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FREQUENCY LIMITS FOR CONDUCTING GRAPHENE CHANNEL CAUSED BY QUANTUM CAPACITANCE AND KINETIC INDUCTANCE

PACS 72.80.Vp, 73.63.-b

By analyzing the Boltzmann kinetic equation for mesosystems, it is shown that the quantum capacitance and the kinetic inductance, which are analogs of the electrostatic capacitance and the magnetic inductance, respectively, have to be taken into consideration while studying the dynamic conductivity of a graphene channel, despite their different physical nature. The account of the quantum conductance and the kinetic inductivity leads to the appearance of a maximum of the impedance. In the case where the graphene channel is an ideal Landauer resistor crossed by an electron without scattering, this maximum corresponds to the THz range (therefore, the effect does not worsen the frequency characteristics of graphene FETs operating in the GHz range). However, for massive graphene channels fabricated with the use of the CVD method, where the electron transport has the diffusive nature, this maximum corresponds to the kHz or MHz range depending on the carrier mobility and the channel length.

Keywords: graphene conducting channel, Boltzmann kinetic equation, quantum capacitance, kinetic inductance.

The dynamic conductivity of quantum nanosystems is an important theoretical problem for nanoelectronics (see, e.g., work [1]). The presentation of the main ideas and methods of this "bottom-down" approach proposed by S. Datta for the consideration of phenomena in nanoelectronics can be found in a series of methodological reviews by Yu.M. Kruglyak and M.V. Strikha published in the journal "Sensorna Elektronika i Mikrosystemni Tekhnologii" (issues from No. 4 (2012) [2] to No. 1 (2015)).

There are a considerable number of works devoted to the conductivity in metallic carbon nanotubes, where the Luttinger liquid theory is used (see, e.g., works [3–5]). However, this theory imposes rigid constraints on analyzed systems. Therefore, a general theory was developed in work [6] with the use of the current-carrying-mode formalism (see, e.g., works [1, 2]), which is applicable to the systems with various dimensions: from three-dimensional mesoscopic conductors to one-dimensional quantum wires. In work [6], it was shown that the correct solution of the Boltzmann kinetic equation for a mesoscopic conductor includes, in addition to the ordinary electrostatic capacitance and electromagnetic inductance, the quantum capacitance and the kinetic inductance. The new quantities can play a considerable role in special cases and govern the specific features of the dynamic conductivity in a system.

The role of the indicated effects in graphene was studied, e.g., in work [7] in the framework of the problem concerning the high-frequency limits for a graphene field-effect transistor. By applying the procedure of self-consistent quantum-mechanical simulation, the cutoff frequency of this transistor was shown to be close to the frequency $\frac{1}{\sqrt{LC}}$ associated with the kinetic inductance and the quantum capacitance of an ideal Landauer graphene resistor (an electron passes through such a resistor without scattering [1, 2]) and amounts to about 100 GHz for a graphene channel 1 μm in length. However, the reduction of the channel length to 10 nm allowed experimenters to reach the terahertz frequency range for today (see work [8] and the reference therein).

At the same time, the issue of characteristic frequencies for relatively massive graphene channels far from the perfect Landauer resistor (their fabrication became possible owing to the discovery of a cheap technology of graphene production by the chemical vapor deposition (CVD) technique [9]) remained beyond the scope of consideration till now. At the same

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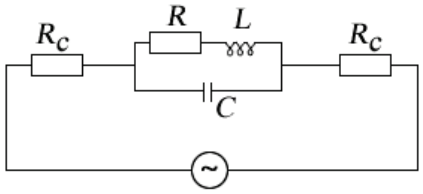

Equivalent circuit for the conducting graphene channel: R the ohmic resistance of the channel, R_C's are contact resistances, C the quantum capacitance, and L the kinetic inductance

time, such issues are challenging because, for example, it is on the CVD-graphene with large dimensions (800×200 μm^2) that the standard of quantum resistance was implemented [10] (according to the conclusion of the cited authors, this standard has advantages in comparison with modern GaAs-based devices). It is important that the cited authors observed a pronounced resonance at a frequency of 3.3 kHz in the frequency dependence of the Hall resistance against a weak descending linear background, which they considered as an artifact of the unknown origin.

Below, we will consider an electric circuit with a graphene resistor, which is connected to a source of ac voltage. Pay attention that, in the majority of problems concerning the graphene physics, the current-voltage characteristic is understood as the dependence of the current in a graphene channel on the gate voltage used for graphene "doping" with electrons or holes [11]. However, in our case, we consider the gate to be grounded. As will be shown below, the equivalent circuit scheme including the graphene channel can be represented as in Figure, where R_c are contact resistances (for simplicity, we consider them to be identical), R is the ohmic resistance of the graphene channel, C the quantum capacitance, and L the kinetic inductance, which emerge owing to the solution of Boltzmann kinetic equation following the procedure similar to that described in work [6].

Let us write down the Boltzmann kinetic equation for an electron that moves in the graphene channel between the contacts along the axis x (the graphene channel lies in the xy plane) in the standard form

$$\frac{\partial f}{\partial t} + \mathbf{v}_x \cdot \boldsymbol{\nabla}_x f - \frac{e}{\hbar}\mathbf{E}_x \cdot \boldsymbol{\nabla}_{k_x} f = \hat{S}_{\rm int} f, \tag{1}$$

where $\hat{S}_{\rm int}$ is the collision integral, and the electric field is related to the electrostatic potential by the expression $\mathbf{E}_x = \hat{x}\,\partial(U/e)/\partial x$ (e is the electron charge). Then Eq. (1) can be rewritten in the form

$$\frac{\partial f}{\partial t} + \mathbf{v}_x \cdot \boldsymbol{\nabla}_x f - \frac{\partial U}{\partial x}\frac{\partial f}{\partial p_x} = \hat{S}_{\rm int} f. \tag{2}$$

In Eqs. (1) and (2),

$$f \equiv \frac{1}{\exp\left[\dfrac{E_m(p,x,t)}{kT}\right] + 1}$$

is the nonequilibrium Fermi distribution function for electrons moving along the channel at the velocity $\mathbf{v}_x$ and with the two-dimensional momentum with the absolute value p, and $E_m(p,x,t)$ is the electron energy that makes allowance for both the linear graphene spectrum and the presence of discrete levels enumerated by the subscript m, which follow from the quantization of the electron motion along the axis z,

$$E_m(p,x,t) = E_m \pm v_{\rm F} p + U(x,t) - \mu(x,t), \tag{3}$$

where $U(x,t)$ is the electrostatic potential generated by the voltage applied to the channel, $\mu(x,t)$ the electrochemical potential (see, e.g., works [1, 2]), and $v_{\rm F} = 10^6$ m/s [11].

Taking Eq. (3) into account in the weak signal approximation (it means that $\frac{\partial f}{\partial E} \approx \frac{\partial f_o}{\partial E}$, where f_o is the equilibrium Fermi distribution function), we can rewrite the second and third terms on the left-hand side of Eq. (2) as follows:

$$\begin{aligned} v_x \frac{\partial f}{\partial x} &= v_x \frac{\partial f_o}{\partial E}\frac{\partial (U-\mu)}{\partial x}, \\ \frac{\partial U}{\partial x}\frac{\partial f}{\partial p_x} &= \frac{\partial U}{\partial x}\frac{\partial f_0}{\partial E}\frac{\partial E}{\partial p_x} = v_x \frac{\partial f_0}{\partial E}\frac{\partial U}{\partial x}. \end{aligned} \tag{4}$$

Then, Eq. (2) looks like

$$\frac{\partial f}{\partial t} + v_x \left(-\frac{\partial f_o}{\partial E}\right)\frac{\partial \mu}{\partial x} = \hat{S}_{\rm int} f. \tag{5}$$

Now, let us multiply Eq. (5) by $\frac{(-e)v_x}{l}$, where l is the length of the graphene channel between the contacts, sum up the result over all momenta p and quantum levels m, and introduce the relaxation time τ by the formula

$$\frac{I}{\tau} = -\frac{e}{l}\sum_{m,p} v_x \hat{S}_{\rm int} f. \tag{6}$$

In the regime of quasiequilibrium ballistic transport, the current is proportional to the chemical potential difference between the contacts and the average number of conductance modes in an energy interval of the order of kT in a vicinity of the equilibrium potential value (see, e.g., works [1, 2]):

$$I = \frac{2e}{h}\langle M\rangle d\mu, \tag{7}$$

where

$$\langle M\rangle = \frac{\pi}{2}\sum_{m,p}\hbar|v_x|\left(-\frac{\partial f_0}{\partial E}\right). \tag{8}$$

From whence, the current can be written in the form

$$I = \frac{e}{l}\sum_{m,v_x>0} v_x\left(-\frac{\partial f_o}{\partial E}\right)d\mu, \tag{9}$$

whereas the kinetic equation (5) looks like

$$\frac{\partial I}{\partial t} + \frac{I}{\tau} = \frac{1}{L}\frac{\partial(\mu/e)}{\partial x}, \tag{10}$$

where L is the kinetic inductance introduced in the weak current regime,

$$\frac{1}{L} = \frac{e^2}{l}\sum_{m,p} v_x^2\left(-\frac{\partial f_o}{\partial E}\right). \tag{11}$$

According to Eq. (10), the quantity L–its dimensionality in the SI system is H (henry)–can be interpreted as an inductance, which is of the kinetic rather than electromagnetic origin, because it follows from the alternative representation of the Boltzmann kinetic equation for a mesoscopic system. In the case where the chemical potential changes linearly along the graphene channel and the relaxation time is large enough, Eq. (10) can be rewritten in a simple form that connects the current with the chemical potential drop along the channel:

$$L\frac{\partial I}{\partial t} = \frac{\Delta\mu}{e}. \tag{12}$$

On the other hand, the total charge of the graphene channel with a unit width is described by the evident relation

$$Q = -el\sum_{m,p} f(m,p), \tag{13}$$

from whence the quantum capacitance can be introduced, by using the formal relation

$$C = \frac{Qe}{\Delta\mu}. \tag{14}$$

Taking Eq. (13) into account, we obtain

$$C = \frac{e^2}{l}\sum_{m,p}\left(-\frac{\partial f_o}{\partial E}\right). \tag{15}$$

Expressions (12) and (14) give ground to represent the equivalent circuit of the graphene channel in the form depicted in Figure, and expressions (11) and (15) make it possible to evaluate the kinetic inductance and the quantum capacitance, respectively.

Substituting (3) to Eqs. (11) and (15), we can obtain, similarly to what was done in work [6], approximate expressions for the quantum capacitance and the kinetic inductance (in so doing, we considered the thermally averaged velocities of electrons to be identical in each of m subbands):

$$C \approx \frac{2e^2 M\, l}{h v_{\rm F}}, \tag{16}$$

$$\frac{1}{L} \approx \frac{2e^2 M}{h v_{\rm F} l}\left\langle v_x^2\right\rangle. \tag{17}$$

Here, M is the total number of subbands approximately equal to the number of the de Broglie electron half-waves across the graphene channel (the channel cross-section is made in the plane yz normally to the current) [2, 6], and $\langle ...\rangle$ means averaging for electrons moving along the axis x (along the current direction).

The impedance of the electric circuit shown in Figure is described by the known expression

$$Z = 2R_c + \frac{R\,(1-\omega^2 LC) + \omega^2 R\,L\,C}{(1-\omega^2 L\,C)^2 + (\omega RC)^2} + \\ + i\frac{\omega\,L(1-\omega^2 L\,C) - \omega\,R^2 C}{(1-\omega^2 L\,C)^2 + (\omega RC)^2}. \tag{18}$$

For $\omega = 0$, formula (18) reproduces the trivial result

$$Z = 2R_c + R. \tag{19}$$

As the frequency of the applied ac voltage grows, a resonance is observed at the frequency ω_r, where the

imaginary part of Eq. (18) equals zero, and the circuit resistance is maximum. This frequency is determined by the formulas

$$\omega_r = \omega_0\sqrt{1-\frac{R^2C}{L}}; \quad \omega_0 = \frac{1}{\sqrt{LC}}. \tag{20}$$

Taking Eqs. (16) and (17) into account, one can see that if $\langle v_x^2\rangle / v_{\rm F}^2 \ll 1$, the approximate equality $\omega_r \approx \approx \omega_0$ holds true.

Let us estimate the frequencies predicted by expression (20). For a high-quality Landauer graphene resistor [2], where the ballistic regime takes place, and the electron passes between the contacts without scattering, we obtain, with regard for Eq. (3), that $\langle v_x^2\rangle \approx \approx v_{\rm F}^2$. Therefore, from Eq. (20), in view of Eqs. (16) and (17), we have

$$\omega_0 \sim \sqrt{\langle v_x^2\rangle}/l = v_{\rm F}/l. \tag{21}$$

This expression brings about the terahertz frequency for a graphene channel 1 μm in length, which is an order of magnitude larger than the frequency obtained in numerical calculations [7]. The frequency discrepancy is associated with the neglect of the relaxation time in our simple analysis, which we assumed to be large enough, when changing from Eq. (10) to Eq. (12).

However, in the case of a long graphene channel grown up by the CVD method, when the electron motion has a diffusive character, and $\sqrt{\langle v_x^2\rangle} \sim \mu\,\bar{E}_{SD}$ (μ is the electron mobility, and $\bar{E}_{SD}$ the time-averaged electric field between the contacts), if the field $\bar{E}_{SD} \sim \sim 10^3$ V/m and the mobility $\mu \sim 1$ m^2/V s (this value is typical of not too perfect CVD graphene), the frequency has an order of megahertz. For lower fields and mobilities, kilohertz frequencies are obtained easily (probably, this may be an explanation of the resonance in the data of work [10], as well as the resonance at a frequency of about 450 kHz in the dynamic conductivity of a submillimeter-size CVD graphene channel [12]).

Note that the results presented above are estimations and were obtained in the framework of a number of approximations. First of all, the current was considered uniform in the y-direction (the approximation of a wide enough graphene ribbon). The relaxation time in the Boltzmann kinetic equation was assumed to be large enough to change from Eq. (10) to Eq. (12). The velocities of thermalized electrons in different subbands were taken to be identical. In addition, various parasitic effects, which inevitably take place in real systems, were neglected.

However, in the framework of a rather simple model described by the Boltzmann kinetic equation for mesoscopic systems and developed for the first time in work [6], it was shown that the presence of the kinetic inductance and the quantum capacitance is not an obstacle for obtaining terahertz frequencies in graphene field transistors with a micron-size channel length (in this case, restrictions are imposed by relaxation processes [7]). At the same time, in massive graphene channels grown up using the CVD method, the quantum capacitance and the kinetic inductance give rise to frequencies in the kilo/megahertz interval. Hence, by varying the channel length (from hundreds of micrometers to millimeters) and the mobility (within an interval of 0.1–1 m^2/(V $\cdot$ s) typical of CVD graphene), we may hope to obtain effective filters for the corresponding frequency range.

The author is grateful to O. Nazarov, who stimulated this work by reporting his experimental results, and to Yu. Kruglyak, who, first in Ukraine, followed the approach developed by S. Datta for the consideration of mesosystems.

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Received 24.01.15.
Translated from Ukrainian by O.I. Voitenko

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ЧАСТОТНІ МЕЖІ
ДЛЯ ГРАФЕНОВОГО ПРОВІДНОГО КАНАЛУ,
ЗУМОВЛЕНІ НАЯВНІСТЮ КВАНТОВОЇ ЄМНОСТІ
ТА КІНЕТИЧНОЇ ІНДУКТИВНОСТІ

Р е з ю м е

На основі аналізу кінетичного рівняння Больцмана для мезосистем показано, що при розгляді динамічної провідності графенового каналу слід додатково враховувати квантову ємність і кінетичну індуктивність, які є аналогами електростатичної ємності і магнітної індуктивності, але мають іншу фізичну природу. Врахування наявності квантової ємності й кінетичної індуктивності призводить до появи максимуму імпедансу. У випадку, коли графеновий канал є ідеальним резистором Ландауера, який електрон проходить без розсіяння, цей максимум відповідає терагерцовому діапазонові (і отже, ефект не погіршує частотних характеристик польових графенових транзисторів, які працюють у гігагерцовому діапазоні). Однак для масивних графенових каналів, вирощених методом осаджування з парової фази, де транспорт електронів має дифузійну природу, максимум відповідає кілогерцовому або мегагерцовому діапазонові (залежно від рухливості носіїв і довжини каналу).