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RELATIVISTIC INVARIANCE AND MASS RENORMALIZATION IN QUANTUM FIELD THEORY

Starting from the instant form of relativistic quantum dynamics for a system of interacting fields, where only the Hamiltonian and the boost operators carry interactions among ten generators of the Poincaré group, we propose a constructive way of ensuring the relativistic invariance (RI) in quantum field theory (QFT) with cutoffs in the momentum space. Our approach is based on an opportunity to separate a part in the primary Hamiltonian interaction, whose density in the Dirac (D) picture is the Lorentz scalar. In this work, we study the compatibility of the RI requirements as a whole, i.e., the fulfilment of the well-known commutations for these generators with the structure of mass counterterms in the total field Hamiltonian.

K e y w o r d s: mass renormalization, relativistic invariance, quantum field theory.

1. Introduction

After P. Dirac [1], any relativistic quantum theory may be so defined that the generator of time translations (Hamiltonian, H), generators of space translations (linear momentum, $\mathbf{P}$), space rotations (angular momentum, $\mathbf{J}$), and Lorentz transformations (boost operator, $\mathbf{N}$) satisfy the Lie–Poincaré commutations. The basic ideas put forward by P. Dirac with his "front", "instant," and "point" forms of the relativistic dynamics have been realized in many relativistic quantum mechanical models. In this context, survey [2] reflects various aspects and achievements of relativistic direct interaction theories. Among the vast literature on this subject, we would like to note an exhaustive exposition of appealing features of the relativistic Hamiltonian dynamics in [3] with an emphasis on the "light-cone quantization".

Motivations for our endeavor to contribute to this area are exposed in [4]. There, in the framework of the instant form of the relativistic quantum dynamics for a system of interacting fields, we offered an algebraic method to integrate the Poincaré commutators. We have not employed the Lagrangian formalism with its Noether representation of the generators for local fields. Our consideration is focused upon various field models, in which the operator H, being divided into the no-interaction part H_F and the interaction one $H_I = \int H_I(\mathbf{x})d\mathbf{x}$, has the interaction density $H_I(\mathbf{x})$ that consists of the scalar $H_{\rm sc}(\mathbf{x})$ and nonscalar $H_{\rm nsc}(\mathbf{x}$ contributions, viz.,

$$H_I(\mathbf{x}) = H_{\rm sc}(\mathbf{x}) + H_{\rm nsc}(\mathbf{x}). \quad (1)$$

The property to be a scalar means

$$U_F(\Lambda)H_{\rm sc}(x)U_F^{-1}(\Lambda) = H_{\rm sc}(\Lambda x) \quad \forall x = (t, \mathbf{x}) \quad (2)$$

with unitary transformations $U_F(\Lambda)$ that realize an irreducible representation of the Lorentz group L_+: $\Lambda \to U_F(\Lambda)$ $\forall\Lambda \in L_+$ on the corresponding Fock space $\mathcal{F}$. As usual, for any operator $O(\mathbf{x})$ in the

Schrödinger (S) picture, there is its counterpart $O(x) = \exp(iH_F t)O(\mathbf{x})\exp(-iH_F t)$ in the D picture. Such models are typical of the meson theory of nuclear forces, where one has to consider vector-meson exchanges and introduce meson-nucleon vertices with cutoffs in the momentum space (see, e.g., [5]).

Decomposition (2) is our starting point in constructing the boosts in the case of both local field models (e.g., with derivative couplings and spins ≥ 1) and their nonlocal extensions. The finding of analytic expressions for them is simplified in the clothed-particle representation (CPR), in which the so-called bad terms are simultaneously removed from the Hamiltonian and boosts [6–8], so these operators acquire the same sparse structure in $\mathcal{F}$. In what follows, we will show how the mass renormalization terms introduced in the Hamiltonian at the very beginning turn out to be related to certain covariant integrals that are convergent in field models with appropriate cutoff factors.

2. Algebraic Approach within Hamiltonian Formalism

In this context, let us quote from Chapter VII of [9]: "... in theories with derivative couplings or spins $j \geq 1$, it is not enough to take Hamiltonian as the integral over space of a scalar interaction density; we also need to add nonscalar terms to the interaction density to compensate noncovariant terms in the propagators". For example, it is the case where the pseudoscalar (π and η), vector (ρ and ω), and scalar (δ and σ) meson (boson) fields interact with the 1/2 spin (N and $\bar{N}$) fermion ones via the Yukawa-type couplings $V = \sum_b V_b = V_s + V_{ps} + V_{\mathrm{v}}$ in $H_I = V +$ mass and vertex counterterms.

Let us take into account that the first relation in (11) in [4] is equivalent to $[\mathbf{N}_F, H_I] = [H, \mathbf{N}_I]$, and let us consider the operator $H_{\mathrm{sc}}(t) = \int H_{\mathrm{sc}}(x)d\mathbf{x}$ and its similarity transformation

$$e^{i\beta\mathbf{N}_F} H_{\mathrm{sc}}(t) e^{-i\beta\mathbf{N}_F} = \int H_{\mathrm{sc}}(L(\beta)x)d\mathbf{x}, \tag{3}$$

where $L(\beta)$ is any Lorentz transformation with parameters $\beta = (\beta^1, \beta^2, \beta^3)$ that are related to the velocity of a moving frame, $\mathbf{N}_F$ and $\mathbf{N}_I$ are the free and interaction parts of the boost operator $\mathbf{N} = \mathbf{N}_F + \mathbf{N}_I$. One can show [4] that the commutation $[H, \mathbf{N}] = i\mathbf{P}$ is fulfilled if, along with the Belinfante-type relation

$$\mathbf{N}_I = \mathbf{N}_B \equiv -\int \mathbf{x} H_{\mathrm{sc}}(\mathbf{x})d\mathbf{x}, \tag{4}$$

the interaction density meets the equation

$$\int \mathbf{x}d\mathbf{x} \int d\mathbf{x}'[H_{\mathrm{sc}}(\mathbf{x}'), H_{\mathrm{sc}}(\mathbf{x})] = [H_{\mathrm{nsc}}.\mathbf{N}], \tag{5}$$

This implies that, in a model with $H_{\mathrm{nsc}} = 0$, we would arrive to the microcausality condition $[H_I(x'), H_I(x)] = 0$ for $(x'-x)^2 \leq 0$ at equal times. But the latter and Eq. (4) may be incompatible.

It forces us to seek an alternative, by introducing $\mathbf{N}_I = \mathbf{N}_B + \mathbf{D}$ to get

$$[H_F, \mathbf{D}] = [\mathbf{N}_B + \mathbf{D}, H_{\mathrm{sc}}] + [\mathbf{N}_F + \mathbf{N}_B + \mathbf{D}, H_{\mathrm{nsc}}], \tag{6}$$

which replaces the relation $[H, \mathbf{N}] = i\mathbf{P}$ and determines the displacement $\mathbf{D}$. Further, by assuming that the scalar density $H_{\mathrm{sc}}(\mathbf{x})$ is of the first order in the coupling constants involved and by putting

$$H_{\mathrm{nsc}}(\mathbf{x}) = \sum_{p=2}^{\infty} H_{\mathrm{nsc}}^{(p)}(\mathbf{x}), \tag{7}$$

we search the operator $\mathbf{D}$ in the form

$$\mathbf{D} = \sum_{p=2}^{\infty} \mathbf{D}^{(p)}, \tag{8}$$

i.e., as a perturbation expansion in powers of the interaction H_{sc}. This leads to the chain of relations

$$[H_F, \mathbf{D}^{(2)}] = [\mathbf{N}_F, H_{\mathrm{nsc}}^{(2)}] + [\mathbf{N}_B, H_{\mathrm{sc}}], \tag{9}$$

$$H_F, \mathbf{D}^{(3)}] = [\mathbf{N}_F, H_{\mathrm{nsc}}^{(3)}] + [\mathbf{D}^{(2)}, H_{\mathrm{sc}}] + \\ + [[\mathbf{N}_B, H_{\mathrm{nsc}}^{(2)}], ..., \tag{10}$$

$$[P_k, D_j^{(p)}] = i\delta_{kj} H_{\mathrm{nsc}}^{(p)} \quad (p = 2, 3, ...) \tag{11}$$

from $[P_k, D_j] = i\delta_{kj} H_{\mathrm{nsc}}, (j, k = 1, 2, 3)$, *etc.* for a recursive finding of the operators $\mathbf{D}^{(p)}$ $(p = 2, 3, ...)$ (details in [4], where our results are compared with those in [10] and [11]). In particular, keeping in mind the elegant Chandler method and following [12], we note the property of a formal solution Y of the equation $[H_F, Y] = X$ to be any linear functional $F(X)$ of a given operator $X \neq 0$ and its useful consequences. In this context, we recall the proof [6, 7] of the existence of a solution

$$Y = -i \lim_{\eta \to 0+} \int_0^{\infty} X(t) e^{-\eta t} dt. \tag{12}$$

3. Application to a Nonlocal Model

As an illustration, we have considered [4] an extension of the Wentzel model with

$$H_I = V_{\rm nloc} + M_s + M_b, \quad V_{\rm nloc} = V_b + V_b^\dagger, \tag{13}$$

in which

$$V_b = \int V_b(\mathbf{x}) d\mathbf{x} = \int \frac{d\mathbf{k}}{\omega_\mathbf{k}} : F_b^\dagger G(k) F_b : a(k), \tag{14}$$

v.s. the original local Wentzel model with $H_I = \int H_I(\mathbf{x}) d\mathbf{x} = V_{\rm loc} + V_{\rm ren}$, $H_I(\mathbf{x}) = V_{\rm loc}(\mathbf{x}) + V_{\rm ren}(\mathbf{x})$, $V_{\rm loc}(\mathbf{x}) = g\varphi_s(\mathbf{x}) : \psi_b^\dagger(\mathbf{x})\psi_b(\mathbf{x}) :$, $V_{\rm ren}(\mathbf{x}) = \delta\mu_s : \varphi_s^2(\mathbf{x}) : + \delta\mu_b : \psi_b^\dagger(\mathbf{x})\psi_b(\mathbf{x}) :$ and mass shifts $\delta\mu_s = \frac{1}{2}(\mu_{0s}^2 - \mu_s^2)(\delta\mu_b = (\mu_{0b}^2 - \mu_b^2))$ for charged spinless bosons (neutral scalar ones) or, in the particle number representation,

$$V_b = \int V_b(\mathbf{x}) d\mathbf{x} = \int \frac{d\mathbf{k}}{\omega_\mathbf{k}} : F_b^\dagger G_0(k) F_b : a(k). \tag{15}$$

Henceforth, as before [7], we prefer to deal with the matrix form

$$\int \frac{d\mathbf{p'}}{E_{\mathbf{p'}}} \int \frac{d\mathbf{p}}{E_\mathbf{p}} F_b^\dagger(p') X(p',p) F_b(p) \equiv F_b^\dagger X F_b$$

for any 2×2 matrix $X(p',p)$ and the row $F_b^\dagger(p) = \{b^\dagger(p), d(p)\} \equiv \{F_1^\dagger(p), F_2^\dagger(p)\}$ with the commutations $[F_{\varepsilon'}(p'), F_\varepsilon^\dagger(p)] = p_0 \delta(\mathbf{p'} - \mathbf{p})\sigma_{\varepsilon'\varepsilon}$, where $\sigma_{\varepsilon'\varepsilon} = (-1)^{\varepsilon-1}\delta_{\varepsilon'\varepsilon}$ in the space of charge indices $(\varepsilon', \varepsilon = 1, 2)$.

In such notations, the matrix $G(k)$ is composed of elements

$$G_{\varepsilon'\varepsilon}(p', p, k) = g_{\varepsilon'\varepsilon}(p', p, k)\delta\big(\mathbf{k} + (-1)^{\varepsilon'}\mathbf{p'} - (-1)^\varepsilon \mathbf{p}\big), \tag{16}$$

while $G_0(k)$ is obtained from (16) by putting the g coefficients equal to 1 in it. Furthermore, we retain the property

$$U_F(\Lambda) V_{\rm nloc}(x) U_F^{-1}(\Lambda) = V_{\rm nloc}(\Lambda x) \tag{17}$$

for our nonlocal interaction density to be the Lorentz scalar. The latter takes place if the "cutoff" functions $g_{\varepsilon'\varepsilon}(p', p, k)$ have the property $g_{\varepsilon'\varepsilon}(\Lambda p', \Lambda p, \Lambda k) = g_{\varepsilon'\varepsilon}(p', p, k)$. The transition $V_{\rm loc} \Rightarrow V_{\rm nloc}$ can be interpreted as an endeavor to regularize the model. In the context, the introduction of such functions in the momentum space is aimed at removing the ultraviolet divergences typical of local field models with trilinear Yukawa-type interactions. These cutoffs are subject to the additional constraints imposed by different symmetries. For example, we mean the invariance of the Hermitian $V_{\rm nloc}$ with respect to i) space inversion $\mathcal{P}$; ii) time reversal $\mathcal{T}$, and iii) charge conjugation $\mathcal{C}$, which yields $g_{\varepsilon'\varepsilon}(p', p, k) = g_{\varepsilon'\varepsilon}(p, p', k)$, $\varepsilon' \neq \varepsilon$, $g_{\varepsilon'\varepsilon}(p', p, k) = g_{\varepsilon'\varepsilon}(p'_-, p_-, k_-)$ and $g_{11}(p', p, k) = g_{22}(p', p, k)$.

The structure of the "mass renormalization" terms M_s and M_b,

$$M_s = \int \frac{d\mathbf{k}}{\omega_\mathbf{k}^2} \{m_1(k) a^\dagger(k) a(k) + m_2(k)[a^\dagger(k) a^\dagger(k_-) + a(k) a(k_-)]\}, \tag{18}$$

$$M_b = \int \frac{d\mathbf{p}}{E_\mathbf{p}^2} \{m_{11}(p) b^\dagger(p) b(p) + m_{12}(p) b^\dagger(p) d^\dagger(p_-) + m_{21}(p) b(p) d(p_-) + m_{22}(p) d^\dagger(p) d(p)\}, \tag{19}$$

is prompted with the clothing procedure developed in [6–8]. In these formulae, the coefficients $m_{1,2}(k)$ and $m_{\varepsilon'\varepsilon}(p)$, being for the time unknown, may be momentum-dependent.

4. Mass Renormalization and Relativistic Invariance as a Whole after Dirac

We have seen [4] how, in the framework of the nonlocal meson-boson model, one can build the $2 \to 2$ interactions between the clothed mesons and bosons. They appear in a natural way from the commutator $\frac{1}{2}[R_{\rm nloc}, V_{\rm nloc}]$ as the operators $b^\dagger a^\dagger ba$, $d^\dagger a^\dagger da$, $b^\dagger b^\dagger bb$, $b^\dagger d^\dagger bd$, $d^\dagger d^\dagger dd$, $b^\dagger d^\dagger aa$, $a^\dagger a^\dagger bd$ of the class [2.2]. Moreover, this commutator is a spring of the good operators $a^\dagger a$, $b^\dagger b$, and $d^\dagger d$ of the class [1.1] together with the bad operators aa and bd of the class [0.2] (henceforth, for brevity, we omit the subscript "c") and their Hermitian conjugates $a^\dagger a^\dagger$ and $b^\dagger d^\dagger$ of the class [2.0]. These operators may be cancelled by the respective counterterms from

$$H_{\rm nsc}(\alpha) = M_s(\alpha) + M_b(\alpha) \tag{20}$$

on the r.h.s. of Eq. (159) in [4]. Such a cancellation gives rise to certain definitions of the mass coefficients in Eqs. (135) and (136) in [4].

With the help of the same technique as in [7], one can show that

$$\frac{1}{2}[R_{\rm nloc}, V_{\rm nloc}](a^\dagger a) =$$
$$= -\frac{1}{2}\int \frac{d\mathbf{k}}{\omega_{\mathbf{k}}^2}\int \frac{d\mathbf{p}}{E_{\mathbf{p}}E_{\mathbf{p}-\mathbf{k}}}\Big[\frac{g_{21}^2(p,q_-,k_-)}{E_{\mathbf{p}}+E_{\mathbf{p}-\mathbf{k}}+\omega_{\mathbf{k}}}+$$
$$+\frac{g_{12}^2(p,q_-,k)}{E_{\mathbf{p}}+E_{\mathbf{p}-\mathbf{k}}-\omega_{\mathbf{k}}}\Big]a^\dagger(k)a(k), \tag{21}$$

where $q=(E_{\mathbf{p}-\mathbf{k}},\mathbf{p}-\mathbf{k})$. In the same way, we obtain

$$\frac{1}{2}[R_{\rm nloc}, V_{\rm nloc}](aa) =$$
$$= -\frac{1}{2}\int \frac{d\mathbf{k}}{\omega_{\mathbf{k}}^2}\int \frac{d\mathbf{p}}{E_{\mathbf{p}}E_{\mathbf{p}-\mathbf{k}}} g_{12}(p,q_-,k)g_{21}(p,q_-,k_-)\times$$
$$\times\left[\frac{1}{E_{\mathbf{p}}+E_{\mathbf{p}-\mathbf{k}}+\omega_{\mathbf{k}}}+\frac{1}{E_{\mathbf{p}}+E_{\mathbf{p}-\mathbf{k}}-\omega_{\mathbf{k}}}\right]\times$$
$$\times\, a(k)a(k_-) \tag{22}$$

or

$$\frac{1}{2}[R_{\rm nloc}, V_{\rm nloc}](aa) =$$
$$= \int \frac{d\mathbf{k}}{\omega_{\mathbf{k}}^2}\int \frac{d\mathbf{p}}{E_{\mathbf{p}}} g_{12}(p,q_-,k)g_{21}(p,q_-,k_-)\times$$
$$\times\left[\frac{1}{\mu_s^2+2p_-k}+\frac{1}{\mu_s^2-2pk}\right]a(k)a(k_-). \tag{23}$$

We recall that the last transition can be done by means of some trick considered in Appendix A in [7].

Furthermore, assuming that

$$M_s^{(2)}(\alpha)+\frac{1}{2}[R_{\rm nloc},V_{\rm nloc}]_{\rm 2mes}=0 \tag{24}$$

with

$$[R_{\rm nloc},V_{\rm nloc}]_{\rm 2mes}=[R_{\rm nloc},V_{\rm nloc}](a^\dagger a)+$$
$$+[R_{\rm nloc},V_{\rm nloc}](aa)+[R_{\rm nloc},V_{\rm nloc}](a^\dagger a^\dagger),$$

we obtain, e.g.,

$$m_1^{(2)}(k)=\frac{1}{2}\int\frac{d\mathbf{p}}{E_{\mathbf{p}}E_{\mathbf{p}-\mathbf{k}}}\left[\frac{g_{21}^2(p,q_-,k_-)}{E_{\mathbf{p}}+E_{\mathbf{p}-\mathbf{k}}+\omega_{\mathbf{k}}}+\right.$$
$$\left.+\frac{g_{12}^2(p,q_-,k)}{E_{\mathbf{p}}+E_{\mathbf{p}-\mathbf{k}}-\omega_{\mathbf{k}}}\right]. \tag{25}$$

To facilitate the further analysis following [4], we propose to handle the cutoffs $g_{\varepsilon'\varepsilon}(p',p,k) = v_{\varepsilon'\varepsilon}([k+(-1)^{\varepsilon'}p'-(-1)^{\varepsilon}p][k-(-1)^{\varepsilon'}p'+(-1)^{\varepsilon}p])$ that possess necessary properties to reduce similar triple integrals to simple ones. In particular, we obtain

$$m_1^{(2)}(k)=m_2^{(2)}(k)\equiv m_s^{(2)}=$$
$$=8\pi\int\limits_0^\infty\frac{t^2dt}{\sqrt{t^2+\mu_b^2}}\frac{f^2(\mu_s^2-4t^2-4\mu_b^2)}{4t^2+4\mu_b^2-\mu_s^2}, \tag{26}$$

by putting $v_{12}(x)=v_{21}(x)=f(x)$. Our calculation in [4] with the popular form $f(x)=g(\Lambda^2-\mu_s^2)(\Lambda^2+\mu_s^2-4\mu_b^2-x)^{-1}$ showed that $m_s^{(2)}$ values considerably decrease when moving from large Λ values (smeared cutoffs) to smaller Λ's, i.e., the cutoffs more localized in the momentum space. It is equivalent to an effective weakening of the initial nonlocal interaction with its coupling constant g. In addition, it turns out that, at moderate Λ values $\sim$1 GeV (typical of the theory of meson-nucleon interactions), the respective numerical deviations from the free boosts can be small.

At last, one should emphasize that if one starts from expansion (7) with the second-order contribution $H_{\rm nsc}^{(2)}=0$, then RI would be violated at the beginning because of the obvious discrepancy between Eqs. (9) and (11).

5. Conclusions

We have proposed a way of ensuring RI in QFT with cutoffs in the momentum space. In contrast to the traditional approach, where the generators of Π are determined as the Noether integrals of the energy-momentum density tensor, our purpose is to find these generators as elements of the Lie algebra of Π starting from the total Hamiltonian, whose interaction density in the D-picture includes a Lorentz-scalar part $H_{\rm sc}(x)$. In this context, using a purely algebraic means, the boost generators can be decomposed into the Belinfante operator built up of $H_{\rm sc}$ and the operator which accumulates the chain of recursive relations in the second and higher orders in $H_{\rm nsc}$. Thereby, it becomes clear that the Poincaré commutations are not fulfilled if the Hamiltonian does not contain some additional ingredients, which we call mass renormalization terms. We have shown how the method of

UCTs enables us to determine the corresponding operators for a given model.

We see that our approach is sufficiently flexible, being applied to the local field models with derivative couplings and spin $j \geq 1$. Its realization shown here for nonlocal extensions of the well-known Yukawa-type couplings gives us an encouraging impetus, when constructing the interactions between clothed particles simultaneously in the Hamiltonian and the corresponding boost operator.

In our opinion, the approach exposed has promising prospects, e.g., in the theory of decaying states (after evident refinements), certainly in quantum electrodynamics, and, we believe, in quantum chromodynamics as well. Such endeavors are under way.

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ПЕРЕНОРМУВАННЯ МАСИ
ТА РЕЛЯТИВІСТСЬКА ІНВАРІАНТНІСТЬ
В КВАНТОВІЙ ТЕОРІЇ ПОЛЯ

Р е з ю м е

Починаючи з миттєвої форми релятивістської квантової динаміки для системи взаємодіючих полів, де серед десяти генераторів групи Пуанкаре тільки гамільтоніан та оператори бустів несуть взаємодії, ми пропонуємо конструктивний спосіб забезпечення релятивістської інваріантності в квантовій теорії з форм-факторами в імпульсному просторі. Наш підхід заснований на можливості відокремити в первинній густині взаємодії H_I частину, яка є Лоренцскаляром. Слід підкреслити, що, якщо виходити з внеску другого порядку $H^{(2)}_{\rm nsc} = 0$, то релятивістська інваріантність порушувалася б з самого початку через очевидну невідповідність між комутаційними співвідношеннями алгебри Пуанкаре.