}.\tag{6.5}$$

In the case $\mathbf{V_0}\upuparrows z$, the components of the dielectric permittivity tensor are

$$\begin{aligned}\varepsilon_\perp&=\frac{z^2-{\omega_3}^2}{z^2-h^2},\\ \varepsilon_\parallel&=\frac{z^2-1}{z^2}.\end{aligned}\tag{6.6}$$

After some simplifications with regard for Eq. (6.6), Eqs. (6.4) and (6.5) read

$$w_{x1,2}=\frac{w_z}{(w_z^2-\omega_1^2)\,(w_z^2-\omega_2^2)}\times$$
$$\times\left(h^2\sin\alpha\cos\alpha\mp w_z\sqrt{(\omega_3^2-w_z^2)(w_z^2-h^2)(w_z^2-1)}\right),\tag{6.7}$$

$$w_{y0}^2=\frac{\left(w_z^2-\omega_1^2\right)\left(w_z^2-\omega_2^2\right)}{w_z^2\,(\omega_3^2-w_z^2)}\,(w_x-w_{x1})(w_x-w_{x2}).\tag{6.8}$$

Expressions (6.7) and (6.8) include the characteristic quantities of the problem,

$$\omega_{1,2}^2=\frac{1}{2}(\omega_3^2\mp\sqrt{\omega_3^4-4h^2\sin^2\alpha}),\tag{6.9}$$

where $\omega_3^2=1+h^2$. The dimensionless frequencies ω_1 and ω_2 are plasma resonances. Plasma resonances play a substantial role in the propagation of electromagnetic waves in plasma. In particular, the wave attenuation and the noise level drastically grow in their vicinity. The refractive index of electromagnetic waves is large ($n\gg1$) near those resonances, and the phase velocity is much less than the light one; i.e. the waves become slow, and, accordingly, the interaction between charged particles is the most effective near plasma resonances [13, 24]. The frequency ω_1 corresponds to lower hybrid frequencies, whose spectrum spans the interval $0\leqslant w\leqslant\min[1,h]$, whereas the frequency ω_2 corresponds to upper hybrid frequencies with the spectrum at $\max[1,h]\leqslant w\leqslant\omega_3$.

The quantities w_{x1}, w_{x2}, ω_1, ω_2, and ω_3 define the integration region. If $h>1$, the obtained characteristic quantities are related to one another as follows:

$$\omega_3\geqslant\omega_2\geqslant h>1\geqslant\omega_1.$$

Let us integrate in Eq. (6.2) over the variable w_y. According to the theory of residues, we obtain

$$\tilde{S}=\frac{1}{2\pi}\iint\limits_\sigma dw_z\frac{w_z^2(w_z^2-h^2)}{\sqrt{(\omega_3^2-w_z^2)(w_z^2-\omega_2^2)(w_z^2-\omega_1^2)}}\times$$
$$\times\frac{dw_x}{\sqrt{(w_x-w_{x1})(w_x-w_{x2})}}\tag{6.10}$$

The integral differs from zero if

$$w_{y0}^2(w_x,w_z)\geqslant0.\tag{6.11}$$

The cross-section of the integration region at $w_{y0}(w_x,w_z)=0$ is depicted in Figs. 3 and 4. At the points $w_z=h$, $w_z=1$, and $w_z=\omega_3$, the curves $w_{x1}(w_z)$ and $w_{x2}(w_z)$ transform into each other.

The integration region σ is determined by inequality (6.11). On the (w_x,w_z)-plane, this region is confined by the curves $w_{x1}(z)$ and $w_{x2}(z)$. Note that the functions $x_1(z)$ and $x_2(z)$ are real-valued in the frequency intervals

$$0\leqslant\omega_1\leqslant\min(1,h),\ \max(1,h)\leqslant\omega_2\leqslant\omega_3.$$

In the case of longitudinal motion ($\alpha=0$), the intervals $w_1\leq w\leq1$ and $h\leq w\leq w_2$ contribute to the energy losses; in the case of transverse one ($\alpha=\pi/2$), these are the intervals $0\leq w\leq w_1$ and

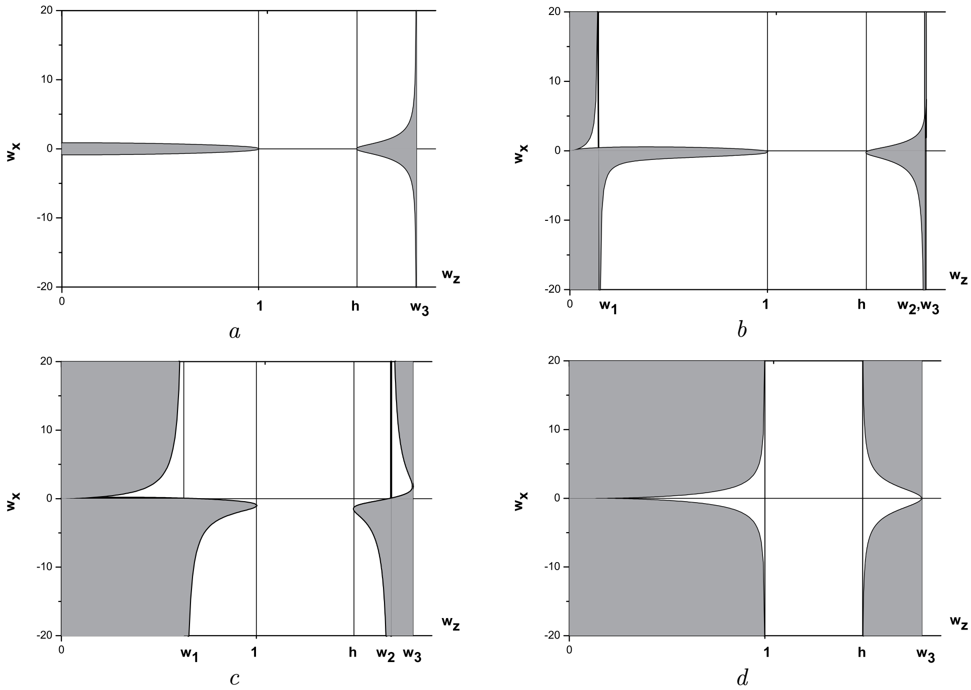


Fig. 3. Cross-section of the integration region ($w_{y0}(w_x, w_z) = 0$), $h = \omega_H/\omega_\mathrm{P} = 0.7$: ($a$) $\alpha = 0$, (b) $\alpha = \pi/18$, (c) $\alpha = \pi/4$, and (d) $\alpha = \pi/2$. Regions with real roots are colored

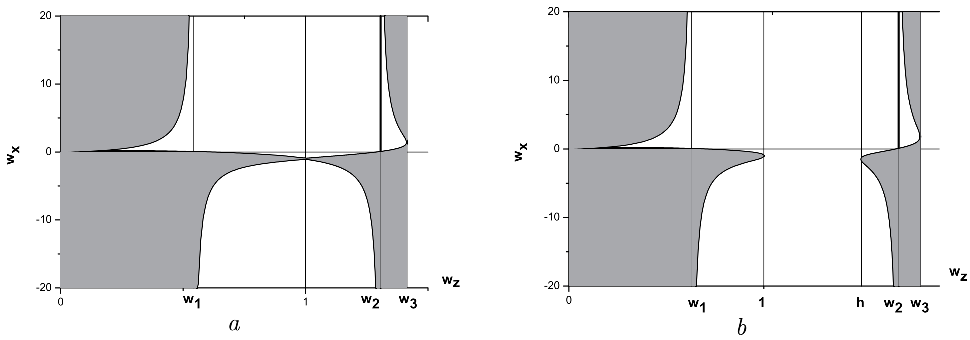


Fig. 4. Cross-section of the integration region ($w_{y0}(w_x, w_z) = 0$), $\alpha = 60$: (a) $h = 0$ and (b) $h = 1.7$. Regions with real roots are colored

$w_2 \le w \le w_3$. In the general case, the energy losses are a sum of contributions from all four intervals,

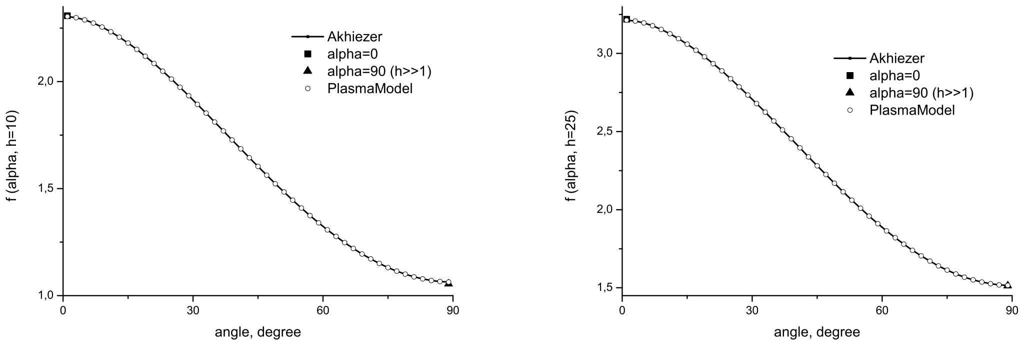
$$\tilde{S} = \frac{1}{\pi} \sum_{\beta=1}^{4} I_\beta, \tag{6.12}$$

where

$$I_1 = \int_0^{\omega_1} dw_z \frac{w_z^2(h^2 - w_z^2)}{\sqrt{P(w_z)}} \times \left(\int_{-\infty}^{w_{x1}} \frac{dw_x}{\sqrt{R(w_x, w_z)}} + \int_{w_{x2}}^{\infty} \frac{dw_x}{\sqrt{R(w_x, w_z)}} \right), \tag{6.13}$$

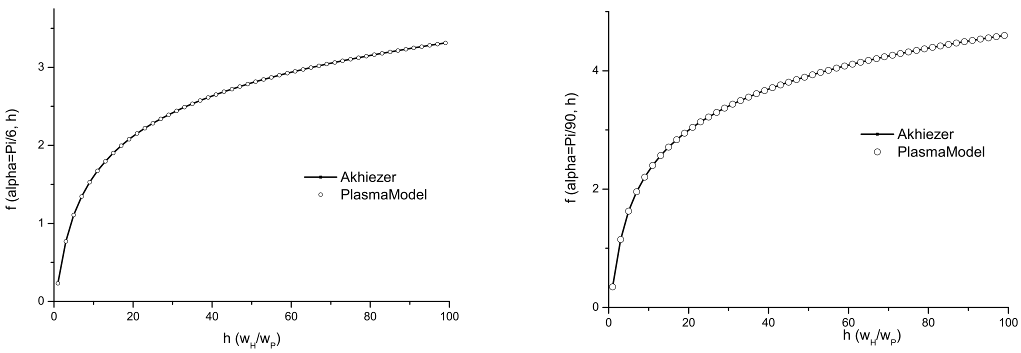
$$I_2 = \int_{\omega_1}^{\min(1,h)} dw_z \frac{w_z^2(h^2 - w_z^2)}{\sqrt{-P(w_z)}} \int_{w_{x2}}^{w_{x1}} \frac{dw_x}{\sqrt{-R(w_x, w_z)}}, \tag{6.14}$$

$$I_3 = \int_{\max(1,h)}^{\omega_2} dw_z \frac{w_z^2(w_z^2 - h^2)}{\sqrt{-P(w_z)}} \int_{w_{x2}}^{w_{x1}} \frac{dw_x}{\sqrt{-R(w_x, w_z)}}, \tag{6.15}$$



a *b*

Fig. 5. Angular dependences of the function $f(\alpha, h)$ at (a) $\omega_H/\omega_{\rm P} = 10$ and (b) $\omega_H/\omega_{\rm P} = 10$. The solid curve corresponds to the results of quantum field theory [13], squares to $\alpha = 0$ and expression (4.13), triangles to $\alpha = \pi/2$ and expression (5.13) at $h \gg 1$, and circles to the dielectric model



a *b*

Fig. 6. Dependenced of the function $f(\alpha, h)$ on the magnetic field parameter $h = \omega_H/\omega_{\rm P}$ at (a) $\alpha = \pi/4$ and (b) $\alpha = \pi/2$. The solid curve corresponds to the results of quantum field theory [13], and circles to the dielectric model

$$I_4 = \int\limits_{\omega_2}^{\omega_3} dw_z \frac{w_z^2(w_z^2 - h^2)}{\sqrt{P(w_z)}} \times$$
$$\times \left(\int\limits_{-\infty}^{w_{x1}} \frac{dw_x}{\sqrt{R(w_x, w_z)}} + \int\limits_{w_{x2}}^{\infty} \frac{dw_x}{\sqrt{R(w_x, w_z)}} \right), \qquad (6.16)$$

$P(w_z) = (\omega_3^2 - w_z^2)(w_z^2 - \omega_2^2)(w_z^2 - \omega_1^2)$, and $R(w_x, w_z) = (w_x - w_{x1})(w_x - w_{x2})$. Equations (6.13) and (6.14) describe the contributions from lower hybrid oscillations, and Eqs. (6.15) and (6.16) from upper ones. Using the correspondence principle, Eq. (6.12) can be rewritten in the form

$$-\frac{d\mathcal{E}}{dt} = \frac{q^2\omega_{\rm P}^2}{V_0} \left(\ln w_{\max} - f(h, \alpha)\right). \qquad (6.17)$$

The exact expression (6.12) was analyzed numerically to find the normalized polarization losses, its dependence on the magnetic field parameter $h = \omega_H/\omega_{\rm P}$, and the arrival angle of a test particle. The numerical results obtained for the additional term $f(h, \alpha)$ are depicted in Figs. 5 and 6. In particular, Fig. 5 demonstrates that, if the angle between the motion direction of the test charged particle and the force lines of the external magnetic field increases, the additive $f(h, \alpha)$ decreases. In other words, the transverse energy losses by the particle are larger than the longitudinal ones. Figure 6 illustrates the dependence of the function $f(h, \alpha)$ on the external magnetic field strength. At low field values, $h \ll 1$, the additional term tends to zero in accordance with the correspondence principle. At large field values, $h \gg 1$, the correction monotonously grows. The results of numerical calculations were compared with the analytical results obtained in the cases of both transverse ($\alpha = \pi/2$) and longitudinal

($\alpha = 0$) motions of a particle with respect to the magnetic field.

7. Conclusions

The dielectric model of energy losses by a charged particle at its motion in magnetized electron plasma has been developed. A new criterion for the verification of the reliability of the results obtained has been proposed in the accordance with the correspondence principle, which does not demand engaging other theoretical models. Analytical expressions were derived for the cases of no magnetic field and a magnetic field with a particle moving along and across it. The results obtained are confirmed by the numerical calculations carried out for the cases of a charged particle moving at an arbitrary angle with respect to the magnetic field and an arbitrary magnetic field strength. The validity of the dielectric model for the description of energy losses by collective effects is ultimately confirmed by a comparison of the results obtained in the framework of this model and the quantum field theory.

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ДІЕЛЕКТРИЧНА МОДЕЛЬ ЕНЕРГЕТИЧНИХ ВТРАТ ВАЖКОЇ ЗАРЯДЖЕНОЇ ЧАСТИНКИ ПРИ РУСІ В ХОЛОДНОМУ ЗАМАГНІЧЕНОМУ ЕЛЕКТРОННОМУ ГАЗІ

Р е з ю м е

У рамках діелектричної моделі знайдено втрати енергії для зарядженої частинки у нескінченній замагніченій електронній плазмі. В роботі використано принцип відповідності, що дало змогу не залучати феноменологічного параметра обрізання для зшивки з теорією парних зіткнень. Отримано аналітичні вирази для втрат енергії у випадку поздовжнього та поперечного магнітному полю рухів зарядженої частинки. Аналітичні результати підтверджені чисельними розрахунками, проведеними для випадку руху зарядженої частинки під довільним кутом до магнітного поля, довільної напруженості. Проведено порівняння втрат енергії зарядженою частинкою, отриманих в рамках діелектричної моделі, з результатами квантової теорії поля.