UDC 517.95:519.86:539.3

K. Dvirnychuk, PhD student
Kyiv National Taras Shevchenko University, Kyiv

ABOUT PROBLEM OF CONTROL THREE-DIMENSIONAL FIELD TRANSVERSE DYNAMIC DISPLACEMENTS OF THICK ELASTIC PLATE

The problems managing three-dimensional cross-field dynamic displacement of thick elastic plates discretely observed for the initial boundary condition. Formulated terms of accuracy and uniqueness of the solution of the problem.

Keywords: differential model, integrated model, three-dimensional problem.

Introduction. Research and robust control of production, socio-economic and technical processes are not possible without quality mathematical model. The nature of these processes is not always possible to construct an adequate model that would fit into a set of mathematical models available for the study of classical methods of mathematical physics, computational mathematics and control theory. Problems [1-3] associated with the core processes of different nature, deep relationships that are difficult to formalize, and the inability to obtain the information necessary to build accurate mathematical model. Often the considered process is described mathematically model only partially and it goes incomplete.

One of these processes is the classic process control three-dimensional field transverse dynamic displacements of thick elastic plates. The background for it is the differential model of transverse dynamic displacement of thick elastic layer. There are many classical [4, 5] and not classical [6, 7] approaches to building such models, however, are not without some mechanical hypotheses. Differential elastic layer model without hypotheses proposed in [8], limited to the static case. Built as our generalization [9] results [8] applies to the dynamics of thick elastic layer.

In solving the problem of controlling three-dimensional field transverse dynamic displacements of thick elastic plates with in [9] results in the presence of limited information about their initial boundary value condition and sent this scientific research.

Differential model of transverse dynamic displacement of thick elastic layer. For the task of controlling three-dimensional field transverse dynamic displacements of thick elastic plates look for it original differential model and some of its features are given in [9]. To suppose that a thick elastic layer thickness $2h$ is in the Cartesian coordinate system x, y, z so that its surface defined planes $z=\pm h$. Assuming that the surface of this layer under the action of normal $q_1^{\pm}(x,y,t)$ and tangential $q_2^{\pm}(x,y,t)$ unknown external dynamic forces ($t\in[0,T]$ – time), through $u(x,y,z,t)$, $v(x,y,z,t)$, $w(x,y,z,t)$ denote the shift points via the layer in the coordinate axes Ox, Oy, Oz respectively. Dynamic transverse displacements $w(x,y,z,t)$ at this present amount

$$w(x,y,z,t)=\sum_{l=1}^{2} w^{(l)}(x,y,z,t)\,. \tag{1}$$

Differential same model under consideration layer constructed in [9], has the form

$$Q^{(l)}(\partial_x,\partial_y,\partial_t)w^{(l)}(x,y,z,t)=d_1^{(l)}(\partial_x,\partial_y,z,\partial_t)q_1^{(l)}(x,y,t)+d_2^{(l)}(\partial_x,\partial_y,z,\partial_t)q_2^{(l)}(x,y,t)\ \ (l=\overline{1,2})\,, \tag{2}$$

Here and further

$$q_1^{(1)}(x,y,t)=\frac{1}{2}(q_1^{+}(x,y,t)+q_1^{-}(x,y,t))\,,\ q_2^{(1)}(x,y,t)=\frac{1}{2}(q_2^{+}(x,y,t)-q_2^{-}(x,y,t))\,,$$

$$q_1^{(2)}(x,y,t)=\frac{1}{2}(q_1^{+}(x,y,t)-q_1^{-}(x,y,t))\,,\ q_2^{(2)}(x,y,t)=\frac{1}{2}(q_2^{+}(x,y,t)+q_2^{-}(x,y,t))\,,$$

$$Q^{(1)}(\partial_x,\partial_y,\partial_t)=(\Delta+D_2^2)((\lambda+2\mu)D_1^2-\lambda\Delta)\cos(hD_1)\frac{\sin(hD_2)}{D_2}-4\mu\Delta D_1^2\frac{\sin(hD_1)}{D_1}\cos(hD_2)\,,$$

$$d_1^{(1)}(\partial_x,\partial_y,z,\partial_t)=D_1^2\left[(\Delta+D_2^2)\frac{\sin(zD_1)}{D_1}\frac{\sin(hD_2)}{D_2}-2\Delta\frac{\sin(hD_1)}{D_1}\frac{\sin(zD_2)}{D_2}\right],$$

$$d_2^{(1)}(\partial_x,\partial_y,z,\partial_t)=2d\left[\frac{1}{\mu}((\lambda+2\mu)D_1^2-\lambda\Delta)\cos(hD_1)\frac{\sin(zD_2)}{D_2}-2D_1^2\frac{\sin(zD_1)}{D_1}\cos(hD_2)\right],$$

$$d_1^{(2)}(\partial_x,\partial_y,z,\partial_t)=(\Delta+D_2^2)\cos(zD_1)\cos(hD_2)-2\Delta\cos(hD_1)\cos(zD_2)\,,$$

$$d_2^{(2)}(\partial_x,\partial_y,z,\partial_t)=2d\left[2D_2^2\cos(zD_1)\frac{\sin(hD_2)}{D_2}+\frac{1}{\mu}(\lambda\Delta-(\lambda+2\mu)D_1^2)\frac{\sin(hD_1)}{D_1}\cos(zD_2)\right], \tag{3}$$

at Δ, D_1^2, D_2^2 that ratios

$$\Delta=d(\partial_x+\partial_y)\,,\ D_1^2=\Delta_1+\frac{1}{c_1^2}\partial_t^2\,,\ D_2^2=\Delta_2-\frac{1}{c_2^2}\partial_t^2$$

determined through

$$\Delta_1=\frac{\lambda+\mu}{\lambda+2\mu}\Delta+\frac{\mu}{\lambda+2\mu}\Delta_2\,,\ \Delta_2=\partial_x^2+\partial_y^2\,,$$

Lame constants λ and μ, speeds $c_2=\sqrt{\frac{\mu}{\rho}}$ and $c_1=\sqrt{\frac{\lambda+2\mu}{\rho}}$ propagation of elastic waves of expansion and shift in the considered environment (ρ – density material) and operator d, for which $d(u+v)=\partial_x u+\partial_y v$.

After of returning trigonometric components $\frac{\sin(zD_m)}{D_m}$, $\frac{\sin(hD_m)}{D_m}$, $\cos(zD_m)$, $\cos(hD_m)$ $(m=\overline{1,2})$ operators (3) of their differential content equations (2) and will describe the three-dimensional field of dynamic displacements of thick elastic layer.

Note that equation (2), recorded with varying degrees of accuracy include classical equation [4, 5], two-dimensional theory of deflection plates, and are known for their generalization [7], are based on non-classical models of mechanics of solid deformable body.

Integrated model of transverse dynamic displacement thick elastic layer. Here are useful for studying the dynamics of three-dimensional fields of transverse dynamic displacements of thick elastic plates equivalent integral differential model (2), which is built by us in [10] and has the form

$$w^{(l)}(x,y,z,t)=\int_{-\infty}^{+\infty} q^{(l)}(x',y',t')G^{(l)}(x-x',y-y',z,t-t')dx'dy'dt', \tag{4}$$

where

$$q^{(l)}(x',y',t')=\sum_{k=1}^{2} q_k^{(l)}(x',y',t'),$$

$$G^{(l)}(x-x',y-y',z,t-t')=\frac{1}{(2\pi i)^3}\int_{-i\infty}^{+i\infty}\left(\frac{\sum_{k=1}^{2} d_k^{(l)}(p_1,p_2,z,q)}{Q^{(l)}(p_1,p_2,q)}e^{p_1(x-x')+p_2(y-y')+q(t-t')}\right)dp_1dp_2dq. \tag{5}$$

We note in passing that the integrated model (4) is specified and refined axial symmetric case study of the dynamics layer when $d=\partial_x+\partial_y$. Given the recent and structure of operators $Q^{(l)}(\partial_x,\partial_y,\partial_t)$, $d_k^{(l)}(\partial_x,\partial_y,z,\partial_t)$ $(l,k=\overline{1,2})$ given in (3), we conclude that the functions $Q^{(l)}(p_1,p_2,q)$ і $d_k^{(l)}(p_1,p_2,z,q)$ $(l,k=\overline{1,2})$ meet axial symmetric case of integrated mathematical model (4), written with the degree of accuracy h^3 are as follows:

$$Q^{(1)}(p_1,p_2,q)=\mu h\left\{\frac{4(\lambda+\mu)}{\lambda+2\mu}(p_1^2+p_2^2)-\frac{\rho}{\mu}q^2\right\}-$$
$$-\mu\frac{h^3}{3!}\left\{\frac{8(\lambda+\mu)}{\lambda+2\mu}(p_1^2+p_2^2)^2-\frac{4(2\lambda^2+8\lambda\mu+7\mu^2)}{(\lambda+2\mu)^2}(p_1^2+p_2^2)\frac{\rho}{\mu}q^2+\frac{\lambda+5\mu}{\lambda+2\mu}\left(\frac{\rho}{\mu}q^2\right)^2\right\},$$
$$Q^{(2)}(p_1,p_2,q)=h\rho q^2+\mu\frac{h^3}{3!}\left\{\frac{8(\lambda+\mu)}{\lambda+2\mu}(p_1^2+p_2^2)^2-\frac{4(3\lambda+4\mu)}{\lambda+2\mu}(p_1^2+p_2^2)\frac{\rho}{\mu}q^2+\frac{3\lambda+7\mu}{\lambda+2\mu}\left(\frac{\rho}{\mu}q^2\right)^2\right\}, \tag{6}$$
$$d_1^{(1)}(p_1,p_2,z,q)=zh\left\{p_1^2+p_2^2-\frac{\rho}{\lambda+2\mu}q^2\right\},\ d_2^{(1)}(p_1,p_2,z,q)=z(p_1+p_2)\left\{\frac{\lambda}{\lambda+2\mu}+\frac{1}{2}h^2\left[p_1^2+p_2^2-\frac{\rho}{\lambda+2\mu}q^2\right]\right\}-$$
$$-\frac{1}{3!}z^3(p_1+p_2)\left\{\frac{3\lambda+2\mu}{\lambda+2\mu}(p_1^2+p_2^2)-\frac{\lambda^2+4\lambda\mu+2\mu^2}{(\lambda+2\mu)^2}\frac{\rho}{\mu}q^2\right\},$$
$$d_1^{(2)}(p_1,p_2,z,q)=1-\frac{1}{2}h^2\left\{\frac{3\lambda+4\mu}{\lambda+2\mu}(p_1^2+p_2^2)-\frac{\rho}{\mu}q^2\right\}+\frac{1}{2}z^2\left\{\frac{\lambda}{\lambda+2\mu}(p_1^2+p_2^2)+\frac{\rho}{\lambda+2\mu}q^2\right\},$$
$$d_2^{(2)}(p_1,p_2,z,q)=h(p_1+p_2)\left\{1-\frac{1}{3!}h^2\left[\frac{3\lambda+4\mu}{\lambda+2\mu}(p_1^2+p_2^2)-\frac{2\lambda+3\mu}{\lambda+2\mu}\frac{\rho}{\mu}q^2\right]\right\}+\frac{1}{2}z^2h(p_1+p_2)\left\{\frac{\lambda}{\lambda+2\mu}\left(p_1^2+p_2^2-\frac{\rho}{\mu}q^2\right)\right\}.$$

Method of calculating integrals (5), recorded with regard to (6), we examined in detail in [11].

Criterion and features three-dimensional problem solving control field transverse dynamic displacements of thick elastic plates. Consider the dynamics of plate cylinder $\Gamma(x,y)=0$ of the examined cut above the elastic layer. Points of such plate assign to some spatial area S_0.

Pose and solve the problem of the determination of the control function $q^{(l)}(x,y,t)$ $(l=\overline{1,2})$, which would be a function $w(x,y,z,t)$ of the plate, the average square coordinated with discretely defined initial and boundary observations

$$L_r^0(\partial_t)w(s)\Big|_{\substack{t=0\\ \sigma=\sigma_j^0\in S_0}}=Y_{rj}^0\ \ (j=\overline{1,J_r},\ r=\overline{1,R_0}), \tag{7}$$

$$L_\rho^\Gamma(\partial_\sigma)w(s)\Big|_{\substack{t=t_j\in[0,T]\\ \sigma=\sigma_j^\Gamma\in S_\Gamma}}=Y_{\rho j}^\Gamma\ \ (j=\overline{1,J_\rho},\ \rho=\overline{1,R_\Gamma}), \tag{8}$$

point area S_0 examining the plates exasperated in neighborhood Y_{ij} $(i=\overline{1,I},\ j=\overline{1,J})$ preset ratios

$$L_i(\partial_s)w(s)\big|_{s=s_{ij}\in S_0\times[0,T]}=Y_{ij},\ \ (j=\overline{1,J_i},\ i=\overline{1,I}). \tag{9}$$

Here and further $S_\Gamma=\Gamma(x,y)\times[-h,h]$, $\sigma=(x,y,z)$, $s=(\sigma,t)$, ∂_t and $\partial_\sigma=(\partial_x,\partial_y,\partial_z)$ – derivatives with respect to time and spatial coordinates x, y, z, $L_r^0(\partial_t)$ $(r=\overline{1,R_0})$, $L_\rho^\Gamma(\partial_\sigma)$ $(\rho=\overline{1,R_\Gamma})$, $L_i(\partial_s)$ $(i=\overline{1,I})$ – linear differential operators. On the number of initial R_0 and boundary R_Γ observations displacements $w(s)$ will not impose any restrictions.

Criterion control function $q^{(l)}(x,y,t)$ $(l=\overline{1,2})$ written in the form

$$\Phi=\sum_{r=1}^{R_0}\sum_{j=1}^{J_r}\left(L_r^0(\partial_t)w(s)\Big|_{\substack{t=0\\ \sigma=\sigma_j^0\in S_0}}-Y_{rj}^0\right)^2+\sum_{\rho=1}^{R_\Gamma}\sum_{j=1}^{J_\rho}\left(L_\rho^\Gamma(\partial_\sigma)w(s)\Big|_{\substack{t=t_j\in[0,T]\\ \sigma=\sigma_j^\Gamma\in S_\Gamma}}-Y_{\rho j}^\Gamma\right)^2+$$
$$\sum_{i=1}^{I}\sum_{j=1}^{J_i}\left(L_i(\partial_s)w(s)\big|_{s=s_{ij}\in S_0\times[0,T]}-Y_{ij}\right)^2\to\min_{q^{(l)}(x,y,t)\ (l=\overline{1,2})}. \quad (10)$$

To solve this problem we use the method proposed in [1] and developed in [2, 3]. According to this function will submit the amount

$$w(s)=w^{(1)}(s)+w^{(2)}(s)=\sum_{l=1}^{2}(w_\infty^{(l)}(s)+w_0^{(l)}(s)+w_\Gamma^{(l)}(s)), \quad (11)$$

which

$$w_\infty^{(l)}(s)=\int_S G^{(l)}(\xi-\xi',z)q^{(l)}(\xi')d\xi', \quad (12)$$

$$w_0^{(l)}(s)=\int_{S^0} G^{(l)}(\xi-\xi',z)u_0^{(l)}(\xi')d\xi', \quad (13)$$

$$w_\Gamma^{(l)}(s)=\int_{S^\Gamma} G^{(l)}(\xi-\xi',z)u_\Gamma^{(l)}(\xi')d\xi' \ (l=\overline{1,2}), \quad (14)$$

$S=(S_0\cap((z=h)\cup(z=-h)))\times[0,\mathrm{T}]$, $S^0=(S_0\cap((z=h)\cup(z=-h)))\times(-\infty,0]$, $S^\Gamma=((R^3\setminus S_0)\cap((z=h)\cup(z=-h)))\times[0,T]$, $\xi=(x,y,t)$, $\xi'=(x',y',t')$.

Functions $u_0^{(l)}(\xi)$, $u_\Gamma^{(l)}(\xi)$ defined outside the considered area $S_0\times[0,\mathrm{T}]$, affecting the function of the transverse dynamic displacement $w(s)$ through the function $G^{(l)}(\xi-\xi',z)$ call a further modeling.

The task of controls a three-dimensional field transverse dynamic displacement of thick elastic plates through the function surface distributed external dynamic disturbances. The problem (10) to find a control function $q^{(l)}(\xi)$ $(l=\overline{1,2})$ to synchronize finding vector function

$$\overline{u}(\xi)=col(col(u_0^{(l)}(\xi)\ (\xi\in S^0),\ u_\Gamma^{(l)}(\xi)\ (\xi\in S^\Gamma),\ q^{(l)}(\xi)\ (\xi\in S)),\ l=\overline{1,2}), \quad (15)$$

such that

$$\Phi\to\min_{\overline{u}(\xi)}. \quad (16)$$

After substituting relations (11) – (14) in the initial boundary conditions (7), (8) and taking into account the desired state defined in (9), we obtain the system of integral equations

$$\int A(\xi')\overline{u}(\xi')d\xi'=\overline{Y} \quad (17)$$

at a known vector

$$\overline{Y}=col(Y^0,Y^\Gamma,Y)$$

and matrix functions

$$A(\xi)=str((A_1^{(l)}(\xi)\ (\xi\in S^0),\ A_2^{(l)}(\xi)\ (\xi\in S^\Gamma),\ A_3^{(l)}(\xi)\ (\xi\in S)),\ l=\overline{1,2}),$$
$$A_n^{(l)}(\xi)=col(A_{kn}^{(l)}(\xi),\ k=\overline{1,3})\ (n=\overline{1,3},\ l=\overline{1,2}),$$

in which

$$Y^0=col((Y_{rj}^0,j=\overline{1,J_r}),r=\overline{1,R_0}),\ Y^\Gamma=col((Y_{\rho j}^\Gamma,j=\overline{1,J_\rho}),\rho=\overline{1,R_\Gamma}),\ Y=col((Y_{ij},j=\overline{1,J_i}),i=\overline{1,I}),$$

$$A_{1n}^{(l)}(\xi')=col\left(\left(L_r^0(\partial_t)G^{(l)}(\xi-\xi',z)\Big|_{\substack{t=0\\ \sigma=\sigma_j^0}},\ j=\overline{1,J_r}\right),\ r=\overline{1,R_0}\right),$$

$$A_{2n}^{(l)}(\xi')=col\left(\left(L_\rho^\Gamma(\partial_\sigma)G^{(l)}(\xi-\xi',z)\Big|_{\substack{t=t_j\\ \sigma=\sigma_j^\Gamma}},\ j=\overline{1,J_\rho}\right),\ \rho=\overline{1,R_\Gamma}\right),$$

$$A_{3n}^{(l)}(\xi')=col\left(\left(L_i(\partial_s)G^{(l)}(\xi-\xi',z)\Big|_{s=s_{ij}},\ j=\overline{1,J_i}\right),\ i=\overline{1,I}\right)\ (l=\overline{1,2},\ n=\overline{1,3}).$$

Here the integration is performed over the region change argument integrand and $\sigma_j^0\in S_0$, $t_j\in[0,T]$, $\sigma_j^\Gamma\in S_\Gamma$, $s_{ij}\in S_0\times[0,T]$.

As a result pseudo inverse system (17) such that

$$\left\|\int A(\xi)\overline{u}(\xi)d\xi-\overline{Y}\right\|^2\to\min_{\overline{u}(\xi)}, \quad (18)$$

controlling-modeling functions $u_0^{(l)}(\xi)$ $(\xi \in S^0)$, $u_\Gamma^{(l)}(\xi)$ $(\xi \in S^\Gamma)$, $q^{(l)}(\xi)$ $(\xi \in S)$ define [3] ratios (at $l = \overline{1,2}$):

$$u_0^{(l)}(\xi) \in \Omega_0^{(l)} = \{u_0^{(l)}(\xi): \; u_0^{(l)}(\xi) = (A_1^{(l)}(\xi))^T P^+(\overline{Y} - A_v) + v_0^{(l)}(\xi), \; \forall v_0^{(l)}(\xi)\} \; (\xi \in S^0),$$

$$u_\Gamma^{(l)}(\xi) \in \Omega_\Gamma^{(l)} = \{u_\Gamma^{(l)}(\xi): \; u_\Gamma^{(l)}(\xi) = (A_2^{(l)}(\xi))^T P^+(\overline{Y} - A_v) + v_\Gamma^{(l)}(\xi), \; \forall v_\Gamma^{(l)}(\xi)\} \; (\xi \in S^\Gamma),$$

$$q^{(l)}(\xi) \in \Omega_q^{(l)} = \{q^{(l)}(\xi): \; q^{(l)}(\xi) = (A_3^{(l)}(\xi))^T P^+(\overline{Y} - A_v) + v^{(l)}(\xi), \; \forall v^{(l)}(\xi)\} \; (\xi \in S), \tag{19}$$

in which for arbitrarily defined and integrated in the change of its argument functions $v_0^{(l)}(\xi)$ $(\xi \in S^0)$, $v_\Gamma^{(l)}(\xi)$ $(\xi \in S^\Gamma)$, $v^{(l)}(\xi)$ $(\xi \in S)$

$$A_v = \sum_{l=1}^{2}\left(\int_{S^0} A_1^{(l)}(\xi) v_0^{(l)}(\xi) d\xi + \int_{S^\Gamma} A_2^{(l)}(\xi) v_\Gamma^{(l)}(\xi) d\xi + \int_S A_3^{(l)}(\xi) v^{(l)}(\xi) d\xi \right),$$

P^+ – matrix pseudo inverse to

$$P = \sum_{l=1}^{2}\left(\int_{S^0} A_1^{(l)}(\xi)(A_1^{(l)}(\xi))^T d\xi + \int_{S^\Gamma} A_2^{(l)}(\xi)(A_2^{(l)}(\xi))^T d\xi + \int_S A_3^{(l)}(\xi)(A_3^{(l)}(\xi))^T d\xi \right) \; (l = \overline{1,2}).$$

Note that the average quadratic precision with which the control-modeling functions $u_0^{(l)}(\xi)$, $u_\Gamma^{(l)}(\xi)$ and $q^{(l)}(\xi)$ $(l = \overline{1,2})$ satisfy (17), and found their use the function $w(\xi)$ is consistent with initial and boundary conditions controlling (7) – (9), will be determined by size [3]

$$\varepsilon^2 = \min_{\overline{u}(\xi)} \left\| \int A(\xi)\overline{u}(\xi) d\xi - \overline{Y} \right\|^2 = \overline{Y}^T \overline{Y} - \overline{Y}^T P P^+ \overline{Y}. \tag{20}$$

Constituents $w^{(l)}(s)$ $(l = \overline{1,2})$ function of transverse dynamic displacement $w(s)$ will be determined at the same ratios (11) – (14) taking into account found under (19) control-modeling functions $u_0^{(l)}(\xi)$, $u_\Gamma^{(l)}(\xi)$ and $q^{(l)}(\xi)$ $(l = \overline{1,2})$.

The condition for the uniqueness $(v_0^{(l)}(\xi) = v_\Gamma^{(l)}(\xi) = v^{(l)}(\xi) = 0)$ of the obtained solution is

$$\lim_{N \to \infty} \det[A^T(\xi_i) A(\xi_j)]_{i,j=1}^N > 0.$$

The task of controls a three-dimensional field transverse dynamic displacement of thick elastic plates via a discrete set of surface distributed external dynamic disturbances. Problem (10) can be successfully resolved if the modeling $u_0^{(l)}(\xi)$, $u_\Gamma^{(l)}(\xi)$ $(l = \overline{1,2})$ and control $q^{(l)}(\xi)$ $(l = \overline{1,2})$ functions determine the vectors

$$u_0^{(l)} = col\,(u_{0m}^{(l)} = u_0^{(l)}(\xi_m^{0(l)}), m = \overline{1, M_0^{(l)}}), \tag{21}$$

$$u_\Gamma^{(l)} = col\,(u_{\Gamma m}^{(l)} = u_\Gamma^{(l)}(\xi_m^{\Gamma(l)}), m = \overline{1, M_\Gamma^{(l)}}), \tag{22}$$

$$q_*^{(l)} = col\,(q_m^{(l)} = q^{(l)}(\xi_m^{(l)}), m = \overline{1, M^{(l)}}) \quad (l = \overline{1,2}) \tag{23}$$

values $u_{0m}^{(l)}$ $(m = \overline{1, M_0^{(l)}})$, $u_{\Gamma m}^{(l)}$ $(m = \overline{1, M_\Gamma^{(l)}})$ and $q_m^{(l)}$ $(m = \overline{1, M^{(l)}})$ $(l = \overline{1,2})$ at points $\xi_m^{0(l)} \in S^0$, $\xi_m^{\Gamma(l)} \in S^\Gamma$ and $\xi_m^{(l)} \in S$ respectively.

In this case the functions $w_0^{(l)}(\xi)$, $w_\Gamma^{(l)}(\xi)$ $(l = \overline{1,2})$ and criteria for solving the problem in contrast to (13), (14) and (16) represent the ratios

$$w_0^{(l)}(s) = \sum_{m=1}^{M_0^{(l)}} G^{(l)}(\xi - \xi_m^{0(l)}, z) u_{0m}^{(l)}, \tag{23}$$

$$w_\Gamma^{(l)}(s) = \sum_{m=1}^{M_\Gamma^{(l)}} G^{(l)}(\xi - \xi_m^{\Gamma(l)}, z) u_{\Gamma m}^{(l)} \quad (l = \overline{1,2}), \tag{24}$$

$$\Phi \to \min_{\substack{u_0^{(l)} \in R^{M_0^{(l)}}, \; u_\Gamma^{(l)} \in R^{M_\Gamma^{(l)}}, \\ q_*^{(l)} \in R^{M^{(l)}} \; (l = \overline{1,2})}}. \tag{25}$$

The problem is (25) is equivalent to the average square inversion of linear algebraic equations

$$A\overline{u} = \overline{Y}, \tag{26}$$

in which the vector $\overline{Y}$ defined above and

$$\overline{u} = col(col(u_0^{(l)}, \; u_\Gamma^{(l)}, \; q_*^{(l)}), \; l = \overline{1,2}),$$

$$A = str((A_1^{(l)}, \; A_2^{(l)}, \; A_3^{(l)}), \; l = \overline{1,2}), \; A_n^{(l)} = col(A_{kn}^{(l)}, \; k = \overline{1,3}) \quad (n = \overline{1,3}, \; l = \overline{1,2}),$$

where

$$A_{1n}^{(l)} = col\left(\left(\left(str\left(L_r^0(\partial_t) G^{(l)}(\xi - \xi_{nm}^{(l)}, z)\Big|_{\substack{t=0 \\ \sigma = \sigma_j^0 \in S_0}}, \; m = \overline{1, M_n^{(l)}}\right), \; j = \overline{1, J_r}\right), \; r = \overline{1, R_0}\right),$$

$$A_{2n}^{(l)} = col\left(\left(\left(str\left(L_\rho^\Gamma(\partial_\sigma) G^{(l)}(\xi - \xi_{nm}^{(l)}, z)\Big|_{\substack{t = t_j \in [0,T] \\ \sigma = \sigma_j^\Gamma \in S_\Gamma}}, \; m = \overline{1, M_n^{(l)}}\right), \; j = \overline{1, J_\rho}\right), \; \rho = \overline{1, R_\Gamma}\right),$$

$$A_{3n}^{(l)} = col\left(\left(str\left((L_i(\partial_s)G^{(l)}(\xi - \xi_{nm}^{(l)}, z)\big|_{s=s_{ij}\in S_0\times[0,T]}\ ,\ m=\overline{1,M_n^{(l)}}\right),\ j=\overline{1,J_i}\right),\ i=\overline{1,I}\right)$$

at $l=\overline{1,2}$, $n=\overline{1,3}$, $\xi_{1m}^{(l)}=\xi_m^{0(l)}$, $\xi_{2m}^{(l)}=\xi_m^{\Gamma(l)}$, $\xi_{3m}^{(l)}=\xi_m^{(l)}$, $M_1^{(l)}=M_0^{(l)}$, $M_2^{(l)}=M_\Gamma^{(l)}$, $M_3^{(l)}=M^{(l)}$.

After average square inversion (26) such that

$$\left\|A\overline{u}-\overline{Y}\right\|^2 \to \min_{\overline{u}},$$

by analogy with the previous case we find that the control-modeling vectors $u_0^{(l)}$, $u_\Gamma^{(l)}$, $q_*^{(l)}$ determined [3] ratios (at $l=\overline{1,2}$):

$$u_0^{(l)} \in \Omega_0^{(l)} = \{u_0^{(l)}:\ u_0^{(l)} = (A_1^{(l)})^T P^+(\overline{Y}-A\overline{v})+v_0^{(l)}\}\ ,$$

$$u_\Gamma^{(l)} \in \Omega_\Gamma^{(l)} = \{u_\Gamma^{(l)}:\ u_\Gamma^{(l)} = (A_2^{(l)})^T P^+(\overline{Y}-A\overline{v})+v_\Gamma^{(l)}\}\ ,$$

$$q_*^{(l)} \in \Omega_q^{(l)} = \{q_*^{(l)}:\ q_*^{(l)} = (A_3^{(l)})^T P^+(\overline{Y}-A\overline{v})+v_*^{(l)}\}$$

for arbitrary $\overline{v}=col(col(v_0^{(l)}\in R^{M_0^{(l)}}, v_\Gamma^{(l)}\in R^{M_\Gamma^{(l)}}, v_*^{(l)}\in R^{M^{(l)}}),\ l=\overline{1,2})$, where P^+ – matrix pseudo inverse to $P=AA^T$.

The accuracy of the solution of the problem is defined [3] ratio

$$\varepsilon = \min_{\overline{u}} \Phi = \overline{Y}^T\overline{Y} - \overline{Y}^T PP^+\overline{Y}$$

and by the uniqueness $(v_0^{(l)}=v_\Gamma^{(l)}=v_*^{(l)}=0)$ of the obtained solution is $\det A^T A>0$.

References

1. Skopetskiy V.V., Stoyan V.A., Kryvonos Yu.G. Mathematical modeling of direct and inverse problems of distributed parameters systems dynamics.– Kyiv: Naukova dumka, 2002. – 361 p. [in Ukrainian]
2. Skopetskiy V.V., Stoyan V.A. Zvarydchuk V.B. Mathematical modeling of the dynamics of distributed space-time processes. – Kyiv: publisher "Steel", 2008. – 316 p. [in Ukrainian]
3. Stoyan V.A. Mathematical modeling of dynamics of linear, quasilinear and nonlinear systems. – Kyiv: STPC "University of Kiev", 2011. – 319 p. [in Ukrainian]
4. Timoshenko S.P., Voynovskyy-Krieger S. Plates and shells. – Moscow: FIZMATLIT, 1966. – 635 p. [in Russian]
5. Donnell L.G. Beams, plates and shells. – Moscow: Nauka, 1982. – 568 p. [in Russian]
6. Grigorenko Ya.M., Vlaycov G.G., Grigorenko A.Ya. Numerical-analytical solution of problems in the mechanics of shells on the basis of various models. – Kyiv: Academperiodika, 2006. – 472 p. [in Russian]
7. Grigolyuk E.I., Selezov I.T. Solid mechanics. Volume 5: Non-classical theory of vibrations of rods, plates and shells. – Moscow: VINITI, 1973. – 272 p. [in Russian]
8. Lurie A.I. Dimensional problem of elasticity. – Moscow: Gostekhizdat, 1955. – 370 p. [in Russian]
9. Stoyan V.A., Dvirnychuk K.V. On the construction of a differential model of dynamic transverse displacement of thick elastic layer // Problems of cybernetics and Informatics. – 2012. – № 4. – P.74-83. [in Russian]
10. Stoyan V.A., Dvirnychuk K.V. Integral models of dynamic transverse displacement of thick elastic layer // Problems of cybernetics and Informatics. – 2013. – № 1.
11. [in Russian] Stoyan V.A., Dvirnychuk K.V. By building the equivalent integral linear differential models // Reports of NAS of Ukraine. – 2012. – № 9. – P.36-43. [in Ukrainian]

Надійшла до редколегії 07.06.13

К. Двірничук, асп.
КНУ імені Тараса Шевченка, Київ

ПРО ПРОБЛЕМУ КЕРУВАННЯ ТРИВИМІРНИМ ПОЛЕМ ПОПЕРЕЧНОГО ДИНАМІЧНОГО ЗМІЩЕННЯ ТОВСТОЇ ПРУЖНОЇ ПЛАСТИНИ

Розглянуті задачі керування трьохвимірним полем поперечних динамічних зміщень товстих пружних плит, дискретно спостережуваних за початково-крайовим станом. Сформульовані умови точності та однозначності розв'язку поставленої задачі.

Ключові слова: диференціальна модель, інтегрована модель, тривимірна задача.

К. Двирничук, асп.
КНУ имени Тараса Шевченко, Киев

О ПРОБЛЕМЕ УПРАВЛЕНИЯ ТРЕХМЕРНЫМ ПОЛЕМ ПОПЕРЕЧНОГО ДИНАМИЧЕСКОГО СМЕЩЕНИЯ ТОЛСТОЙ УПРУГОЙ ПЛАСТИНЫ

Рассмотрены задачи управления трехмерным полем поперечных динамических смещений толстых упругих плит, дискретно наблюдаемых за начально-краевым состоянием. Сформулированные условия точности и однозначности решения поставленной задачи.

Ключевые слова: дифференциальная модель, интегрированная модель, трехмерная задача.