

Нехай функція $g^k \in C^\infty([0, T])$, $k = \overline{1, M}$, такі, що $g^k(T) = 0$. Розглянемо (7) для $f = g^k$. Просумуємо (7) по k від 1 до $g^k \in C^\infty([0, T])$, $k = \overline{1, M}$. Позначимо $\Phi^M(x, t) = \sum_{k=1}^M \psi^k(x) g^k(t)$. Отримуємо

$$\int_{G_T} \left\{ a_{ij} u_{x_i}^h \Phi_{x_j}^M - h b_i u_{x_i}^h \Phi^M - u^h \Phi_s^M \right\} dx ds = \int_G \varphi^h(x) \Phi^M(x, 0) dx. \quad (8)$$

Функції $\Phi^M(x, t)$ складають щільну множину серед елементів $W_{1,1}^2(G_T)$, які дорівнюють нулю при $s = T$. Умови теореми забезпечують можливість граничного переходу в (8) по послідовностям $\{\Phi^M\}_{M=1}^\infty$. В результаті отримуємо (3). Лему доведено.

Теорема. Припустимо, що виконуються умови А), В), С), Д). Тоді перша крайова задача для рівняння (1) не може мати двох узагальнених розв'язків.

Доведення. Припустимо, що u_1 і u_2 – два узагальнених розв'язки першої крайової задачі для (1) з $\overset{\circ}{SV}$. Тоді u_1^h і u_2^h – розв'язки задачі (3) з $\overset{\circ}{V}$. Але, як встановлено в [2], задача (3) не може мати двох різних розв'язків з $\overset{\circ}{V}$. Це означає, що для $h \in C^\infty([0, T])$ має місце умова $E\varepsilon(h)(u_1(x, s, \cdot) - u_2(x, s, \cdot)) = 0$ на множині G_T^h повної $(\lambda_d \times \lambda)$ – міри циліндра G_T . Розглянемо множину $h \in C^\infty([0, T])$ всюди щільну в $L^2([0, T])$. Наприклад, це можуть бути многочлени з раціональними коефіцієнтами. Тоді $\{\varepsilon(h_i)\}_{i=1}^\infty$ утворює тотальну множину в $L^2(\Omega)$. Позначимо $\tilde{G}_T = \bigcap_{i=1}^\infty G_T^{h_i}$. Множина $\tilde{G}_T \subseteq G_T$ має повну $(\lambda_d \times \lambda)$ – міру циліндра G_T . Для всіх $h \in \{h_i\}_{i=1}^\infty$ будемо мати: $E\varepsilon(h)(u_1 - u_2)|_{\tilde{G}_T} = 0$.

З цього безпосередньо випливає, що $u_1 - u_2 = 0$ за мірою $\lambda_d \times \lambda \times P$. Отже, u_1 і u_2 не відрізняються як елементи $\overset{\circ}{SV}$. Теорему доведено.

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ЕДИНСТВЕННОСТЬ РЕШЕНИЯ СТОХАСТИЧЕСКИХ ДИФФЕРЕНЦІАЛЬНИХ УРАВНЕНИЙ В ЧАСТИХ ПРОІЗВОДНИХ ПАРАБОЛИЧЕСКОГО ТИПА С ОПЕРЕЖЕНИЕМ

Получены условия единственности обобщенного решения стохастических дифференциальных уравнений в частных производных параболического типа с опережением.

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THE UNIQUENESS OF THE SOLUTION OF ANTICIPATING PARTIAL STOCHASTIC DIFFERENTIAL EQUATIONS OF PARABOLIC TYPE

Conditions for the uniqueness of the generalized solution of anticipating partial stochastic differential equations of parabolic type are obtained.

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INVARIANT SUBALGEBRA OF UNIVERSAL ENVELOPING ALGEBRA FOR ORTHOGONAL MATRIX LIE ALGEBRA

The structure and properties of invariant Gelfand-Zetlin subalgebra of universal enveloping algebra for the orthogonal complex Lie algebra. This algebra is considered as subalgebra of polynomials on groups of variables, depending on two indices that are invariant with respect to the action of the Weyl group. The Gelfand-Zetlin algebra is realized as some finite extension of the algebra of symmetric polynomials on groups of variables.

INTRODUCTION. The aim of this paper is to study the structure of the invariant subalgebra of the universal embedding algebra of complex orthogonal matrix Lie algebra O_n . We use Gelfand-Zetlin formal construction for orthogonal Lie algebras [1–3, 6]. We construct the orthogonal operator algebra associated with Gelfand-Zetlin formulae. The orthogonal Lie algebra

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O_n is realized as operator Lie algebra on the set of tableau according to parity of n [3]. Thus sufficient to determine the action of generators of the Lie algebra O_n . The center Z of universal embedding algebra U_n [4] is generated by independent invariant operators, which can be determined by the S-theorem Poxa [3], they are Kasimir operators for O_n invariant with respect to the action of the Weyl group. The Gelfand-Zetlin approach interprets the center Z of universal embedding algebra U_n of orthogonal Lie algebra O_n as invariant Gelfand-Zetlin subalgebra Γ [4–6], and which is studied in this paper.

DEFINITIONS AND STATEMENT OF THE PROBLEM. Let K denotes the basic field of complex numbers. For $n \in N, n > 1$, we will denote by $e_{ij} \in Mat_{n \times n}(K)$, $1 \leq i < j \leq n$, the matrix units. We denote by O_n the Lie algebra of complex skew symmetric $n \times n$ -matrix and call it the orthogonal matrix Lie algebra. Denote by U_n the universal embedding algebra for O_n , and by Z_n the center of U_n . The matrices $E_{ji} = e_{ji} - e_{ij}$, $1 \leq i < j \leq n$, form a standard generator set for O_n , and the set of matrices $E_{j+1,j}$, $1 \leq j < n$ is a minimal system of generators.

We identify O_n , $1 \leq m \leq n$, with an orthogonal Lie subalgebra of O_n spanned by $\{e_{ji} - e_{ij} \mid 1 \leq i < j \leq m\}$. So that we have the following chain of orthogonal Lie algebras $O_1 \subset O_2 \subset \dots \subset O_n$, embedded in the left upper corner. It induces the embeddings of the universal enveloping algebras $U_1 \subset U_2 \subset \dots \subset U_n$.

We use the Gelfand-Zetlin formal construction concerning the the orthogonal Lie algebra [1, 6]. The simple finite-dimensional modules of the orthogonal Lie algebra O_{n+1} are parametrized by the vectors $m = (m_1, \dots, m_p)$, $p = \left[\frac{n+1}{2} \right]$, with complex coefficients, satisfying

$$\begin{cases} m_1 \geq m_2 \geq \dots \geq m_{p-1} \geq m_p \mid n = 2p, \\ m_1 \geq \dots \geq m_{p-1} \geq m_p \geq 0 \quad n = 2p+1. \end{cases}$$

These vectors represent the highest weight of the corresponding simple module. This construction uses the notion of tableau. By a tableau, $[t]$, we mean a doubly-indexed complex vector

$$O_{2p} : \begin{array}{ccccccccc} t_{2p-1,j} & \dots & \dots & t_{2p-1,p} & & t_{2p,j} & \dots & \dots & t_{2p,p} \\ & t_{2p-2,1} & \dots & t_{2p-2,p-1} & & t_{2p-1,1} & \dots & t_{2p-1,p} \\ & \dots & \dots & & & \dots & \dots & \dots & \dots \\ & & & t_{1,1} & & & & & t_{1,1} \end{array}$$

The Gelfand-Zetlin formalism provides that O_{n+1} is realized as an operator Lie algebra on the set of all patterns or on the attached polynomial ring. For this, it is sufficient to determine the action of the generators E_{ji} of O_{n+1} . We denote

$$\begin{cases} l_{2p,j} = t_{2p,j} + p - j + 1/2, \\ l_{2p-1,j} = t_{2p-1,j} + p - j, \end{cases} \quad j = 1, \dots, p. \quad (1)$$

The following lemma summarizes the results [2,3].

Lemma 1. In terms of variables $l_{2p,1}, \dots, l_{2p,p}$ (1), any element of the Weyl group W_{2p+1} for O_{2p+1} can be represented as a product of permutation of these variables and of an arbitrary number of inversions of a type:

$$\varepsilon_{2p,j} : \begin{cases} l_{2p,j} \mapsto -l_{2p,j}, \\ l_{2p-1,k} \mapsto l_{2p-1,k}, k \neq j. \end{cases}$$

The center Z_{2p+1} of universal enveloping algebra U_{2p+1} is generated by the symmetric polynomials of even order $z_k = \sum_{j=1}^p l_{2p,j}^k$, $k \in 2N$. Let $Z_2^p = \langle \varepsilon_{2p,1} \rangle \times \dots \times \langle \varepsilon_{2p,p} \rangle$, $\varepsilon_{2p,j}^2 = 1$, $j = 1, \dots, p$ be an elementary 2-group. The symmetric group Sym_p acts on Z_2^p by the permutation of components. Then the Weyl group W_{2p+1} of odd orthogonal Lie algebra O_{2p+1} as an abstract group is isomorphic to the semi direct product $Sym_p \rtimes Z_2^p$.

Lemma 2. In terms of variables $l_{2p-1,1}, \dots, l_{2p-1,p}$ (1), any element of the Weyl group W_{2p} for O_{2p} can be represented as a product of permutation of these variables and of an arbitrary number of inversions of a type:

$$\varepsilon_{2p-1,i} \varepsilon_{2p-1,j} : \begin{cases} l_{2p-1,i} \mapsto -l_{2p-1,i}, \\ l_{2p-1,j} \mapsto -l_{2p-1,j}, \\ l_{2p-1,k} \mapsto l_{2p-1,k}, k \neq i, j. \end{cases}.$$

The center Z_{2p} of universal enveloping algebra U_{2p} is generated by the product $pl_{2p-1,1} \dots l_{2p-1,p}$ and by the symmetric polynomials of even order of a type $z_k = \sum_{j=1}^p l_{2p-1,j}^k$, $k \in 2N$.

Let $\varphi: Z_2^p \rightarrow \{\pm 1\}$ be a canonical epimorphism. We denote by E_p the sub group $\text{Ker } \varphi$ of index 2 in Z_2^p , i.e. $\varphi(\varepsilon) = 1$ if and only if ε contains even number of non unit components. We denote W_{2p}^+ the semi direct product of groups Sym_p and Z_2^p . The Weyl group W_{2p} of even orthogonal Lie algebra O_{2p} as an abstract group is isomorphic to the semi direct product of groups Sym_p and E_p , it is a sub group of index 2 in W_{2p}^+ .

METHODS. We fix some integer positive n and define the action of the Weyl group $W = W_{n+1}$ on the polynomial ring Λ . For any m , we put $p(m) = \left[\frac{n+1}{2} \right]$ and $c_m = 1$ if m is even, $c_m = 0$ for m odd. Let $p = p(m)$, we consider the following numbers:

$$N_{[1;n]} = \sum_{m=1}^{n-1} p(m) \text{ i } N_{[1;n]} = \sum_{m=1}^n p(m) = N_{[1;n]} + p .$$

Then $N_{[1;n]} + N_{[1;n]} = \frac{n(n-1)}{2}$. We denote by $J_m = [1; p(m)]$ an integer interval, and we indicate the following index sets: $J_{[1;n]} = \{mj\}_{m \in [1;n], j \in J_m}$, $J_{[1;n]} = \{mj\}_{m \in [1;n], j \in J_m}$. Then $|J_{[1;n]}| = N_{[1;n]}$ i $|J_{[1;n]}| = N_{[1;n]}$.

Let $\Lambda_m = K \left[\left\{ \lambda_{mj} \right\}_{j \in J_m} \right]$ be the polynomial K -algebra in $p(m)$ variables, denote by Λ the polynomial K -algebra $\Lambda = K[\Lambda_1, \dots, \Lambda_n]$ in $N_{[1;n]}$ variables, where

$$\begin{cases} \lambda_{2q,j} = l_{2q,j} + 1/2 & = t_{2q,j} + q + 1 - j, \quad 2q \leq n; \\ \lambda_{2q-1,j} = l_{2q-1,j} & = t_{2q-1,j} + q - j, \quad 2q-1 \leq n; \end{cases}$$

or, equivalently, $\lambda_{mj} = t_{mj} + \left[\frac{m}{2} \right] + 1 - j$, $j \in [1; p(m)]$.

For any $m, q = p(m)$, let Z_2^q be a 2-group defined above, and let $E_m = Z_2^q$ for m even, E_m be a sub group of index 2 in Z_2^q for m odd. Thereafter, we consider the transformation groups acting on the m -th level: the symmetric group S_m , and the groups $G_m = S_m \times Z_2^2$, $W_m = S_m \times E_m$ with \times be a semi-direct product. We have the following groups acting on Λ :

$$S = \prod_{m \in [1;n]} S_m, \quad E = \prod_{m \in [1;n]} E_m, \quad G = \prod_{m \in [1;n]} G_m = S \times \prod_{m \in [1;n]} Z_2^q, \quad W = \prod_{m \in [1;n]} W_m = S \times E .$$

Under the construction, Weyl group of the orthogonal group O_{n+1} is isomorphic to W .

We determine the action of G and W on Λ . The group S acts naturally by the permutations on the variables having

the same first index. The involution $\varepsilon_{mj} \in E_m$ is given by the formulae $\begin{cases} \lambda_{mj}^{\varepsilon_{mj}} = \bar{\lambda}_{mj}, \\ \lambda_{ki}^{\varepsilon_{mj}} = \lambda_{ki}, \quad ki \neq mj, \end{cases}$ with

$$\bar{\lambda}_{mj} = c_m - \lambda_{mj} = \begin{cases} -\lambda_{mj}, & c_m = 0, \\ 1 - \lambda_{mj}, & c_m = 1, \end{cases}$$

be the conjugate element for λ_{mj} in Λ . For m odd, any generator $\varepsilon_{mj}\varepsilon_{mk}$ of E_m acts on Λ as a composition.

We establish some useful identities in Λ . For any $i, j \in J_m$, there holds $\lambda_{mj} + \bar{\lambda}_{mj} = \lambda_{mi} + \bar{\lambda}_{mi} (= c_m)$. Equality

$$\lambda_{mj} + \bar{\lambda}_{mj} + \lambda_{ki} + \bar{\lambda}_{ki} = c_m + c_k = 1 .$$

hold for all $m, k \in [1;n]$ provided $|m - k| = 1$.

For any $m \leq n$ and any $j \in J_m$, introduce a new variable $\gamma_{mj} = \lambda_{mj}\bar{\lambda}_{mj}$. The action of the group G is transferred to this variables in the natural way while E acts identically. We denote by $\Gamma_m^s = Sym \left[\left\{ \gamma_{mj} \right\}_{j \in J_m} \right] = K \left[\left\{ \gamma_{mj} \right\}_{j \in J_m} \right]^{S_m}$ the K -algebra of symmetric polynomials in q variables $\left\{ \gamma_{mj} \right\}_{j \in J_m}$. Given $0 \leq k \leq q = p(m)$ we denote by $\sigma_k^{(m)} = e_k(\gamma_{m1}, \dots, \gamma_{mq})$ the k -th elementary polynomial with $\sigma_0^{(m)} = 1$. Then $\Gamma_m^s = K \left[\left\{ \sigma_k^{(m)} \right\}_{k \in [1;q]} \right]$ is an algebra of symmetric polynomials.

Given $m = 2q-1$, we define the product $\lambda_{(2q-1)} = \lambda_{2q-1,1} \dots \lambda_{2q-1,q} \in \Lambda_m^{G_m}$. Then $(\lambda_{(2q-1)})^2 = \prod_{j=1}^q \gamma_{2q-1,j} \in \Gamma_{2-1}^s$. We set

$$\gamma_{(2q-1)} = \gamma_{2q-1,1} \dots \gamma_{2q-1,q} = \sigma_q^{(m)} \in \Gamma_m^s .$$

Here are some statements from the theory of symmetric polynomials. For some set of variables $\{x_1, \dots, x_r\}$ consider the ring of symmetric polynomials $Sym_r = Sym(x_1, \dots, x_r) = K[x_1, \dots, x_r]^{S_r}$, the symmetric group S_r acts naturally by permutation of variables.

We denote by $s_{r,t} = s_{r,t}(x_1, \dots, x_r)$ t -th elementary symmetric polynomial of the first r variables, $1 \leq t \leq r \leq q$. We assume $s_{r,0} = 1$ и $s_{r,t} = 0$ for $t > r$. Then $Sym_r = K[s_{r,1}, \dots, s_{r,r}]^{S_q}$. Let $Sym'_r \subset Sym_r$ denotes the sub-ring of symmetric polynomials over K , generated by the elementary polynomials $s_{r,t}$, $t = 1, \dots, r-1$.

Lemma 3. Let $1 \leq r \leq q$. Then for any $0 \leq t \leq q$ the following holds: $s_{r-1,t} = \sum_{k=0}^t (-1)^k x_r^k s_{r,t-k}$. In particular, there is equality $\sum_{k=0}^r (-1)^k x_r^k s_{r,t-k} = 0$.

Proof. We prove the first equality by induction on $t > 0$. It is true for $t = 1$ and $r > t$, because $s_{r-1,1} = s_{r,1} - x_r s_{r-1,0}$. For $t > 1$, we have: $s_{r-1,t} = s_{r,t} - x_r s_{r-1,t-1} = s_{r,t} - x_r \sum_{k=0}^{t-1} (-1)^k x_r^k s_{r,t-k-1} = s_{r,t} + x_r \sum_{l=1}^t (-1)^l x_r^l s_{r,t-l} = \sum_{l=0}^t (-1)^l x_r^l s_{r,t-l}$. The statement of Lemma 3 is obtained by substitution $t = r$.

RESULTS. We set $\Gamma_{2q} = \Gamma_{2q}^s$, $2q \leq n$. For $2q-1 \leq n$, we denote by Γ_{2q-1} the subalgebra in Λ_{2q-1} , generated by Γ_{2q-1}^s and $\lambda_{(2q-1)}$. Then $\Gamma_{2q-1}^s \subset \Gamma_{2q-1}$ is a square extension. The constructed Γ_{2q} and Γ_{2q-1} are the polynomial subalgebras in q variables.

Lemma 4. For any $m \in [1; n]$ the center Z_{m+1} of universal embedding algebra U_{m+1} is a subalgebra $\Gamma_m \subset \Lambda_m$.

Proof. Let $m = 2q$. For any j we obtain: $\gamma_{2q,j} = \lambda_{2q,j} \bar{\lambda}_{2q,j} = (l_{2q,j} + 1/2)(-l_{2q,j} + 1/2) = 1/4 - l_{2q,j}^2$. Therefore, $\gamma_{2q} = \sum_{j=1}^q l_{2q,j}^2 = \sum_{j=1}^q (1 - \gamma_{2q,j})^2$ is a symmetric polynomial on $\gamma_{2q,1}, \dots, \gamma_{2q,q}$. The proof in this case follows from Lemma 1. Similarly, for the case $m = 2q-1$, using Lemma 2, we obtain, that the center Z_{2q} is generated by the symmetric polynomials in variables $\gamma_{2q-1,1}, \dots, \gamma_{2q-1,q}$ and by the polynomial $\lambda_{(2q-1)}$. Besides, $\lambda_{(2q-1)}^2 = (-1)^q \prod_{j=1}^q \gamma_{2q-1,j}$ because $\prod_{j=1}^q \lambda_{2q-1,j} = (-1)^q \prod_{j=1}^q \bar{\lambda}_{2q-1,j}$.

By the definition, the Gelfand-Zetlin subalgebra $\Gamma \subset U_{n+1}$ is generated by all centers Z_{m+1} for $m \in [1; n]$. By the construction, Γ is a commutative integral domain, generated by all Γ_m . Let $\Gamma^s \subset \Gamma$ be a subalgebra, generated by all Γ_m^s , $m \in [1; n]$.

Лема 5. Following identities are valid: $\Gamma = \Lambda^W$ and $\Gamma^s = \Lambda^G$.

Proof. Obviously, $\Gamma^s \subset \Lambda^G$. We prove the opposite inclusion by induction on n . Let $\Lambda_{[1;n]} \subset \Lambda$ be a sub ring in the variables λ_{mj} , $m \in [1; n]$, and $\Gamma_{[1;n]} \subset \Gamma$ be a subalgebra, generated by all Γ_m , $m \in [1; n]$. By assumption of induction, $\Lambda_{[1;n]}^G = \Gamma_{[1;n]}^s$.

It is easy to see that $\Lambda^G \subset \Lambda_{[1;n]}^G[\lambda_{n1}, \dots, \lambda_{np}] = \Gamma_{[1;n]}[\lambda_{n1}, \dots, \lambda_{np}]$ where $p = p(n)$. Suppose that some $a \in \Lambda^G \setminus \Gamma_{[1;n]}$ depends on the variable λ_{nj} . Then there exists a polynomial $b(t) \in \Gamma_{[1;n]}[\lambda_{n1}, \dots, \lambda_{np-1}][t]$ such that $a = b(\lambda_{nj})$. According to the assumption, $b(\bar{\lambda}_{nj}) = a^{\pi_{nj}} = b(\lambda_{nj})$. Note that the polynomial $b(\bar{\lambda}_{nj}) + b(\lambda_{nj})$ is symmetric with respect to two variables $\bar{\lambda}_{nj}, \lambda_{nj}$, and thus depends on $\bar{\lambda}_{nj} \lambda_{nj} = \gamma_{nj}$ and $\bar{\lambda}_{nj} + \lambda_{nj} = c_n \in K$. Hence, $a = b(\gamma_{nj})$ for some $b(t) \in \Gamma_{[1;n]}[\lambda_{n1}, \dots, \lambda_{np-1}][t]$. From this we obtain the following inclusion: $b(t) \in \Gamma_{[1;n]}[\gamma_{n1}, \dots, \gamma_{np-1}][t]$.

From the theory of symmetric polynomials, it follows that every polynomial $a \in \Gamma_{[1;n]}[\gamma_{n1}, \dots, \gamma_{np}]$ can be presented as a polynomial $a = b(\gamma_{nj}) \in \Gamma^s[\gamma_{nj}]$ for some $j \in [1; p]$ of degree less than p , and this presentation is unique. Because $a^{\pi_{ni,nj}} = a$ with $\pi_{ni,nj}$ be a transposition, this means that the polynomial $b(t)$ has at least p roots $\gamma_{n1}, \dots, \gamma_{np}$, contrary to the assertion that the power does not exceed p . With the proven $a \in \Gamma^s$, from where we obtain the inclusion $\Lambda^G \subset \Gamma^s$.

Now we assume $\Lambda_{[1;n]}^W = \Gamma_{[1;n]}$ and show that $\Gamma = \Lambda^W$. It is trivial for n even. If n is odd, then $\Gamma^s = \Lambda^G \subset \Lambda^W$ because $W \subset G$. Note that $\Gamma^s[\gamma_{n1}, \dots, \gamma_{np}]^G = \Gamma^s[\gamma_{n1}, \dots, \gamma_{np}]^G = \Gamma^s$. Therefore, any element $F \in \Lambda^W$ can be represented as $a \in \Gamma^s[\lambda_{n1}, \dots, \lambda_{np}]$, moreover the degree of any λ_{nj} is not more than 1, while for n odd, there is $\lambda_{nj}^2 = -\gamma_{nj}$. Hence,

if F has a factor $c\lambda_{nj}$, then $c\lambda_{nj} = c'\lambda_{ni}$ for any i , because $a^{\pi_{ni,nj}} = a$. From this we obtain $a = a'\lambda_{ni}$ with $a' \in \Gamma^s$. The proof is complete.

Consider the orthogonal Lie algebra O_{n+1} and its universal embedding algebra U_{n+1} . The corresponding polynomial K -algebra Λ and the Gelfand-Tsetlin algebra are defined above. We denote by F the field of fractions of Γ , and by L the field of fractions of Λ . The action of the group W on F and L is determined by the action of W on Λ . By the Lemmas 3 and 4, we obtain the following theorems.

Theorem 1. The natural monomorphism $\Gamma \rightarrow \Lambda$ can be extended to an embedding of the fields $K \rightarrow L$. The subalgebra $\Gamma = \Lambda^W$ is a Galois order in the sense of [5], $L^W = K$ and $G = G(L/K)$ is a Galois group of the field extension $K \subset L$. Besides, the global dimension of Γ equals $N_{[1;n]}$.

Theorem 2. $\Gamma^s = \Lambda^G$ is a ring of multi symmetric polynomials in groups of variables $\{\gamma_{mj} \mid j \in J_m\}_{m \in [1;n]}$, and F^s is a field of fractions of Γ^s . Then $F \supset F^s$ is an extension of fields of degree $2^{\left[\frac{n-1}{2}\right]}$. Moreover, $F = F^s[t_1, t_3, \dots, t_k]/J$, where k is a maximal odd integer less or equal $n+1$, an ideal J is generated by $t_r^2 - \sigma_r^{p(r)}$, $r = 1, 3, \dots, k$.

CONCLUSIONS. There is considered an invariant subalgebra Γ of universal embedding algebra U_n of complex orthogonal matrix Lie algebra O_n . Were introduced new variables, in terms of which Γ is a subalgebra of polynomial algebra, which is invariant under the action of Weyl group W . We describe algebra Γ as subalgebra of the algebra of symmetric polynomials in groups of variables.

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ІНВАРИАНТНА ПІДАЛГЕБРА УНІВЕРСАЛЬНОЇ ОБГОРТОЮЧОЇ АЛГЕБРИ ДЛЯ ОРТОГОНАЛЬНОЇ МАТРИЧНОЇ АЛГЕБРИ ЛІ

Досліджено структуру та властивості інваріантної підалгебри Гельфанда-Цетліна універсальної обгортоючої алгебри для ортогональної комплексної алгебри Лі. Ця під алгебра розглядається як під алгебра алгебри поліномів від груп змінних, залежних від двох індексів, яка є інваріантною відносно дії групи Вейля. В роботі показано, що під алгебра Гельфанда-Цетліна реалізується як деяке скінчене розширення алгебри симетричних поліномів від груп змінних.

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ИНВАРИАНТНАЯ ПОДАЛГЕБРА УНИВЕРСАЛЬНОЙ ОБЕРТЫВАЮЩЕЙ АЛГЕБРЫ ДЛЯ ОРТОГОНАЛЬНОЙ МАТРИЧНОЙ АЛГЕБРЫ ЛИ

Исследованы структура и свойства инвариантной подалгебры Гельфанда-Цетліна универсальной обертывающей алгебры для ортогональной комплексной алгебры Ли. Эта под алгебра рассматривается как под алгебра полиномов от групп переменных, зависящих от двух индексов, которая является инвариантной относительно действия группы Вейля. Алгебра Гельфанда-Цетліна реализуется как некоторое конечное расширение алгебры симметричных полиномов от групп переменных.

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ANNIHILATOR SUBALGEBRAS IN LIE ALGEBRAS OF DERIVATIONS

Let K be an algebraically closed field of characteristic zero and $K(x_1, \dots, x_n)$ be the field of rational functions. The set $\text{Ann}(S)$ of all K -derivations of $K(x_1, \dots, x_n)$ which annihilate a set $S \subseteq K(x_1, \dots, x_n)$ is a subalgebra of the Lie algebra $W_n(K)$. The structure of the subalgebra $\text{Ann}(S)$ is connected with centralizers of elements of $W_n(K)$. A characterization of the subalgebra $\text{Ann}(S)$ is given, some sets of generators of $\text{Ann}(S)$ are pointed out in the Lie algebra of all K -derivations of the polynomial ring $K[x_1, \dots, x_n]$.

INTRODUCTION. Let K be an algebraically closed field of characteristic zero and $R = K(x_1, \dots, x_n)$ be the field of rational functions in n variables. The set $\text{Der}_K R$ of all K -derivations of R , i.e. K -linear operators D on the field R satisfying the Leibniz's rule: $D(fg) = D(f)g + fD(g)$ for all $f, g \in R$ is a Lie algebra over K and a vector space over R in a natural way: if we take $f \in R, D \in \text{Der}_K R$, the derivation fD sends any element $g \in R$ to $fD(g)$. The structure of the Lie