if F has a factor $c\lambda_{nj}$, then $c\lambda_{nj} = c'\lambda_{ni}$ for any i, because $a^{\pi_{ni,nj}} = a$. From this we obtain $a = a'\lambda_{ni}$ with $a' \in \Gamma^s$. The proof is complete.

Consider the orthogonal Lie algebra O_{n+1} and its universal embedding algebra U_{n+1}. The corresponding polynomial K -algebra Λ and the Gelfand-Tsetlin algebra are defined above. We denote by F the field of fractions of Γ, and by L the field of fractions of Λ. The action of the group W on F and L is determined by the action of W on Λ. By the Lemmas 3 and 4, we obtain the following theorems.

Theorem 1. The natural monomorphism $\Gamma \to \Lambda$ can be extended to an embedding of the fields $K \to L$. The subalgebra $\Gamma = \Lambda^W$ is a Galois order in the sense of [5], $L^W = K$ and $G = G(L/K)$ is a Galois group of the field extension $K \subset L$. Besides, the global dimension of Γ equals $N_{[1;n]}$.

Theorem 2. $\Gamma^s = \Lambda^G$ is a ring of multi symmetric polynomials in groups of variables $\left\{ \gamma_{mj} \mid j \in J_m \right\}_{m\in[1;n]}$, and F^s is a field of fractions of Γ^s. Then $F \supset F^s$ is an extension of fields of degree $2^{\left[\frac{n-1}{2}\right]}$. Moreover, $F = F^s[t_1, t_3, \ldots, t_k]/J$, where k is a maximal odd integer less or equal $n+1$, an ideal J is generated by $t_r^2 - \sigma_r^{p(r)}$, $r = 1, 3, \ldots, k$.

CONCLUSIONS. There is considered an invariant subalgebra Γ of universal embedding algebra U_n of complex orthogonal matrix Lie algebra O_n. Were introduced new variables, in terms of which Γ is a subalgebra of polynomial algebra, which is invariant under the action of Weyl group W. We describe algebra Γ as subalgebra of the algebra of symmetric polynomials in groups of variables.

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ІНВАРІАНТНА ПІДАЛГЕБРА УНІВЕРСАЛЬНОЇ ОБГОРТУЮЧОЇ АЛГЕБРИ ДЛЯ ОРТОГОНАЛЬНОЇ МАТРИЧНОЇ АЛГЕБРИ ЛІ

Досліджено структуру та властивості інваріантної підалгебри Гельфанда-Цетліна універсальної обгортуючої алгебри для ортогональної комплексної алгебри Лі. Ця під алгебра розглядається як під алгебра алгебри поліномів від груп змінних, залежних від двох індексів, яка є інваріантною відносно дії групи Вейля. В роботі показано, що під алгебра Гельфанда-Цетліна реалізується як деяке скінчене розширення алгебри симетричних поліномів від груп змінних.

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ИНВАРИАНТНАЯ ПОДАЛГЕБРА УНИВЕРСАЛЬНОЙ ОБЕРТЫВАЮЩЕЙ АЛГЕБРЫ ДЛЯ ОРТОГОНАЛЬНОЙ МАТРИЧНОЙ АЛГЕБРЫ ЛИ

Исследованы структура и свойства инвариантной подалгебры Гельфанда-Цетлина универсальной обертывальной алгебры для ортогональной комплексной алгебры Ли. Эта под алгебра рассматривается как под алгебра полиномов от групп переменных, зависящих от двух индексов, которая является инвариантной относительно действия группы Вейля. Алгебра Гельфанда-Цетлина реализуется как некоторое конечное расширение алгебры симметричных полиномов от групп переменных.

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ANNIHILATOR SUBALGEBRAS IN LIE ALGEBRAS OF DERIVATIONS

Let K be an algebraically closed field of characteristic zero and K(x_1 , …, x_n) be the field of rational functions. The set Ann(S) of all K-derivations of K(x_1 , …, x_n) which annihilate a set S $\subseteq$ K(x_1 , …, x_n) is a subalgebra of the Lie algebra W_n(K). The structure of the subalgebra Ann(S) is connected with centralizers of elements of W_n(K). A characterization of the subalgebra Ann(S) is given, some sets of generators of Ann(S) are pointed out in the Lie algebra of all K-derivations of the polynomial ring K[x_1 , …, x_n].

INTRODUCTION. Let K be an algebraically closed field of characteristic zero and $R = K(x_1, ..., x_n)$ be the field of rational functions in *n* variables. The set $Der_K R$ of all K-derivations of R, i.e K-linear operators D on the field R satisfying the Leibniz's rule: $D(fg) = D(f)g + fD(g)$ for all $f, g \in R$ is a Lie algebra over K and a vector space over R in a natural way: if we take $f \in R, D \in Der_K R$, the derivation fD sends any element $g \in R$ to $fD(g)$. The structure of the Lie algebra $Der_K R$ is of great interest because one can consider derivations as vector fields on geometric objects. If we take any element $D \in Der_K R$ then D can be written in the form:

$$D = f_1 \frac{\partial}{\partial x_1} + ... + f_n \frac{\partial}{\partial x_n}, f_i \in R,$$

and D is a vector field with rational coefficients. Such Lie algebras were studied by many authors (see, for example [1], [2], [3]). If we take any set $S \subseteq R = K(x_1,...,x_n)$, then one can pose a question about the structure of the annihilator $Ann_{\widetilde{W}_n(K)}(S) = \{D \in \widetilde{W}_n(K) \mid D(s) = 0 \textit{ for all } s \in S\}$ (here $\widetilde{W}_n(K) = Der_K R$). This subset $Ann(S) \subseteq \widetilde{W}_n(K)$ is obviously a subalgebra of $\widetilde{W}_n(K)$ and a subspace of the R-space $\widetilde{W}_n(K)$. We point out explicitly a basis (over R) of $Ann(S)$ (see Theorem 1). Note that the annihilator $Ann(S)$ is contained in a larger subalgebra $N \subseteq \widetilde{W}_n(K)$, which consists of all derivations D of R such that $D(K(S)) \subseteq K(S)$ where $K(S)$ is the subfield of R generating by the set S and by the subfield K. We give also the basis of N over R. These results are applied to the Lie algebra $W_n(K)$ of all derivations of the polynomial ring $K[x_1,...,x_n]$. Here we consider systems of generators over $K[x_1,...,x_n]$ since $Ann_{W_n(K)}(S)$ is a submodule of the free $K[x_1,...,x_n]$-module $W_n(K)$ (Theorem 2).

We use standard notations, the ground field is denoted by K and is algebraically closed of characteristic zero. Recall that a polynomial *g* is a slice for a derivation $D \in W_n(K)$ if $D(g) = 1$.

ANNIHILATORS OF SETS OF RATIONAL FUNCTIONS. The next two lemmas contain some preliminary results used in the sequal.

Lemma 1. Let $D_1, D_2 \in \widetilde{W}_n(K)$ and $a, b \in R$. Then

1) $[aD_1, bD_2] = ab[D_1, D_2] + aD_1(b)D_2 - bD_2(a)D_1$;

2) If in addition $a, b \in Ker D_1 \cap Ker D_2$ then $[aD_1, bD_2] = ab[D_1, D_2]$.

Lemma 2. Let D be a K-derivation of the field $K(x_1,...,x_n)$ and F be an algebraic extension of $K(x_1,...,x_n)$. Then there exists a unique extension $\overline{D}$ of D on F, i.e. such a derivation of F that $\overline{D}|_{K(x_1,...,x_n)} = D$

Lemma 3. Let S be a subset of $K(x_1,...,x_n)$ and $K(S)$, the subfield generated by K and S in $K(x_1,...,x_n)$. If $\overline{K(S)}$ is the algebraic closure of $K(S)$ in $K(x_1,...,x_n)$ then:

1) $Ann_{\widetilde{W}_n(K)}(S) = Ann_{\widetilde{W}_n(K)}(K(S)) = Ann_{\widetilde{W}_n(K)}(\overline{K(S)})$;

2) $Ann_{\widetilde{W}_n(K)}(S)$ is a subalgebra of $\widetilde{W}_n(K)$ and a vector-space over R of dimension $m = n - tr.\deg_K K(S)$;

3) If in addition $m \geq 1$ then the Lie algebra $Ann_{\widetilde{W}_n(K)}(S)$ is simple.

Proof. 1) Every K-derivation $D \in \widetilde{W}_n(K)$ annihilating the set S annihilates obviously the subfield $K(S)$. Since the subfield $\overline{K(S)}$ is an algebraic extension of $K(S)$ then every derivation of R annihilating $K(S)$ annihilates also $\overline{K(S)}$ by the Lemma 2. Therefore $Ann_{\widetilde{W}_n(K)}(\overline{K(S)}) = Ann_{\widetilde{W}_n(K)}(K(S))$. The converse inclusion is obvious, so we have $Ann_{\widetilde{W}_n(K)}(K(S)) = Ann_{\widetilde{W}_n(K)}(\overline{K(S)})$.

2) Let $k = tr.\deg_K K(S)$ and $y_1,...,y_k$ be a transcendence basis for $K(S)$ over K (the elements $y_1,...,y_k$ can be chosen from the set S). Take a complement of the set $\{y_1,...,y_k\}$ to a transcendence basis of the field $K(x_1,...,x_n)$ consisiting of arbitrarily chosen elements $y_{k+1},...,y_s$ and consider the derivations $\frac{\partial}{\partial y_i}, i = 1,...,n$, of the subfield $K(y_1,...,y_n)$ of the field $R = K(x_1,...,x_n)$. Let us extend the derivations $\frac{\partial}{\partial y_i}, i = 1,...,n$, to the derivations of the field $K(x_1,...,x_n)$ using Lemma 2, and save the same notations.

Then $\frac{\partial}{\partial y_{k+1}},...,\frac{\partial}{\partial y_n}$ annihilate obviously the subfield $K(S)$ (because $\frac{\partial}{\partial y_i}(y_j) = 0, i = k+1,...,n, j = 1,...,k$). Therefore $R\frac{\partial}{\partial y_{k+1}} + ... + R\frac{\partial}{\partial y_n} \subseteq Ann_{\widetilde{W}_n(K)}(S)$. Let us show that the latest inclusion is equality. Take any $D \in Ann_{\widetilde{W}_n(K)}(S)$. Then D can be written in the form:

$$D = f_1 \frac{\partial}{\partial y_1} + ... + f_k \frac{\partial}{\partial y_k} + f_{k+1} \frac{\partial}{\partial y_{k+1}} + ... + f_n \frac{\partial}{\partial y_n}$$

for some $f_i \in K(x_1,...,x_n)$ (because the derivations $\frac{\partial}{\partial y_1},...,\frac{\partial}{\partial y_n}$ are linearly independent over $K(x_1,...,x_n)$).

Since $f_{k+1}\frac{\partial}{\partial y_{k+1}} + ... + f_n \frac{\partial}{\partial y_n} \in Ann_{\widetilde{W}_n(K)}(S)$, we see that $D_1 = f_1 \frac{\partial}{\partial y_1} + ... + f_k \frac{\partial}{\partial y_k} \in Ann_{\widetilde{W}_n(K)}(S)$.

The obvious equalities $D_1(y_i)=0, i=1,...,k$ imply $f_i=0$, i.e. $D_i=0$ (by the equality $\frac{\partial}{\partial y_i}(y_j)=\delta_{ij}$). Hence $Ann_{\widetilde{W}_n(K)}(S)=R\frac{\partial}{\partial y_1}+...+R\frac{\partial}{\partial y_n}$.

3) Let $m=n-k\geq 1$. We have by the above proven

$$Ann_{\widetilde{W}_n(K)}(S)=R\frac{\partial}{\partial y_1}+...+R\frac{\partial}{\partial y_n},$$

i.e. the subalgebra $Ann_{\widetilde{W}_n(K)}(S)$ is a vector space of dimension $m\geq 1$ over R. Let I be a nonzero proper ideal of $Ann_{\widetilde{W}_n(K)}(S)$. Then I contains a nonzero ideal J which is a vector space over R. The latter is impossible because of results of the paper [4].

Let S be a non-empty subset of $K(x_1,...,x_n)$ and $\{y_1,...,y_k\}$ be a maximal algebraically independent over K subsystem of S. We can assume without loss of generality that $S=\{y_1,...,y_k\}$ because $Ann_{\widetilde{W}_n(K)}(S)=Ann_{\widetilde{W}_n(K)}(\{y_1,...,y_k\})$ by the previous lemma. Complement the set S to a transcendental basis $\{x_1,...,x_{n-k},y_1,...,y_k\}$ of the field $K(x_1,...,x_n)$ over K (it is possible after renumbering of variables). Denote by J_{S,x_i} the jacobian derivation of the subfield $K(x_1,...,x_{n-k},y_1,...,y_k)$ defined by the rule: $J_{S,x_1}(h)=\det J(x_1,...,x_{i-1},h,x_{i+1},...,x_{n-k},y_1,...,y_k), i=1,...,n-k$, where the latter determinant is as following:

$$\begin{vmatrix} 1 & 0 & ... & ... & 0 \\ ... & ... & ... & ... & ... \\ \frac{\partial h}{\partial x_1} & \frac{\partial h}{\partial x_2} & ... & ... & \frac{\partial h}{\partial y_k} \\ ... & ... & ... & ... & ... \\ 0 & 0 & ... & 1 & 0 \\ 0 & 0 & ... & 0 & 1 \end{vmatrix} \qquad (1)$$

Extend these derivations on the whole field $K(x_1,...,x_n)$ (it is possible by Lemma 2) and save the same notations for them. Denote by $\Delta=\det J(x_1,...,x_{n-k},y_1,...,y_k)$.

Lemma 4. The derivations $J_{S,x_i}, i=1,...,k$, are linearly independent over $K(x_1,...,x_n)$ and each of them contains the subfield $\overline{K(S)}$ in its kernel and the derivation $J_{S,y_i}\bullet\frac{1}{\Delta}$ maps the subfield $\overline{K(S)}$ into itself.

Proof. Assume that $f_1J_{S,x_1}+...+f_{n-k}J_{S,x_{n-k}}=0$ for some $f_i\in R$. Since $J_{S,x_i}(x_i)\neq 0$ (because this is the jacobian of algebraically independent over K rational functions $x_1,...,x_{n-k},y_1,...,y_k$) and $J_{S,x_i}(x_j)=0, i\neq j$, we get $f_1=0,...,f_{n-k}=0$ and therefore $J_{S,x_1},...,J_{S,x_{n-k}}$ are linearly independent over R. Further $J_{S,x_i}(y_j)=0$ for $j=1,...,k$ and $i=1,...,n-k$ and therefore $K(S)\subseteq KerJ_{S,x_i}, i=1,...,n-k$. But then $\overline{K(S)}\subseteq KerJ_{S,x_i}$ by Lemma 2 .

The derivation $\frac{1}{\Delta}J_{S,y_i}$ agrees on $K(y_1,...,y_k)$ with the derivation $\frac{\partial}{\partial y_i}$. So $\frac{1}{\Delta}J_{S,y_i}$ maps $K(y_1,...,y_n)$ into itself. By Lemma 2 the derivation $\frac{1}{\Delta}J_{S,y_i}$ can be uniquely extended on the algebraic closure $\overline{K(S)}$ of the subfield $K(S)$ in R and hence $\frac{1}{\Delta}J_{S,y_i}$ maps $\overline{K(S)}$ into itself.

Remark. The derivations $\frac{1}{\Delta}J_{S,x_i}$ and $\frac{1}{\Delta}J_{S,y_j}$ can be easily written in the standard basis $\{\frac{\partial}{\partial x_1},...,\frac{\partial}{\partial x_n}\}$ of the field $K(x_1,...,x_n)$ in the form $J_{S,x_1}=f_1\frac{\partial}{\partial x_1}+...+f_n\frac{\partial}{\partial x_n}$, where $f_i\in K(x_1,...,x_n)$. The coefficients are minors of order $n-1$ of the determinant (1)

Theorem 1. Let K be a field, S be a subset of the field $K(x_1,...,x_n)$ of rational functions and $y_1,...,y_k$ be a maximal set of algebraically independent over K elements of S. Then the annihilator $Ann_{\widetilde{W}_n(K)}(S)$ is a vector space over $K(x_1,...,x_n)$ with the basis $J_{S,x_1},...,J_{S,x_{n-k}}$. The set N_1 of all elements of $\widetilde{W}_n(K)$ which map $K(S)$ into itself is a semidirect sum $N_1=M\times Ann_{W_n(K)}(S)$ where M is a subalgebra of $\widetilde{W}_n(K)$ with the basis $\frac{1}{\Delta}J_{S,y_1},...,\frac{1}{\Delta}J_{S,y_k}$.

Proof. As $\frac{1}{\Delta}J_{S,y_i}(y_i)=1, J_{S,y_i}(y_j)=0, i\neq j$, we see that $\frac{1}{\Delta}J_{S,y_1},...,\frac{1}{\Delta}J_{S,y_k}$ are linearly independent over R.

Every derivation J_{S,y_i} maps $\overline{K(S)}$ into itself by Lemma 4. Denote by *M* the subalgebra $M=\overline{K(S)}J_{S,y_1}+...+\overline{K(S)}J_{S,y_k}$ of the Lie algebra $\widetilde{W}_n(K)$. It can be easily shown that $Ann_{\widetilde{W}_n(K)}(S)$ is an ideal of the Lie algebra N_1. Take any element $D\in N_1$. Then

$D(y_i)=f_i$ for some $f_i \in \overline{K(S)}$ and $(D-\sum_{i=1}^{k} f_i J_{S,y_i})(y_i)=0$ for all $i=1,...,k$. But then $D-\sum_{i=1}^{k} f_i J_{S,y_i} \in Ann_{\widetilde{W}_n(K)}(S)$ and therefore $N_1 = M \times Ann_{W_n(K)}(S)$ the semidirect sum of the subalgebra M and the ideal $Ann_{W_n(K)}(S)$ (it is obvious that the intersection of these subalgebras of N_1 is zero).

Example. Consider more detailed the case $tr.\deg_K K(S)=1$. We can assume without loss of generality that $S=\{y_1\}$. The annihilator of the subset S consists of linear combinations over R of the derivations $J_{S,x_i}=\frac{\partial}{\partial x_i}, i=1,...,n-1$. The subalgebra M is of the form $M=\overline{K(y_1)}(\frac{\partial y_1}{\partial x_n})^{-1}\frac{\partial}{\partial y_1}$.

ANNIHILATORS OF RATIONAL FUNCTIONS IN THE LIE ALGEBRA $W_n(K)$**.** We will denote by $W_n(K)$ the Lie algebra of all K-derivations of the polynomial ring $K[x_1,...,x_n]$. It consists of the derivations of the form $D=f_1\frac{\partial}{\partial x_1}+....+f_n\frac{\partial}{\partial x_n}$, $f_i \in K[x_1,...,x_n]$. The Lie algebra $W_n(K)$ is contained in $\widetilde{W}_n(K)$ and $W_n(K)$ is a free $K[x_1,...,x_n]$-module. This Lie algebra acts naturally on $K(x_1,...,x_n)$ and we consider annihilators of elements of $K(x_1,...,x_n)$ in the subalgebra $W_n(K)$. If $\varphi \in K(x_1,...,x_n)$ then $Ann_{W_n(K)}(\varphi)$ is a submodule of the module $W_n(K)$ over the ring $K[x_1,...,x_n]$. We point out generators of $Ann_{W_n(K)}(\varphi)$ in some cases.

Lemma 5. Let $\varphi \in K(x_1,...,x_n)$ and $Ann_{W_n(K)}(\varphi)$ be the annihilator of φ in $W_n(K)$. Then:

1) $Ann_{W_n(K)}(\varphi)$ is a submodule of rank $n-1$ of $W_n(K)$;

2) If in addition $n=2$, then the $Ann_{W_n(K)}(\varphi)$ is a free submodule of rank 1 of $W_2(K)$**.**

Proof. 1) See Lemma 4.
2) See [3].

Lemma 6. Let $D=\sum_{i=1}^{n} p_i \frac{\partial}{\partial x_i} \in W_n(K)$ with $p_i \in K[x_1,...,x_n]$ and $D(\varphi)=0$ for some $\varphi \in K(x_1,...,x_n)$. Then the derivations $D_j^{(i)}=-\frac{\partial\varphi}{\partial x_i}\frac{\partial}{\partial x_j}+\frac{\partial\varphi}{\partial x_j}\frac{\partial}{\partial x_i}, i,j=1,...,n, i\neq j$, satisfy the equality $\sum_{j=1,j\neq i}^{n} p_j D_j^{(i)}=(-\frac{\partial\varphi}{\partial x_i})D$.

Proof. It is obvious that $D_j^{(i)}(\varphi)=0$ for $i,j=1,...,n$. Further we have

$$\sum_{j=1,j\neq i}^{n} p_j D_j^{(i)} = \sum_{j=1,j\neq i}^{n} p_j\left(-\frac{\partial\varphi}{\partial x_i}\frac{\partial}{\partial x_j}+\frac{\partial\varphi}{\partial x_j}\frac{\partial}{\partial x_i}\right)=\left(-\frac{\partial\varphi}{\partial x_i}\right)\sum_{j=1,j\neq i}^{n} p_j\frac{\partial}{\partial x_j}+\left(\sum_{j=1,j\neq i}^{n} p_j\frac{\partial\varphi}{\partial x_j}\right)\frac{\partial}{\partial x_i}$$

The latter sum can be written in the form

$$\left(-\frac{\partial\varphi}{\partial x_i}\right)\sum_{j=1}^{n} p_j\frac{\partial}{\partial x_j}+p_i\frac{\partial\varphi}{\partial x_i}\frac{\partial}{\partial x_i}+\left(\sum_{j=1}^{n} p_j\frac{\partial\varphi}{\partial x_j}\right)\frac{\partial}{\partial x_i}-p_i\frac{\partial\varphi}{\partial x_i}\frac{\partial}{\partial x_i}=\left(-\frac{\partial\varphi}{\partial x_i}\right)D,$$

since $\sum_{j=1}^{n} p_j\frac{\partial\varphi}{\partial x_j}=D(\varphi)=0$ $\cdot$

Theorem 2. Let $\varphi=\frac{u}{v}\in K(x_1,...,x_n)$ be such a rational function with coprime polynomials u,v that there exist polynomials $f_1,...,f_n$ with $f_1\frac{\partial\varphi}{\partial x_1}+....+f_n\frac{\partial\varphi}{\partial x_n}=\frac{1}{v^2}$. Then $Ann_{W_n(K)}(\varphi)$ is a submodule of rank $n-1$ over $K[x_1,...,x_n]$ with generators $\overline{D}_j=\sum_{i=1,i\neq j}^{n} f_i D_j^{(i)}, j=1,...,n$, where $D_j^{(i)}=\left(-\frac{\partial\varphi}{\partial x_i}\frac{\partial}{\partial x_j}+\frac{\partial\varphi}{\partial x_j}\frac{\partial}{\partial x_i}\right)v^2$

Proof. Take any derivation $D\in Ann_{W_n(K)}(\varphi)$, let $D=p_1\frac{\partial}{\partial x_1}+....+p_n\frac{\partial}{\partial x_n}, p_i\in K[x_1,...,x_n]$. Consider the derivation $D_j^{(i)}=-\frac{\partial\varphi}{\partial x_i}\frac{\partial}{\partial x_j}+\frac{\partial\varphi}{\partial x_j}\frac{\partial}{\partial x_i}$. Since $D(\varphi)=0$ then by Lemma 6 it holds $\sum_{j=1,j\neq i}^{n} p_j D_j^{(i)}=(-\frac{\partial\varphi}{\partial x_i})D, i=1,...,n$. Multiply both sides of these equalities by f_i and add them. Then we obtain in the right hand side $\sum_{i=1}^{n} f_i(-\frac{\partial\varphi}{\partial x_i})D=-\frac{1}{v^2}D$. The left hand side is of the form $\sum_{i=1}^{n} f_i\left(\sum_{j=1,j\neq i}^{n} p_j D_j^{(i)}\right)=\sum_{j=1}^{n} p_j\left(\sum_{i=1,i\neq j}^{n} f_i D_j^{(i)}\right)$. Denote $\widetilde{D}_j=\sum_{i=1,i\neq j}^{n} f_i D_j^{(i)} v^2$. Then we get $-D=\sum_{j=1}^{n} p_j\widetilde{D}_j$. It follows from this equality that $Ann_{W_n(K)}(\varphi)$ has generators $\widetilde{D}_1,...,\widetilde{D}_n$. It can be easily shown that $\widetilde{D}_1,...,\widetilde{D}_n\in W_n(K)$.

Corollary 1. Let f be a polynomial from the polynomial ring $K[x_1,...,x_n]$. If the ideal of $K[x_1,...,x_n]$ generated by the partial derivatives $\frac{\partial f}{\partial x_1},...,\frac{\partial f}{\partial x_n}$ coincides with $K[x_1,...,x_n]$, then the annihilator $Ann_{W_n(K)}(f)$, can be generated by *n* generators.

Corollary 2. If a polynomial $f \in K[x_1,...,x_n]$ is a slice for a derivation $D \in W_n(K)$, then $Ann_{W_n(K)}(f)$ can be generated by *n* elements.

Corollary 3. Let $f \in K[x_1,...,x_n]$ be such a polynomial that at least one of the partial derivatives $\frac{\partial f}{\partial x_i}, i=1,...,n$ is a nonzero constant. Then $Ann_{W_n(K)}(f)$ is a free submodule of $W_n(K)$ of rank $n-1$.

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АНУЛЯТОРНІ ПІДАЛГЕБРИ В АЛГЕБРАХ ЛІ ДИФЕРЕНЦІЮВАНЬ

Нехай K алгебраїчно замкнене поле характеристики нуль і K(x₁ , …, xₙ) - поле раціональних функцій від n змінних. Множина Ann(S) всіх K-диференціювань K(x₁ , …, xₙ), яка анулює підмножину S ⊆ K(x₁ , …, xₙ) є підалгеброю алгебри Лі Wₙ(K). Структура підалгебри Ann(S) пов'язана з централізаторами елементів в Wₙ(K). Дано характеризацію підалгебри Ann(S), вказано систему твірних для підалгебри Ann(S) в алгебрі Лі всіх K-диференціювань кільця многочленів K[x₁,…, xₙ].

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АННУЛЯТОРНЫЕ ПОДАЛГЕБРЫ В АЛГЕБРАХ ЛИ ДИФФЕРЕНЦИРОВАНИЙ

Пусть K алгебраически замкнутое поле характеристики нуль і K(x₁ , …, xₙ) поле рациональных функций от n переменнных. Множество Ann(S) всех K-дифференцирований K(x₁ , …, xₙ), аннулирующих подмножество S ⊆ K(x₁ , …, xₙ) является подалгеброй алгебры Ли Wₙ(K). Структура подалгебры Ann(S) связана с централизаторами элементов в Wₙ(K). Дано характеризацию подалгебры Ann(S), указано систему образующих для подалгебры Ann(S) в алгебре Ли всех K-диференцирований кольца многочленов K[x₁ , …, xₙ].

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НАБЛИЖЕНЕ ОПТИМАЛЬНЕ КЕРУВАННЯ В ФОРМІ ОБЕРНЕНОГО ЗВ'ЯЗКУ ДЛЯ ПАРАБОЛІЧНОЇ СИСТЕМИ ЗІ ШВИДКО КОЛИВНИМИ КОЕФІЦІЄНТАМИ

Розглядається задача знаходження оптимального керування в формі оберненого зв'язку (синтезу) в лінійно-квадратичної задачі, яка складається з параболічної системи зі швидко коливними коефіцієнтами та напіввизначеного цільового функціоналу. Знайдено точну формулу синтезу та за допомогою переходу до усереднених параметрів обґрунтовано його наближену форму.

ВСТУП. Важливою задачею в теорії оптимального керування нескінченновимірними еволюційними системами є побудова оптимального керування в формі оберненого зв'язку (синтезу) [1], [4]. Для широкого класу задач як з розподіленим [1], так і з зосередженим керуванням [5] вдається знайти замкнену форму оптимального синтезу, параметри якої виражаються через власні функції та числа відповідного диференціального оператора. При цьому якщо коефіцієнти оператора є швидко осцилюючими [2], то виникає задача обґрунтування наближеного усередненого синтезу [5]. У даній статті така задача розв'язана для параболічної лінійної системи з зосередженим керуванням в правій частині та квадратичним критерієм якості. На основі знайденої формули точного синтезу обґрунтовано форму наближеного усередненого оптимального керування в формі оберненого зв'язку.

ПОСТАНОВКА ЗАДАЧІ. Нехай $\Omega \subset R^p, p \ge 1$, – обмежена область, $\varepsilon \in (0,1)$ – малий параметр, $T > 0$. В циліндрі $Q_T = (0,T)\times\Omega$ для вектор-функцій $y = y(t,x)$, $u = u(t)$ розглядається задача

$$\begin{cases} \frac{\partial y}{\partial t} = A\Delta^{\varepsilon} y + By + C^{\varepsilon}(x)u, \\ y(t,x) = 0,\ x \in \partial\Omega, \\ y(0,x) = y_0^{\varepsilon}(x), \\ u \in U = \left(L^2[0,T]\right)^n, \end{cases} \tag{1}$$

$$J(y,u) = \sum_{i=1}^{n} \alpha_i \left(\int_{\Omega} y_i(T,x) q(x)\, dx \right)^2 + \sum_{i=1}^{n} \gamma_i \int_0^T u_i^2(t) dt \to \inf. \tag{2}$$