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### ПРЕДСТАВЛЕННЯ КЛАСИЧНОГО РОЗВ'ЯЗКУ ЛІНІЙНОГО ХВИЛЬОВОГО РІВНЯННЯ З ЧИСТИМ ЗАПІЗНЮВАННЯМ

*Розглянуто лінійне диференціальне рівняння теплопровідності з запізнюванням.*

*Ключові слова: динамічна система, різниці рівняння, точки спокою, асимптотична стійкість, фазовий портрет.*

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*Рассмотрено линейное дифференциальное уравнение теплопроводности с запаздыванием.*

*Ключевые слова: динамическая система, разностные уравнения, точки покоя, асимптотическая устойчивость, фазовый портрет.*

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## APPROACH FOR SOLVING OF TRANSPORTATION PROBLEM WITH FUZZY RESOURCES

*In this paper, a method is proposed to find the fuzzy optimal solution of fuzzy transportation model by representing all the parameters as triangle fuzzy numbers. To illustrate the proposed method a fuzzy transportation problem is solved by using the proposed method and the results are obtained. The proposed method is easy to understand, and to apply for finding the fuzzy optimal solution of fuzzy transportation models in real life situations.*

*Keywords: fuzzy transportation problem, triangle fuzzy numbers, optimal solution.*

### INTRODUCTION

The transportation problem which, is one of network integer programming problems is a problem that deals with distributing any commodity from any group of 'sources' to any group of destinations or 'sinks' in the most cost effective way with a given 'supply' and 'demand' constraints. Depending on the nature of the cost function, the transportation problem can be categorized into linear and nonlinear transportation problem.

Transportation problem is a linear programming (LP) problem stemmed from a network structure consisting of a finite number of nodes and arcs attached to them. In a typical problem a production is to be transported from  $m$  sources to  $n$  destinations and their capacities are  $a_1, a_2, \dots, a_m$  and  $b_1, b_2, \dots, b_n$ , respectively. There is a penalty  $C_{ij}$  and variable  $X_{ij}$  associated with transporting unit of production and unknown quantity to be shipped from source  $i$  to destination  $j$ .

Efficient algorithms have been developed for solving the transportation problem when the cost coefficients and the supply and demand quantities are known exactly. However, there are cases that these parameters may not be presented in a precise manner. For example, the unit shipping cost may vary in a time frame. The supplies and demands may be uncertain due to some uncontrollable factors.

Bellman and Zadeh [1] proposed the concept of decision making in fuzzy environment. Lai and Hwang [2] considered the situation where all parameters are fuzzy. In 1979, Isermann [3] introduced algorithm for solving this problem which provides effective solutions. The Ringuest and Rinks [4] proposed two iterative algorithms for solving linear, multi criteria transportation problem. S.Chanas and D.Kuchta [6] the approach based on interval and fuzzy coefficients had been elaborated. Tien Fuling [7] applied the method of interactive fuzzy multi-objective linear programming to transportation planning decisions. A new approach called fuzzy modified computational procedure to find the optimal solution was discussed in [8]. The new arithmetic operations of trapezoidal fuzzy numbers are employed to get the fuzzy optimal solutions. In this work, the fuzzy transportation problems using triangle fuzzy numbers are discussed. Here after, we have to propose the method of fuzzy modified distribution to be finding out the optimal solution for the total fuzzy transportation minimum cost.

There are also studies discussing the fuzzy transportation problem. Chanas et al. [6] investigate the transportation problem with fuzzy supplies and demands and solve them via the parametric programming technique in terms of the Bellman and Zadeh criterion. Their method is to derive the solution which simultaneously satisfies the constraints and the goal to a maximal degree. In this paper fuzzy transportation problem is discussed with constraints where the supply and demand are triangle fuzzy numbers. This paper aims to find out the best compromise solution among the set of feasible solutions for fuzzy transportation problem.

### FUZZY TRANSPORTATION MODEL FORMULATION

We deal with the production and transportation planning of a certain manufacturer that has production facilities and central stores for resellers. Each store can receive products from all production plants and it is not necessary that all products are produced in all production units.

Assume that a logistics center seeks to determine the transportation plan of a homogeneous commodity from  $m$  sources to  $n$  destinations. Each source has an available supply of the commodity to distribute to various destinations, and each destination has a forecast demand of the commodity to be received from various sources. This work focuses on developing a fuzzy linear programming (FLP) method for optimizing the transportation plan in fuzzy environments.

Let's consider

– *index sets*:

$i$  – index for source, for all  $i = 1, 2, \dots, m$ ,

$j$  – index for destination, for all  $j = 1, 2, \dots, n$ ;

– *decision variables*:

$x_{ij}$  – units transported from source  $i$  to destination  $j$  (units);

– *objective functions*:

$Z$  – total transportation costs;

– *parameters*:

$c_{ij}$  – transportation cost per unit delivered from source  $i$  to destination  $j$ ,

$a_i$  – total available supply at each source  $i$  (units),

$b_j$  – total forecast demand at each destination  $j$  (units).

Then the transportation problem is formulated as:

minimize total transportation costs

$$Z = \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij}, \quad (1)$$

with constraints on total available supply for each source  $i$

$$\sum_{j=1}^n x_{ij} = a_i, \quad i = \overline{1, m}, \quad (2)$$

and constraints on total forecast demand for each destination  $j$

$$\sum_{i=1}^m x_{ij} = b_j, \quad j = \overline{1, n}, \quad (3)$$

$$x_{ij} \geq 0, \quad i = \overline{1, m}, \quad j = \overline{1, n}.$$

Matrix  $X = \|x_{ij}\|_{\substack{i=\overline{1, m}, \\ j=\overline{1, n}}} = \begin{pmatrix} x_{11} & \dots & x_{1n} \\ \dots & x_{ij} & \dots \\ x_{m1} & \dots & x_{mn} \end{pmatrix}$ , that satisfy the conditions (2), (3), is called by transportation plan. The balance

condition

$$\sum_{i=1}^m a_i = \sum_{j=1}^n b_j, \quad (4)$$

is necessary and sufficient for transportation problem solving.

If any of the parameters  $a_i$ , or  $b_j$  is fuzzy, the total transportation cost  $Z$  becomes fuzzy as well. The conventional transportation problem defined then turns into the fuzzy transportation problem (FTP).

### FUZZY TRANSPORTATION MODEL SOLUTION

We propose to explore the solution of transportation problem with fuzzy distributing any commodity resources which defines as triangle fuzzy numbers [9]  $\tilde{a}_i, i = \overline{1, m}, \tilde{b}_j, j = \overline{1, n}$ .

In this case we consider transportation problem as linear programming problem that may be written in the next form:

$$\min Z = \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij}, \quad (5)$$

subject to

$$\sum_{j=1}^n x_{ij} = \tilde{a}_i, \quad i = \overline{1, m}, \quad (6)$$

$$\sum_{i=1}^m x_{ij} = \tilde{b}_j, \quad j = \overline{1, n}, \quad (7)$$

and

$$\sum_{i=1}^m \tilde{a}_i = \sum_{j=1}^n \tilde{b}_j. \quad (8)$$

Fuzzy values  $\tilde{a}_i, i = \overline{1, m}, \tilde{b}_j, j = \overline{1, n}$ , are consider as Triangular Fuzzy Numbers (TFN)  $\tilde{a}_i = (a_i - a_i^l, a_i, a_i + a_i^r), i = \overline{1, m}$ ,

with tolerances  $a_i^l (< a_i), a_i^r (> 0)$  for  $\sum_{j=1}^n x_{ij} = \tilde{a}_i, i = \overline{1, m}$  and  $\tilde{b}_j = (b_j - b_j^l, b_j, b_j + b_j^r), j = \overline{1, n}$ , with tolerances  $b_j^l (< b_j),$

$b_j^r (> 0)$  for  $\sum_{i=1}^m x_{ij} = \tilde{b}_j, j = \overline{1, n}$ , respectively.

Triangular Fuzzy Numbers  $\tilde{b}_i = (b_i - b_i^l, b_i, b_i)$ ,  $i = \overline{1, m}$ , are called Left Triangular Fuzzy Number (LTFN) with tolerance  $b_i^l (< b_i)$ ,  $i = \overline{1, m}$ , and Triangular Fuzzy Numbers  $\tilde{b}_i = (b_i, b_i, b_i + b_i^r)$ ,  $i = \overline{1, m}$ , are called Right Triangular Fuzzy Number (RTFN) with tolerance  $b_i^r (> 0)$ ,  $i = \overline{1, m}$ .

The solution of FTP (5)-(8) according to approach [10] is offered. Let  $L_1$  and  $U_1$  are the lower and upper bound for the objective function  $Z$ . When the aspiration levels for objective have been obtained, we form a fuzzy model which is as follows:

find  $x_{ij}$ ,  $i = \overline{1, m}$ ,  $j = \overline{1, n}$ , so as to satisfy

$$Z \geq \tilde{s}, \quad \tilde{s} = (L_1, U_1, U_1), \quad (9)$$

$$\sum_{j=1}^n x_{ij} = \tilde{a}_i, \quad i = \overline{1, m}, \quad \sum_{i=1}^m x_{ij} = \tilde{b}_j, \quad j = \overline{1, n}, \quad \sum_{i=1}^m \tilde{a}_i = \sum_{j=1}^n \tilde{b}_j, \quad (10)$$

$$x_{ij} \geq 0, \quad i = \overline{1, m}, \quad j = \overline{1, n}.$$

The membership functions for fuzzy constraints of (1.3, 1.4) are defined as:

for first constraint (9)

$$\mu^1 \left( \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij} \right) = \begin{cases} 0, & \text{for } \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij} < L_1, \\ \left( \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij} - L_1 \right) / (U_1 - L_1), & \text{for } L_1 \leq \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij} < U_1, \\ 1, & \text{for } \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij} \geq U_1, \end{cases}$$

for the  $i$ -th constrains,  $i = \overline{1, m}$ ,

$$\mu_i^2 \left( \sum_{j=1}^n x_{ij} \right) = \begin{cases} 0, & \text{for } \sum_{j=1}^n x_{ij} < a_i - a_i^l, \\ \left( \sum_{j=1}^n x_{ij} - a_i + a_i^l \right) / a_i^l, & \text{for } a_i - a_i^l \leq \sum_{j=1}^n x_{ij} < a_i, \\ \left( a_i + a_i^r - \sum_{j=1}^n x_{ij} \right) / a_i^r, & \text{for } a_i \leq \sum_{j=1}^n x_{ij} < a_i + a_i^r, \\ 1, & \text{for } \sum_{j=1}^n x_{ij} \geq a_i + a_i^r, \end{cases}$$

for the  $j$ -th constrains,  $j = \overline{1, n}$ ,

$$\mu_j^3 \left( \sum_{i=1}^m x_{ij} \right) = \begin{cases} 0, & \text{for } \sum_{i=1}^m x_{ij} < b_j - b_j^l, \\ \left( \sum_{i=1}^m x_{ij} - b_j + b_j^l \right) / b_j^l, & \text{for } b_j - b_j^l \leq \sum_{i=1}^m x_{ij} < b_j, \\ \left( b_j + b_j^r - \sum_{i=1}^m x_{ij} \right) / b_j^r, & \text{for } b_j \leq \sum_{i=1}^m x_{ij} < b_j + b_j^r, \\ 1, & \text{for } \sum_{i=1}^m x_{ij} \geq b_j + b_j^r. \end{cases}$$

Using the max-min operator (as Zimmermann [3]) linear programming problems for (9), (10) is formulated as follows:

$$\max \lambda \quad (11)$$

subject to

$$\begin{aligned} \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij} - \lambda(U_1 - L_1) &\geq L_1, \\ \sum_{j=1}^n x_{ij} - \lambda a_i^l &\geq a_i - a_i^l, \\ \sum_{j=1}^n x_{ij} + \lambda a_i^r &\leq a_i + a_i^r, \end{aligned} \quad (12)$$

$$\sum_{i=1}^m x_{ij} - \lambda b_j^l \geq b_j - b_j^l,$$

$$\sum_{i=1}^m x_{ij} + \lambda b_j^r \geq b_j + b_j^r,$$

$$0 \leq \lambda \leq 1, \quad x_{ij} \geq 0, \quad i = \overline{1, m}, \quad j = \overline{1, n}.$$

## APPLICATION IN TRANSPORTATION MODELS

The following data adopted from J. Reeb and S. Leavengood [11] is used to show that the above Fuzzy Transportation Problem with fuzzy constraints can be employed to solved.

First, let's formulate our problem and we will solve using the LP software WINQSB. The XYZ Sawmill Company's CEO asks to see next month's log hauling schedule to his three sawmills. He wants to make sure he keeps a steady, adequate flow of logs to his sawmills to capitalize on the good lumber market. Secondary, but still important to him, is to minimize the cost of transportation. The harvesting group plans to move to three new logging sites. The distance from each site to each sawmill is in Table 1. The average haul cost is \$2 per mile for both loaded and empty trucks. The logging supervisor estimated the number of truckloads of logs coming off each harvest site daily. The number of truckloads varies because terrain and cutting patterns are unique for each site. Finally, the sawmill managers have estimated the truckloads of logs their mills need each day. All these estimates are in Table 1.

**Table 1.** Supply and demand of sawlogs for the XYZ Sawmill Company.

Logging site	Distance to mill (miles)			Maximum truckloads/day per logging site
	Mill A	Mill B	Mill C	
1	8	15	50	20
2	10	17	20	30
3	30	26	15	45
Mill demand (truckloads/day)	30	35	30	

The next step is to determine costs to haul from each site to each mill (Table 2).

**Table 2.** Round-trip transportation costs for XYZ Sawmill Company.

Logging site	Mill A	Mill B	Mill C
1	32	60	200
2	40	68	80
3	120	104	60

We can set the LP problem up as a cost minimization; that is, we want to minimize hauling costs and meet each of the sawmills. This problem is formulate as:

let  $x_{ij}$  = amounts transported from Site  $i$  to Mill  $j$ ,  $i = 1, 2, 3$  (logging sites),  $j = 1, 2, 3$  (sawmills),

objective function

$$32x_{11} + 40x_{21} + 120x_{31} + 60x_{12} + 68x_{22} + 104x_{32} + 200x_{13} + 80x_{23} + 60x_{33} \rightarrow \min \quad (13)$$

subject to

$$\begin{aligned} x_{11} + x_{21} + x_{31} &= 30 && \text{Truckloads to Mill A} \\ x_{12} + x_{22} + x_{32} &= 35 && \text{Truckloads to Mill B} \\ x_{13} + x_{23} + x_{33} &= 30 && \text{Truckloads to Mill C} \\ x_{11} + x_{12} + x_{13} &= 20 && \text{Truckloads from Site 1} \\ x_{21} + x_{22} + x_{23} &= 30 && \text{Truckloads from Site 2} \\ x_{31} + x_{32} + x_{33} &= 45 && \text{Truckloads from Site 3} \end{aligned} \quad (14)$$

In this model all right hand side are fuzzy numbers as follow:

$$\tilde{30} = (28, 30, 32), \tilde{35} = (34, 35, 37), \tilde{30} = (29, 30, 31), \tilde{20} = (18, 20, 23), \tilde{30} = (28, 30, 33), \tilde{45} = (44, 45, 46).$$

By using equation (11) the model become

$$\max \lambda$$

subject to

$$32x_{11} + 40x_{21} + 120x_{31} + 60x_{12} + 68x_{22} + 104x_{32} + 200x_{13} + 80x_{23} + 60x_{33} - \lambda(5760 - 5560) \geq 5560,$$

$$\begin{aligned} x_{11} + x_{21} + x_{31} &\geq 28 + 2\lambda, \\ x_{11} + x_{21} + x_{31} &\leq 32 - 2\lambda, \\ x_{12} + x_{22} + x_{32} &\leq 34 + 2\lambda, \\ x_{12} + x_{22} + x_{32} &\leq 38 - 3\lambda, \\ x_{13} + x_{23} + x_{33} &\geq 29 + \lambda, \\ x_{13} + x_{23} + x_{33} &\leq 31 - \lambda, \\ x_{11} + x_{12} + x_{13} &\geq 18 + 2\lambda, \\ x_{11} + x_{12} + x_{13} &\leq 23 - 3\lambda, \\ x_{21} + x_{22} + x_{23} &\geq 28 + 2\lambda, \\ x_{21} + x_{22} + x_{23} &\leq 33 - 3\lambda, \\ x_{31} + x_{32} + x_{33} &\geq 44 + \lambda, \\ x_{31} + x_{32} + x_{33} &\leq 46 - \lambda, \\ x_{ij} &\geq 0, \quad i = 1, 2, 3, j = 1, 2, 3. \end{aligned}$$

The optimal solution is :

$$x_{11} = 0, x_{21} = 26, x_{31} = 0, x_{12} = 15.8, x_{22} = 0, x_{32} = 17.17, x_{13} = 2.17, x_{23} = 0, x_{33} = 25.8, \lambda = 1, z = 5842.8.$$

### FUZZY APPROACH TO SOLVING PROBLEMS OF LINEAR PROGRAMMING BASED ON METHODS FOR DECISION MAKING

The parameter in the formulation of the optimization problem (11) – (12) determines the total guaranteed level of fuzzy defined resources. In practice the tolerance depends from type and specific resources. This allows us to generalize the above proposed approach to the case of using multiple parameters.

Let it be known that the tolerance of demand and supply resources in constraints are independent and can be formalized in the form of inequalities

$$\begin{aligned} \sum_{j=1}^n x_{ij} - \lambda_1 a_i^l &\geq a_i - a_i^l, & \sum_{j=1}^n x_{ij} + \lambda_1 a_i^r &\leq a_i - a_i^r, \\ \sum_{i=1}^m x_{ij} - \lambda_2 b_j^l &\geq b_j - b_j^l, & \sum_{i=1}^m x_{ij} + \lambda_2 b_j^r &\leq b_j + b_j^r, \\ i = \overline{1, m}, \quad j = \overline{1, n}, \quad 0 \leq \lambda_p &\leq 1, \quad p = \overline{1, 2}. \end{aligned} \quad (13)$$

To find the marginal values change resource use way to compare the importance of constraints on the basis of decision-making methods.

Consider the method of constructing membership functions using pairwise comparisons. Let the set of elements  $X = \{x_i \geq 0, i = \overline{1, k}\}$ . The degree of association elements to fuzzy sets can be obtained by comparing the elements together.

Let  $q_{ij}$  denote the element's estimation  $x_i$  in compare with element  $x_j$ ,  $i, j = \overline{1, k}$ . For consistency,  $q_{ij} = 1/q_{ji}$ . Estimations  $q_{ij}$  are define the matrix  $Q = \|q_{ij}\|$ ,  $i, j = \overline{1, k}$ .

Further find the eigenvector  $w = (w_1, \dots, w_m)$  corresponding to maximal eigenvalue  $\lambda(Q)$  of matrix  $Q$ . The values  $w_i \geq 0$ ,  $i = \overline{1, k}$  are considering as levels of elements membership  $x_i$ ,  $i = \overline{1, k}$  to fuzzy set.

Coefficients of the relative importance of elements  $q_{ij}$  derived from estimates of the importance scale (Table 3).

Table 3

Relative importance of elements	Matrix A elements
Equal importance of elements	1
Slightly important	3
More important	5
Essentially important	7
Far more important	9
Intermediate values	2,4,6,8

Comparing the importance of constraints that define the area of solutions of fuzzy linear programming problem, finally we obtain the problem model, taking into account the importance of group constraints:

find  $\lambda_0 \in [0, 1]$  as solution linear programming task

$$\lambda_0 \rightarrow \max \quad (13)$$

subject to

$$\begin{aligned} \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij} - \lambda_0 (U_1 - L_1) &\geq L_1, \\ \sum_{j=1}^n x_{ij} - \lambda_1 a_i^l &\geq a_i - a_i^l, \\ \sum_{j=1}^n x_{ij} + \lambda_1 a_i^r &\leq a_i - a_i^r, \\ \sum_{i=1}^m x_{ij} - \lambda_2 b_j^l &\geq b_j - b_j^l, \\ \sum_{i=1}^m x_{ij} + \lambda_2 b_j^r &\leq b_j + b_j^r, \end{aligned} \quad (14)$$

$$w_1 \leq \lambda_1, \quad w_2 \leq \lambda_2, \quad x_{ij} \geq 0, \quad i = \overline{1, m}, \quad j = \overline{1, n}, \quad \lambda_p \geq \lambda_0, \quad 0 \leq \lambda_p \leq 1, \quad p = \overline{1, 2}.$$

In this case, we assume the matrix  $Q$  is define as

$$Q = \begin{bmatrix} 1 & 5 \\ 1/5 & 1 \end{bmatrix}.$$

The maximal eigenvalue of matrix  $\lambda(Q) = 2$  and eigenvector  $w$  corresponding to this value is  $w = (5/6, 1/6)$ . According to our new approach the model become

$$\max \lambda$$

subject to

$$32x_{11} + 40x_{21} + 120x_{31} + 60x_{12} + 68x_{22} + 104x_{32} + 200x_{13} + 80x_{23} + 60x_{33} - \lambda_0(5760 - 5560) \geq 5560,$$

$$x_{11} + x_{21} + x_{31} \geq 28 + 2\lambda_1,$$

$$x_{11} + x_{21} + x_{31} \leq 32 - 2\lambda_1,$$

$$x_{12} + x_{22} + x_{32} \leq 34 + 2\lambda_1,$$

$$x_{12} + x_{22} + x_{32} \leq 38 - 3\lambda_1,$$

$$x_{13} + x_{23} + x_{33} \geq 29 + \lambda_1,$$

$$x_{13} + x_{23} + x_{33} \leq 31 - \lambda_1,$$

$$x_{11} + x_{12} + x_{13} \geq 18 + 2\lambda_2,$$

$$x_{11} + x_{12} + x_{13} \leq 23 - 3\lambda_2,$$

$$x_{21} + x_{22} + x_{23} \geq 28 + 2\lambda_2,$$

$$x_{21} + x_{22} + x_{23} \leq 33 - 3\lambda_2,$$

$$x_{31} + x_{32} + x_{33} \geq 44 + \lambda_2,$$

$$x_{31} + x_{32} + x_{33} \leq 46 - \lambda_2,$$

$$5/6 \leq \lambda_1, 1/6 \leq \lambda_2, \lambda_1 \geq \lambda_0, \lambda_2 \geq \lambda_0,$$

$$x_{ij} \geq 0, i = 1, 2, 3, j = 1, 2, 3.$$

The optimal solution is :

$$x_{11} = 0.23, x_{21} = 29.42, x_{31} = 0, x_{12} = 19.78, x_{22} = 0, x_{32} = 14.8, x_{13} = 0, x_{23} = 0, x_{33} = 29.83, \lambda_0 = 0.712, \lambda_1 = 0.83, \lambda_2 = 0.712, z = 5699.96$$

### CONCLUSION

Transportation models have wide applications in logistics and supply chain for reducing the cost. Some previous studies have devised solution procedures for fuzzy transportation problems. In this paper we have thus obtained an optimal solution for a fuzzy transportation problem using triangle fuzzy number. A new approach to find the optimal solution of transportation problem with fuzzy resources is discussed. The new method is planned to use for solving the transportation problem with all fuzzy parameters.

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### ПІДХІД ДЛЯ РОЗВ'ЯЗКУ ТРАНСПОРТНОЇ ЗАДАЧІ З НЕЧІТКИМИ РЕСУРСАМИ

*В роботі розглянуто метод пошуку оптимального розв'язку нечіткої транспортної задачі, ресурси в якій представлено нечіткими трикутними числами. Проілюстровано використання методу на прикладі реальної транспортної задачі. Запропоновано залучення розробленого підходу для вирішення нечітких транспортних задач загального вигляду.*

*Ключові слова: нечітка транспортна задача, трикутник з нечіткими числами, оптимальне рішення.*

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### ПОДХОД ДЛЯ РЕШЕНИЯ ТРАНСПОРТНОЙ ЗАДАЧИ С НЕЧЕТКИМИ РЕСУРСАМИ

*В работе рассмотрен метод поиска оптимального решения нечеткой транспортной задачи, ресурсы в которой представлены нечеткими треугольными числами. Проиллюстрировано использование метода на примере реальной транспортной задачи. Предложено привлечение разработанного подхода для решения нечетких транспортных задач общего вида.*

*Ключевые слова: нечеткая транспортная задача, треугольник с нечеткими числами, оптимальное решение.*