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DIPOLE-EXCHANGE SPIN WAVES IN A FERROMAGNETIC NANOTUBE

Spin waves in a cylindrical ferromagnetic nanotube are studied. A nanotube with an external magnetic field applied parallel to its symmetry axis is considered. A linearized Landau–Lifshitz equation in the magnetostatic approximation is used with regard for the magnetic dipole-dipole interaction, exchange interaction, and anisotropy effects. As a result, the dispersion relation and the radial wavenumber spectrum for spin waves in the above-described nanotube are found. From the radial wavenumber spectrum, limitations on the transverse-angular modes are defined.

K e y w o r d s: spin wave, nanomagnetics, ferromagnetic nanotube, dipole-exchange theory.

1. Introduction

Waves of magnetization in magnetically ordered materials, the so-called spin waves [1,2], have been studied extensively both theoretically and experimentally in recent years. Magnetic spin waves are a subject of the study in new fields of research and technology such as magnonics [2] and spintronics [3]. Articles on the topic investigate, in particular, the spectrum and the dispersion relation for spin waves in media of various types [4–6] and the processes of reflection and passage of spin waves on the interface of two media [7, 8]. In numerous articles, the intense study of spin waves in thin ferromagnetic films [9–11], micron-sized magnetic quantum dots [12–14], nanowires [4, 15–17], and other nanostructures is conducted. Spin waves are promising for a variety of practical applications – for creating new data storage devices, data transmission devices, and so on.

One of the results of the nanotechnology advance in recent decades is the synthesis and the application of composite nanostructures. It is known that anomalous magnetic properties are typical ащк nanocomposites that contain a ferromagnet [18–23]. Spin waves in various nanocomposites – multilayer thin films [24, 25], ensembles of ferromagnetic nanoparticles in certain matrices [26], and so on – are widely studied. However, a little attention has been paid to spin waves in composite nanoparticles, which are the subject of a particular research interest. In particular, recently synthesized magnetic nanotubes [27–33] have found a wide range of applications, particularly in magnetobiology [34, 35]. Still, the spin waves in magnetic nanotubes currently attract a little attention, and the available papers on this subject investigate mostly spin solitons [36] and waves on magnetic domains interfaces [37, 38].

In this work, we study spin waves in a ferromagnetic nanotube. We have found a dispersion relation for the spin waves in such structure considering the magnetic dipole-dipole interaction, the exchange interaction, and the anisotropy effects. We have also found the radial wavenumber spectrum for such waves.

2. Statement of the Problem

Let us consider a nanotube, which is composed of a non-magnetic core and a ferromagnetic metallic shell, with the inner radius a and the external radius b. The outside material is also considered non-magnetic.

We consider the nanotube shell comprised of a ferromagnet that has an uniaxial magnetic anisotropy, with the magnetic anisotropy axis directed along the nanotube symmetry axis. We consider the ferromagnet of the "easy axis" type, so the saturation magnetization is also directed along the symmetry axis of the ferromagnetic shell. We assume that the fer-

romagnet is characterized by the following parameters: the uniaxial anisotropy parameter β (it is considered constant), the exchange interaction parameter α (the exchange energy tensor in the general case of an uniaxial crystal is diagonal and has two independent components; we consider the case where these components are equal, which is true for a cubic crystal, for a polycrystalline ferromagnet with small crystals, *etc.*). We assume that the saturation magnetization $\mathbf{M}_0$ of the shell is constant throughout the shell volume and is directed along the nanotube axis. We neglect the dissipation and, therefore, the damping of spin waves in the nanotube shell, discarding the relaxation terms in the Landau–Lifshitz equation. The gyromagnetic ratio γ of the nanotube ferromagnet is considered constant.

Let us consider a spin wave propagating in the shell of the above-described nanotube in parallel to its axis. Considering a typical nanotube length and corresponding wavenumber limitations, we have to take into account both the magnetic dipole-dipole interaction and the exchange interaction in the Landau–Lifshitz equation. We also have to keep the anisotropy addend in this equation, as we consider an uniaxial ferromagnet.

We apply the linearized spin wave theory, so the spin wave magnetization density and the magnetic field are small perturbations of the overall magnetization density and the overall magnetic field, correspondingly. Thus, a perturbation $\mathbf{m}$ of the magnetization density ($\mathbf{M} = \mathbf{M}_0 + \mathbf{m}$, where $\mathbf{M}$ is the overall magnetization) must satisfy the condition $|\mathbf{m}| \ll |\mathbf{M}_0|$.

The goal of this work is to obtain a dispersion relation and a radial wavenumber spectrum of the above-described spin waves.

3. Theoretical Background

Let us write down the Landau–Lifshitz equation for a nanotube described in the previous section. If the deviations of the magnetization $\mathbf{m}$ and of the magnetic field $\mathbf{h}$ inside the ferromagnet from their values in the ground state – $\mathbf{M}_0$ and $\mathbf{H}_0^{(i)}$, respectively – are small, the linearized Landau–Lifshitz equation inside the ferromagnetic tube (after omitting the damping term) has the form [1]

$$\frac{\partial \mathbf{m}}{\partial t} = \gamma\left(\mathbf{M}_0 \times \left(\mathbf{h} + \alpha \sum_i \frac{\partial^2 \mathbf{m}}{\partial x_i^2} + \beta \mathbf{n}\,(\mathbf{mn}) - \right.\right.$$

$$\left.\left. - \frac{1}{M_0^2}\left(\mathbf{M}_0\mathbf{H}_0^{(i)} + \beta\,(\mathbf{M}_0\mathbf{n})^2\right)\mathbf{m}\right)\right), \quad (1)$$

where $\mathbf{n}$ is a unit vector along the anisotropy axis of the system.

Let us direct the axis Oz along the symmetry axis of the system. As follows from the properties of "easy axis" ferromagnets, the saturation magnetization is directed along $\mathbf{n}$ and, consequently, along the axis Oz. Therefore, the ground-state magnetic field inside the ferromagnetic shell is also directed along Oz: $\mathbf{H}_0^{(e)} - 4\pi\hat{N}\mathbf{M}_0 = \mathbf{H}_0^{(i)} || Oz$, where $\mathbf{H}_0^{(e)}$ is an external field (outside the nanotube), $\hat{N}$ is the tensor of demagnetizing coefficients (for the symmetry of our system, $4\pi\hat{N}\mathbf{M}_0 = 0$). Using these relations, let us substitute $\mathbf{m}$ and $\mathbf{h}$ in the periodic-by-time form

$$\mathbf{m}(\mathbf{r}, t) = \mathbf{m}_0(\mathbf{r})\exp(i\omega t), \mathbf{h}(\mathbf{r}, t) = \mathbf{h}_0(\mathbf{r})\exp(i\omega t) \quad (2)$$

into Eq. (1). After considering the fact that $\mathbf{M}_0 \,||\, ||\,\mathbf{H}_0^{(e)}\,||\,\mathbf{n}\,||\,Oz$, $\mathbf{m}_0 \perp \mathbf{e}_z$, we obtain

$$i\omega\mathbf{m}_0 = \gamma\left(M_0\mathbf{e}_z \times \left(\mathbf{h}_0 + \alpha\Delta\mathbf{m}_0 - \left(\beta + \frac{H_0^{(e)}}{M_0}\right)\mathbf{m}_0\right)\right), \quad (3)$$

where $\mathbf{e}_z$ is a unit vector of the axis Oz.

In order to solve the Landau–Lifshitz equation, we need one more relation between the magnetization and the magnetic field. Let us use the magnetostatic approximation [1]. In this approximation, the magnetic field deviation $\mathbf{h}$ is a potential field: $\mathbf{h} = -\nabla\Phi$, $\mathbf{h}_0 = -\nabla\Phi_0$, where Φ is a magnetic potential, and $\Phi = \Phi_0(\mathbf{r})\exp(i\omega t)$. Using this approximation and the Maxwell equation $\mathrm{div}\mathbf{h} = -4\pi\,\mathrm{div}\mathbf{m}$, we obtain the sought relation:

$$\Delta\Phi - 4\pi\mathrm{div}\mathbf{M} = 0. \quad (4)$$

Equations (3) and (4) give us the necessary relationship between $\mathbf{m}$ and $\mathbf{h}$. Using this system of equations, we can find the dispersion relation and the wavenumber spectrum for spin waves in the shell.

4. Dispersion Relation and Wavenumber Spectrum

Let us find the dispersion relation for spin waves in the ferromagnetic nanotube, using the system of equations (3), (4).

In the system of equations (3), (4) (after taking the relation $m_{0z} = 0$ into account), the magnetization perturbation $\mathbf{m}$ can be eliminated. In this way, the system reduces to the following equation for the magnetic potential:

$$\left(\frac{\omega^2}{\gamma^2 M_0^2} - \left(\frac{H_0^{(e)}}{M_0} + \beta - \alpha\Delta\right)\left(\left(\frac{H_0^{(e)}}{M_0} + \beta\right) + \right.\right.$$
$$\left.\left. + 4\pi - \alpha\Delta\right)\right)\Delta\Phi_0 + 4\pi\left(\frac{H_0^{(e)}}{M_0} + \beta - \alpha\Delta\right)\frac{\partial^2\Phi_0}{\partial z^2} = 0. \quad (5)$$

Let us use the cylindrical coordinate system (ρ, θ,z). In these coordinates, Eq. (5) has solutions in the form

$$\Phi = (A_1 J_n(k_\perp \rho) + A_2 N_n(k_\perp \rho)) \exp(i(n\theta + k_\parallel z - \omega t)), \quad (6)$$

where A_1 and A_2 are constants, $J_n(k_\perp \rho)$ is the Bessel function of the order n, $N_n(k_\perp \rho)$ is the Neumann function of the order n, $k_\perp$ is a transverse wavenumber, and n is a transverse-angular mode number. (This form of solution becomes evident if we write down the relation $\Delta\Phi = -\left(k_\perp^2 + k_\parallel^2\right)\Phi$ for the potential given by (6). Note that, by the properties of the Bessel functions, the angular mode number n coincides with the order of the radial Bessel functions $J_n(k_\perp \rho)$, $N_n(k_\perp \rho)$, so we call it the "transverse-angular" mode number.) By substituting solution (6) into Eq. (5), we obtain a dispersion equation in the form

$$\alpha^2(k_\parallel^2 + k_\perp^2)^3 + 2\alpha(\tilde{\beta} + 2\pi)(k_\parallel^2 + k_\perp^2)^2 +$$
$$+ \left(\tilde{\beta}(\tilde{\beta} + 4\pi) - \frac{\omega^2}{\gamma^2 M_0^2} - 4\pi\alpha k_\parallel^2\right) \times$$
$$\times \left(k_\parallel^2 + k_\perp^2\right) - 4\pi\tilde{\beta}k_\parallel^2 = 0, \quad (7)$$

where $\tilde{\beta} = \beta + \frac{H_0^{(e)}}{M_0}$. This equation corresponds to the following dispersion relation:

$$\omega = \gamma M_0 \times$$
$$\times \sqrt{\alpha^2 k^4 + 2\alpha(2\pi + \tilde{\beta})k^2 + \tilde{\beta}(4\pi + \tilde{\beta}) - 4\pi k_\parallel^2\left(\alpha + \frac{\tilde{\beta}}{k^2}\right)}, \quad (8)$$

where the total wavenumber $k^2 = k_\parallel^2 + k_\perp^2$. Note that the result we obtained agrees with the dispersion relation obtained in [4, 39] for cylindrical nanowires. Thus, the transition from nanowires to nanotubes does not change the pattern of spin waves in the system. The dispersion relation we obtained considers the exchange effects.

At this point, we, in the general case, should apply the boundary conditions for the magnetic field (for both vectors $\mathbf{B}$ and $\mathbf{H}$) and solve equations for the magnetic potential with these boundary conditions both inside and outside the nanotube. In this way, we can obtain another necessary relation between the spin wave frequency ω and the wavenumber components k, $k_\parallel$. This method requires numerical calculations; however, as we shall see, for a cylindrical nanotube with a non-magnetic external material, the boundary conditions for the magnetization are sufficient to find this relation (namely, an orthogonal wavenumber spectrum) in an analytical form for volume spin-wave modes.

Let us impose exchange boundary conditions on the magnetization perturbation $\mathbf{m}$ (see, e.g., [1]) on the inner and the outer border of the nanotube. In the absence of the magnetic moment outside the shell, these boundary conditions can be written as $\mathbf{m}|_{\rho=a,b} = 0$, $\left.\frac{\partial \mathbf{m}}{\partial \rho}\right|_{\rho=a,b} = 0$ (as for the strong pinning case, see, e.g., [41]). The Maxwell equation (4) allows us to transform these boundary conditions into conditions for the potential Φ:

$$\Delta\Phi|_{\rho=a,b} = 4\pi\left(\frac{dm_{0\rho}}{d\rho} + \frac{m_{0\rho}}{\rho} + \frac{in}{\rho}m_{0\theta} + ik_\parallel m_{0z}\right)\Big|_{\rho=a,b} \times$$
$$\times \exp(i(n\theta + k_\parallel z - \omega t)) = 0, \quad (9)$$

where $\mathbf{m}$ can be represented in the form $\mathbf{m} = \mathbf{m}_0(\rho)\exp\left(i\left(n\theta + k_\parallel z - \omega t\right)\right)$. On the other hand, the condition $\Delta\Phi|_{\rho=a,b} = 0$ for volume spin waves can be rewritten as $k^2\Phi_0\big|_{a,b} = 0$, using the following property of the Bessel functions:

$$\frac{1}{\rho}\frac{d}{d\rho}\rho\frac{d}{d\rho}\Phi_0 = \left(-k_\perp^2 + \frac{n^2}{\rho^2}\right)\Phi_0. \quad (10)$$

Therefore, for $k \neq 0$, the boundary conditions for the magnetization $\Phi_0|_{a,b} = 0$ yield

$$A_1 J_n(k_\perp a) + A_2 N_n(k_\perp a) =$$
$$= A_1 J_n(k_\perp b) + A_2 N_n(k_\perp b) = 0. \quad (11)$$

In the general case, we have to solve this equation together with other relations (obtained from the

boundary conditions for the magnetic field) between the spin wave amplitudes and the orthogonal wavenumber. However, we note that, in our case (nonmagnetic external material), the boundary conditions for the magnetization in the form (11) can be reduced to a form that does not require such calculations after the division by the amplitude A_1:

$$J_n(k_\perp a)+\frac{A_2}{A_1}N_n(k_\perp a)=J_n(k_\perp b)+\frac{A_2}{A_1}N_n(k_\perp b)=0. \tag{12}$$

This system of two equations for two unknowns, really, allows us to obtain the orthogonal wavenumber spectrum without solving the system of equations with complete boundary conditions. Therefore, the orthogonal wavenumber spectrum is given by the system of transcendental equations (12).

For a wide shell with $k_\perp a \gg 1$ or for a shell that is thin compared to its width, so that $\frac{b-a}{a} \ll 1$, the expression for the transverse wave number spectrum (12) can be simplified significantly. (Since $k_\perp$ is of the same order of magnitude or greater than $\frac{1}{b-a}$, if the condition $\frac{b-a}{a} \ll 1$ is satisfied, the condition $k_\perp a \gg 1$ is also satisfied.) Using the asymptotics of Bessel functions, we can write $\Phi_0(\mathbf{r}) = \frac{C}{\sqrt{\rho}}\sin(k\rho+\delta)\exp\left(i\left(n\theta+k_\parallel z\right)\right)$, where C is a normalization constant and δ is an initial phase. Hence, for a thin shell, $k_\perp$ can be obtained from the boundary conditions (11) in the form

$$k_\perp = \frac{\pi p}{b-a}, \tag{13}$$

where p is any nonnegative integer. (Note that the thin shell condition $\frac{b-a}{a} \ll 1$ that allows for the transverse wavenumber to be written in the form (13) is satisfied for typical nanotubes.)

Note that the transverse wavenumber spectrum for a thin shell (13) is analogous to the spectrum of a particle in a one-dimensional potential well. So, the problem becomes quasi-one-dimensional for a thin shell.

5. Discussion

Let us analyze the dispersion relation (given by (8)) and the transverse wavenumber spectrum (given by (12) and (13)) for spin waves in a ferromagnetic nanotube.

First, we note that if the ferromagnetic shell is thin compared to the characteristic length of the exchange interaction ($b-a \ll l_{\text{ex}}$), so we can consider $k_\perp = 0$ (radial dependence of the magnetization on the shell thickness can be neglected), the dispersion relation (8) transforms into the following form:

$$\omega = \gamma M_0\left(\alpha k_\parallel^2 + \frac{H_0^{(e)}}{M_0} + \beta_j\right) \Leftrightarrow$$

$$\Leftrightarrow k_\parallel = \sqrt{\frac{1}{\alpha}\left(\frac{\omega}{\gamma M_0} - \frac{H_0^{(e)}}{M_0} - \beta\right)}. \tag{14}$$

The dispersion relation (14) for a thin shell agrees with the dispersion relation for a thin ferromagnetic film and for a thin cylindrical nanowire (see, e.g., [4, 39, 40]). Therefore, we can say that, for small enough (by one or two dimensions) ferromagnetic nanoobjects, the spin wave pattern is similar and becomes quasi-one-dimensional.

Second, let us make the numerical evaluations of the spin wave frequency given by (8) in the absence of an external magnetic field, by assuming that the longitudinal wavenumber is restricted, on the one hand, by the nanotube length (which makes unities or tens of micrometers for typical nanotubes), and, on the other hand, by the exchange interaction length (has the order of several nanometers for typical ferromagnets). Similar restrictions are imposed on the transverse wavenumber, with the addition that the transverse wavenumber nullifies when $n = 0$. Thus, both the longitudinal wavenumber $k_\parallel$ and the total wavenumber k for a typical nanotube change from 102 cm^{-1} to 106 cm^{-1} by the order of magnitude. For a typical ferromagnetic nanotube, $\beta \sim 1$ and $\alpha \sim 10^{-12}$ cm^{-2}. So, for a nanotube consisting of a material with the gyromagnetic ratio $\gamma = 107$ Hz/Gs and the saturation magnetization $M_0 = 103$ Gs (typical values for ferromagnets used in experiments), a spin wave frequency calculated with the use of (8) has the order of magnitude of 1010 Hz throughout the whole range of wavenumbers. (Note that if $k//$ and $k_\perp$ both tend to zero, which corresponds to the zero transverse-angular mode in an infinitely long nanotube, the frequency of oscillations $\omega = \gamma M_0\sqrt{\beta(4\pi+\beta)}$ also has an order of 1010 Hz.)

As one can see from (13), the transverse wavenumber increases, as the shell thickness decreases. Since typical shells are thin, this fact puts a limitation on the mode number n (because of a limitation on the

transverse wavenumber $k_\perp$ related to the exchange length). In particular, when the nanotube thickness is small compared to the exchange length ($b-a \ll l_{\text{ex}}$), the magnetization is uniform through the thickness of the shell, so we can consider $k_\perp = 0$. Therefore, for very thin tubes ($b-a < l_{\text{ex}}$), only a zero transverse-angular mode is possible ($n = 0$). A typical nanotube have a thickness of tens of nanometers; for such nanotubes, the number of possible transverse-angular modes has an order of $\frac{b-a}{l_{\text{ex}}} \sim 10$.

6. Conclusions

Therefore, we have developed a theory of dipole-exchange spin waves in ferromagnetic nanotubes. For a spin wave in a cylindrical nanotube composed of an "easy axis" ferromagnet, we have obtained a dispersion relation and a radial (transverse) wavenumber spectrum.

We have shown that the above-mentioned dispersion relation in the case of a nanotube, which is thin compared to the characteristic exchange interaction length, transforms into the dispersion relation for spin waves in a thin ferromagnetic nanowire and a thin ferromagnetic film. The dispersion relation for a nanotube that is thin compared to the exchange length becomes quadratic in the wavenumber.

We have also shown that, for a thin nanotube (the nanotube thickness is much less than its inner radius, which is true for a typical nanotube), the wavenumber levels become equidistant, so the wavenumber spectrum becomes quasi-one-dimensional. The distance between these levels is inversely proportional to the nanotube thickness.

The analysis of the exchange limitations shows that, for spin waves in the above-mentioned nanotubes, only the first N transverse-angular modes can be excited, where the number $N \sim \frac{b-a}{l_{\text{ex}}}$ (here, a and b are the inner and the outer radii of the nanotube, respectively, and l_{ex} is the characteristic exchange interaction length) has an order of 10 for a typical nanotube.

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ДИПОЛЬНО-ОБМІННІ СПІНОВІ
ХВИЛІ У ФЕРОМАГНІТНІЙ НАНОТРУБЦІ

Р е з ю м е

У роботі досліджено спінові хвилі у циліндричній феромагнітній нанотрубці. Розглянуто нанотрубку у зовнішньому магнітному полі, прикладеному паралельно до її осі симетрії. Використано лінеаризоване рівняння Ландау–Ліфшица у магнітостатичному наближенні з урахуванням магнітної диполь-дипольної взаємодії, обмінної взаємодії та ефектів анізотропії. В результаті знайдено дисперсійне відношення та спектр радіальних хвильових чисел для спінових хвиль у описаній вище нанотрубці. Зі спектра радіальних хвильових чисел визначено обмеження на поперечно-кутові моди.