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FLEXURAL VIBRATIONS AND DISSIPATIVE HEATING OF A NANOCOMPOSITE BEAM UNDER STATIC AND CYCLIC LOADING

Forced resonance vibrations and dissipative heating of viscoelastic beam made of polymeric nanocomposite reinforced by unidirectionally aligned nanofibers made of straight single-walled carbon nanotubes (CNTs) are investigated. Geometrical nonlinearity of the second order as well as temperature dependence of the complex moduli of nanocomposite materials are taken into account. To solve the coupled nonlinear problem of thermovisoelasticity under cyclic loading, the quasi-linearization technique (for complex moduli determination) is used in combination with the discrete-orthogonalization method and iterative procedure. Orthogonal discretization is used at each iteration to integrate the complex-value analogues of beam motion equations. The explicit finite-difference scheme is used to solve the heat-conduction equation with a heating source caused by dissipation. The influence of the dissipative heating and nonlinearity of physical properties as well as geometrical nonlinearity on the dynamic characteristics, heating temperature and damping of the forced vibrations for the nanocomposite beam with different volume fraction of CNTs fibers under combined uniform transverse harmonic and static pressure are investigated.

Introduction. The polymeric composite elements are widely used in current engineering applications, but these elements are sensitive to various kinds of loading. Thus, the response of such structure elements to actual force and kinematic loading must be evaluated [1]. The forced vibration analysis of structural elements occupies an important place in the dynamics of deformable systems. Especially of nonlinear systems, the applied researches in this field show the need for a broader theoretical analysis in this field of engineering for new materials [1, 2].

There are several factors that influence the behavior of a structure under high level of dynamic loading (resonance vibrations). Some of them are inertia effects, non-linearity of material properties and the coupling of the mechanical and thermal fields [3]. In this situation, apart from purely mechanical fatigue failure, structural elements may undergo thermal failure, i.e., softening or even melting due to vibrational or self-heating, which occurs because of high hysteresis losses and low heat conductivity of polymer materials [4]. Indeed, the self-heating may alter the strength features of the structural element, and degrade its performance. The interaction of the mechanical and thermal fields in viscoelastic bodies is studied within the framework of a coupled thermomechanics [1–4]. Therefore, it is necessary to evaluate thermomechanical behavior of the material with taking into account the effects of physical and geometric nonlinearities. In recent years, the evaluation of both geometric and physical nonlinearity as well as their mutual influence on behavior of thin-walled structural elements has attracted increasing research efforts [5, 6]. It must be mentioned that, in general, to derive the governing equations of motion of thin-walled structural elements, the von Karman type of geometrically nonlinear strain–displacement relationships is the most widely used. It is well known that, the solution of the coupled problem of thermoviscoelasticity is more complicated especially in the case of long-term inelastic deformation, because of the necessity of storing a large body of data and performing extensive computations to take into account the deformation history. To overcome these difficulties in the specific case of harmonic loading, a simplified model of thermomechanically coupled processes was developed in [1, 3]. This model is based on the concept of complex moduli, and specified by a modified technique of equivalent linearization [3].

The main aim of this investigation is to use the simplified model of the behavior of viscoelastic nanocomposite beam under combined cyclic and static loading to give an approximate formulation to the coupled thermomechanical problem. In the framework of approximate formulation [1, 7], the laws governing the forced vibrations and self-heating for nanocomposite beam with unidirectionally aligned CNTs fibers under considered conditions are derived. Also, the influence of geometric and physical nonlinearity at the different volume fraction of CNTs fibers on the dynamic characteristics of the system is investigated over a wide range of amplitude, frequency and temperature. As mentioned earlier, the approximate formulation is based on concept of complex moduli, therefore, the overall macroscopic properties of nanocomposite material with unidirectionally aligned CNTs fibers under harmonic loading are obtained as complex moduli by using approximate approach for its constituents (CNTs fibers, polymeric matrix and interface) and the homogenization procedure based on the modified Mori – Tanaka method [8]. The prediction procedure of nanocomposite material properties under different conditions of frequency, amplitude and temperature are presented in our previous works [11–13].

Problem formulation. Let consider a one-layer rectangular cross section beam made of the nanocomposite with unidirectionally aligned CNTs fibers. The beam is referred to a Cartesian coordinate system $Oxyz$, so that, $0 < x < l$, $|y| = b/2$ and $|z| = h/2$ which l, b and h are length, width and thickness of beam, respectively. The axis of the beam coincides with the axis Ox. The beam is subjected to bending by the transverse pressure $q^z(x,t) = q^0(x) + q'(x)\cos(\omega t)$ in the plane xOz, which consists of the constant components $q^0(x)$ and harmonic in time, t, excitation with amplitude $q'(x)$ and frequency ω, close to the resonance frequency. We assume that the strains are small, but the beam deflections are such that, it is necessary to take into account the squares of rotation angles in kinematic relations. Therefore, the equations of motion are also nonlinear. We also consider $u=u(x)$, $w=w(x)$ and $v=v(x)$ are displacements of point of middle surface along axes Ox, Oz and Oy, respectively. The beam has a uniform heat distribution $T=T(x)$ over the cross section of beam, and on the lateral surface heat exchange occurs with an environment having the temperature T_C. Also, the material of beam is considered viscoelastic nanocomposite so that, its inelastic response under monoharmonic loading is assumed as function of the intensity of the stress, frequency and temperature [11].

Under considered assumptions above, the displacement field of the Euler – Bernoulli beam theory (EBT) is given by $\bar{x} = u\hat{e}_x + v\hat{e}_y + w\hat{e}_z$ where $u = u(x) - zw_{,x}(x)$, $v = 0$ and $w = w(x)$. Taking into account the von Karman nonlinearity, the strain component, ε_{xx}, will be as follow

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 = \frac{\partial}{\partial x}\left(u(x) - zw_{,x}(x)\right) + \frac{1}{2}\left(\frac{\partial}{\partial x}w(x)\right)^2 = u_{,x}(x) - zw_{,xx}(x) + \frac{1}{2}w_{,x}(x)^2 . \quad (1)$$

In the framework *EBT*, considering the effect of temperature and the van Karman nonlinearity the strain component, the stress component, σ_{xx}, is

$$\sigma_{xx} = E^*\varepsilon_{xx} = E^*\left(\varepsilon_x + zk_x - \alpha^C(T - T_C)\right), \quad k_x = \frac{\partial V_x}{\partial x} = -\frac{\partial^2 w}{\partial x^2}, \quad V_x = -\frac{\partial w}{\partial x}, \quad \varepsilon_x = u_{,x} + \frac{1}{2}V_x^2, \tag{2}$$

where E^*, k_x, V_x, ε_x and α^C are linear viscoelastic Young's operator, curvature, rotation angle, strain of point of middle surface along axis of the beam and coefficient of linear thermal expansion of nanocomposite material, respectively. By substituting Eq. (2), the integral quantities of force and momentum over the thickness of beam are presented as

$$N_x = bN_{xx} = b\int_{-h/2}^{h/2}\sigma_{xx}dz = E^*A\varepsilon_x - E^*A\alpha^C(T - T_C), \; M_x = bM_{xx} = b\int_{-h/2}^{h/2}\sigma_{xx}zdz = \frac{E^*bh^3}{12}k_x. \tag{3}$$

Using Eq (3) the equations of curvature, rotation angle and strain of point of middle surface along axis of the beam in Eq. (1) can rewrite as follow

$$\frac{\partial u}{\partial x} = CN_x - \frac{1}{2}V_x^2 + \alpha^C(T - T_C), \quad k_x = \frac{\partial V_x}{\partial x} = -\frac{\partial^2 w}{\partial x^2} = \frac{12}{bh^3}J^*M_x = DM_x \text{ and } V_x = -\frac{\partial w}{\partial x}, \tag{4}$$

where $J^* = 1/E^*$ is the reverse operator; $C = J^*/A$ and $D = 12/bh^3\, J^*$.

Under considerations above, the equations of nonlinear vibrations of the flexible beam in the Ox and Oz axes can be written as follows [18]

$$\frac{\partial^2\hat{Q}_x}{\partial x^2} + Aq^z = \hat{I}_0\frac{\partial^2 w}{\partial t^2}, \quad \frac{\partial M_x}{\partial x} = \hat{Q}_x + N_xV_x \text{ and } \frac{\partial N_x}{\partial x} = \hat{I}_0\frac{\partial^2 u}{\partial t^2} \quad \text{where} \quad \hat{I}_0 = b\int_{-h/2}^{h/2}\rho dz, \tag{5}$$

where $\hat{Q}_x = Q_x - N_xV_x$. Also, ρ and Q_x are the density and crosscutting force, respectively. In present investigation, we consider the longitudinal vibration of beam is as quasi-static. Thus, the last equation of nonlinear vibration of beam in the Ox axis (Eqs. 5) can be written as $\partial N_x/\partial x = 0.$

With assuming that, the beam ends in the longitudinal direction are fastened rigidly and hingedly in the transverse direction, the mechanical boundary conditions have the form

$$u = 0, \quad w = 0, \quad M_x = 0 \quad \text{for} \quad x = 0, l. \tag{6}$$

The energy balance equation for the beam, averaged over its cross section area and the period of vibrations, can be expressed as

$$C_v^C\frac{\partial T}{\partial t} = \lambda^C\frac{\partial^2 T}{\partial x^2} - \frac{\chi P}{A}(T - T_C) + \langle\hat{D}'\rangle, \text{ where } \langle\hat{D}'\rangle = \int_{-h/2}^{h/2}\langle D'\rangle dz = \int_{-h/2}^{h/2}\langle s_{ij}.\dot{\varepsilon}_{ij}^{in}\rangle dz, \tag{7}$$

where C_v^C, λ^C and χ are the volumetric heat capacity, thermal conductivity of composite material and heat transfer coefficient, respectively. $P = 2(b+h)$ is perimeter of the beam cross section and $\langle\hat{D}'\rangle$ shows the volumetric rate of dissipation averaged over the cross section that can be expressed as function of components of complex strain and stress. In current study, we consider the initial and boundary thermal conditions as follow

$$T = T_0 \text{ at } t = 0 \text{ and } T = T_0(x) \quad \text{for} \quad x = 0, l. \tag{8}$$

It worth to mentioned that, in this study, the volumetric heat capacity, C_v^C, and the thermal expansion coefficients in the longitudinal direction, α^C, of polymeric composites are predicated based on the rule of mixture and also the thermal conductivity, λ^C, can be expressed by Halpin – Tsai model [14–17] as follow:

$$C_v^C = C_v^fV^f + C_v^MV^M, \quad \alpha^C = V^f\alpha^f + V^M\alpha^M, \quad \lambda^C = \lambda^M\left[\frac{1+\zeta\eta V^f}{1-\eta V^f}\right] \text{ and } \eta = \left(\left(\frac{\lambda^f}{\lambda^M}\right) - 1\right)\Big/\left(\left(\frac{\lambda^f}{\lambda^M}\right) + \zeta\right), \tag{9}$$

where $\zeta = 1$.

Construction of the solution of the problem. As mentioned earlier, the solution of the coupled problem of thermoviscoelasticity is more complicated especially in the case of long-term inelastic deformation. To overcome these difficulties in the specific case of harmonic loading, in this section, an Approximate formulation is expanded based on [9, 10]. Let us develop the approximate solution of nonlinear Eqs. (4)–(8) as harmonic series in terms of time. In this solution, the variables w, V_x, $\hat{Q}_x$ and M_x which characterize the beam deflection are assumed as a single-mode approximation while the variables u, N_x, and ε_x which describe the plane deformation of the beam are considered in the second–mode harmonic approximation, so that, we pursue an approximate solution of the problem in the form

$$\begin{aligned} &u = \bar{u} + u_1'\cos\omega t - u_1''\sin\omega t + u_1'\cos 2\omega t - u_2''\sin 2\omega t, \quad N_x = \bar{N} + N_1'\cos\omega t - N_1''\sin\omega t + N_2'\cos 2\omega t - N_2''\sin 2\omega t, \\ &\varepsilon_x = \bar{\varepsilon} + \varepsilon_1'\cos\omega t - \varepsilon_1''\sin\omega t + \varepsilon_2'\cos 2\omega t - \varepsilon_2''\sin 2\omega t, \quad w = \bar{w} + w'\cos\omega t - w''\sin\omega t, \quad V_x = \bar{V}_x + V_x\cos\omega t - V_x\sin\omega t, \\ &\hat{Q}_x = \bar{\hat{Q}}_x + \hat{Q}_x'\cos\omega t - \hat{Q}_x''\sin\omega t, \quad M_x = \bar{M}_x + M_x'\cos\omega t - M_x''\sin\omega t. \end{aligned} \tag{10}$$

In the frame of approximate solution, the reverse operator J^* can be expressed in the complex quantity as form $J^*(t) = J^\infty + Re\left[\tilde{J}e^{ik\omega t}\right]$ for $k = 1, 2$, so that the equilibrium creep compliance under monotonic loading, J^∞, depends on temperature, while, the components of complex creep compliance under harmonic loading, $\tilde{J} = J' - iJ''$, depend on amplitude of stress, frequency and temperature as follow

$$\bar{J} = J^\infty(T), \quad J_1^{','''} = J_1^{','''}(\sigma, \omega, T) \quad \text{and} \quad J_2^{','''} = J_2^{','''}(\sigma, 2\omega, T). \tag{11}$$

By substituting variables of Eq (10) in Eqs (4)–(8), the following system of nonlinear equations can obtained, so that,

$$\begin{gathered}
\frac{d\bar{u}}{dx}=\bar{C}\bar{N}-\frac{1}{2}\bar{V}_x^{\,2}-\frac{1}{4}\left(V_x'^{\,2}+V_x''^{\,2}\right)+\alpha\left(T-T_C\right),\quad \frac{du_1'}{dx}=C_1'N_1'-C_1''N_1''-\bar{V}_xV_x',\quad \frac{du_1''}{dx}=-C_1''N_1'+C_1'N_1''-\bar{V}_xV_x'',\\
\frac{du_2'}{dx}=C_2'N_2'+C_2''N_2''-\frac{1}{4}\left(V_x'^{\,2}-V_x''^{\,2}\right),\quad \frac{du_2''}{dx}=-C_2''N_2'+C_2'N_2''-\frac{1}{2}V_x'V_x'',\quad \frac{d\bar{w}}{dx}=-\bar{V}_x,\quad \frac{dw'}{dx}=-V_x',\quad \frac{dw''}{dx}=-V_x'',\\
\frac{d\bar{V}_x}{dx}=\bar{D}_1\bar{M}_x,\quad \frac{dV_x'}{dx}=D_1'M_x'+D_1''M_x'',\quad \frac{dV_x''}{dx}=D_1'M_x''-D_1''M_x',\quad \frac{d\bar{N}}{dx}=0,\quad \frac{dN_1'}{dx}=0,;\frac{dN_1''}{dx}=0,\quad \frac{dN_2'}{dx}=0,\\
\frac{dN_2''}{dx}=0,\quad \frac{d\bar{\hat{Q}}_x}{dx}=-q^0A,\quad \frac{d\hat{Q}_x'}{dx}=-\rho A\omega^2w'-q'A,\quad \frac{d\hat{Q}_x''}{dx}=-\rho A\omega^2w'',\quad \frac{d\bar{M}_x}{dx}=\bar{\hat{Q}}_x+\frac{1}{2}\left(N_1'V_x'+N_1''V_x''\right)+\bar{N}\bar{V}_x,\\
\frac{dM_x'}{dx}=\hat{Q}_x'+N_1'\bar{V}_x+\bar{N}V_x'+\frac{1}{2}N_2'V_x'+\frac{1}{2}N_2''V_x'',\quad \frac{dM_x''}{dx}=\hat{Q}_x''+N_1''\bar{V}_x+\bar{N}V_x''+\frac{1}{2}N_2''V_x'+\frac{1}{2}N_2'V_x'',
\end{gathered}\tag{12}$$

where based on the Eq. (4), quantities $\bar{C}$, $\bar{D}$, $\tilde{C}_k$ and $\tilde{D}_k$ are defined in (13) in which, subscripts $k=1, 2$ show the frequency ω and 2ω, respectively.

$$\bar{C}=\frac{J^\infty}{A},\quad \bar{D}=\frac{12}{bh^3}J^\infty,\quad \tilde{C}_k=C_k'-iC_k''=\frac{1}{A}\left(J_k'-iJ_k''\right),\ \text{and}\ \tilde{D}_k=D_k'-iD_k''=\frac{12}{bh^3}\left(J_k'-iJ_k''\right).\tag{13}$$

According to the Eq. (6), the boundary conditions are assumed as follows:

$$\bar{u}=u_1'=u_1''=u_2'=u_2''=0,\quad \bar{w}=w_1'=w_1''=0\ \ \text{and}\ \ \bar{M}=M_x'=M_x'=0\ \ \text{for}\ \ x=0,l.\tag{14}$$

In the framework approximate formulation, the dissipative function for considered beam is determined by substituting complex variables into (2) and then (7) as

$$\left\langle\hat{D}'\right\rangle=\frac{\omega b}{2\lambda A}\int_{-h/2}^{h/2}\left(\sigma_{xx}''\varepsilon_{xx}'-\sigma_{xx}'\varepsilon_{xx}''\right)dz=\frac{\omega}{2\lambda A}\left[N_1''\varepsilon_{x,1}'-N_1'\varepsilon_{x,1}''+2\left(N_2''\varepsilon_{x,2}'-N_2'\varepsilon_{x,2}''\right)+M_x''k_x'-M_x'k_x''\right],\ \text{where}\ \varepsilon_{x,1}'=C_1'N_1'+C_1''N_1'',$$
$$\varepsilon_{x,1}''=C_1'N_1''+C_1''N_1',\ \varepsilon_{x,2}'=C_2'N_2'+C_2''N_2'',\ \varepsilon_{x,1}'=C_2'N_2''+C_2''N_2',\ k_x'=D_1'M_x'+D_1''M_x''\ \text{and}\ k_x''=-D_1''M_x'+D_1'M_x''.\tag{15}$$

As a result, we have a system of equations for the average functions represented in (7) and (12)–(15) that must be solved as integration system. Also, the nonlinear coupled thermomechanic problem represented by these equations are solved as two-point boundary value problem (BVP) for ordinary differential equations (ODEs) system. In this investigation, to solve the first-order differential equations system above, the solver based on finite difference method (FDM) is employed. Recognizing that, the linearized system of ordinary differential equations in each approximation variables are integrated by the method of discrete orthogonalization with using a typical program [19].

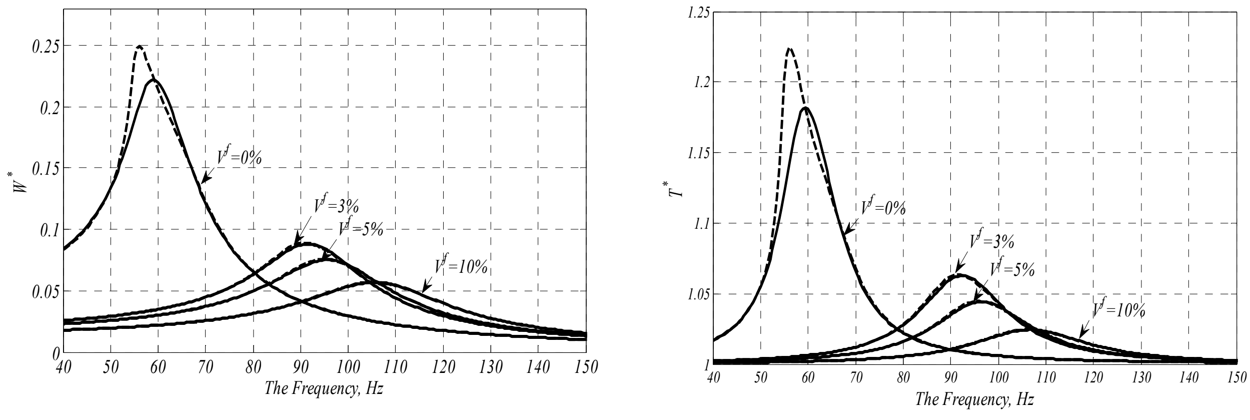
Numerical results and analysis.

Material properties. In order to predict the viscoelastic behavior of materials under monotonic and harmonic loading, an transversely isotropic nanocomposites system with unidirectionaly oriented CNTs fibers is considered. As mentioned earlier, In general, temperature, amplitude of loading, frequency and volume fraction of nanofiber are assumed as controlling parameters. The complete review of studies on the constitutive equations of micro- and macromechanical model of nonlinear viscoelastic behavior of polymeric nanocomposite materials under monoharmonic deformation is presented in previous works [11–13]. In this study,we have used the complex moduli computed in [11], in which the numerical solutions was carried out in a wide range of omplitude of harmonic loading for different volume fraction (3, 5 and 10 percent of CNTs fibers). Also, temperature and frequency are considered 25, 50, 80 °C and 1, 50, 100 Hz, respectively. According to microstructural geometry of CNTs, the nanofiber aspect ratio for the transversely isotropic nanocomposites is chosen to be equal to 3.5.

Amplitude, frequency and temperature characteristics (steady-state response). A numerical analysis has been conducted for the beam with single layer made of epoxy nanocomposite with unidirectionaly oriented CNTs fibers with the following physical parameters for epoxy resin and CNTs fibers [14–17]: $C_v^f=0.629\cdot10^6$ j/m³ K, $C_v^M=1.513\cdot10^6$ j/m³ K, $\lambda^f=2000$ *j/m* K, $\lambda^M=0.47$ j/m K, $\alpha^f=45\cdot10^{-6}$ 1/K, $\alpha^M=3\cdot10^{-6}$ 1/K, $\rho^f=1680$ kg/m³, $\rho^M=1214$ kg/m³ and $\chi=20$ W /m² K. Moreover, the geometry of the beam is assumed l=0.35 m, b=0.01 m and h=0.01.

In this section, the main aim is studing of amplitude and temperature frequency characteristics of the nanocomposite beam under consideration conditions with taking into accont the effects of nonlinear factores consist of geometrical and physically nonlinearity in the region of the first resonance. For this reason, we compare the solutions of the four problems, which are considered as follow: the first problem is a linear vescoelastic problem, in which the geometrical and physically nonlinearity aren't considered. Therefore, in this problem, the quadratic terms in (12) are ignored and also the properties of material are considered to be independent of temperature; the second problem is a nonlinear vescoelastic problem with taking into account the physically nonlinearity, in which thermomechanical coupling is considered; the third problem is a nonlinear viscoelastic problem with taking into account the geometrical nonlinearity, in which the properties of material are considered to be independent of temperature. Indeed, in this problem, the thermomechanical coupling is not considered; and finally, the fourth problem is a nonlinear viscoelastic problem with taking into account the geometrical nonlinearity and physically nonlinearity, so that it is called a completely nonlinear viscoelastic problem.

The effect of geometrical nonlinearity. To study the effects of geometrical nonlinearity on amplitude and temperature frequency characteristics in the vicinity of the first resonance, the results of solution of first and third problems are compared. By defination above, the material of beam in both of these problem is assumed viscoelastic and independent of temperature. The frequency dependencies of the amplitudes and stationary temperature in the region of the first resonance for different volume fraction (0, 3, 5 and 10 %) and constant amplitude of cyclic loading (70 kPa) are presentd In Figs. 1 (a) and (b). In This figure, the maximum value of the normalized deflection , $w^*=\max(w'^{\,2}+w''^{\,2})^{0.5}/h,\ 0\le x\le l$ and dimensionless temperature, $T^*=T_{\max}/T_0$, along the beam for problem *1* and problem *3* are compared. Here, solid and dashed lines correspond to the results of solution of the linear viscoelastic problem (problem *1*) and nonlinear viscoelastic problem with regard for geometric nonlinearity (problem *3*), respectively.



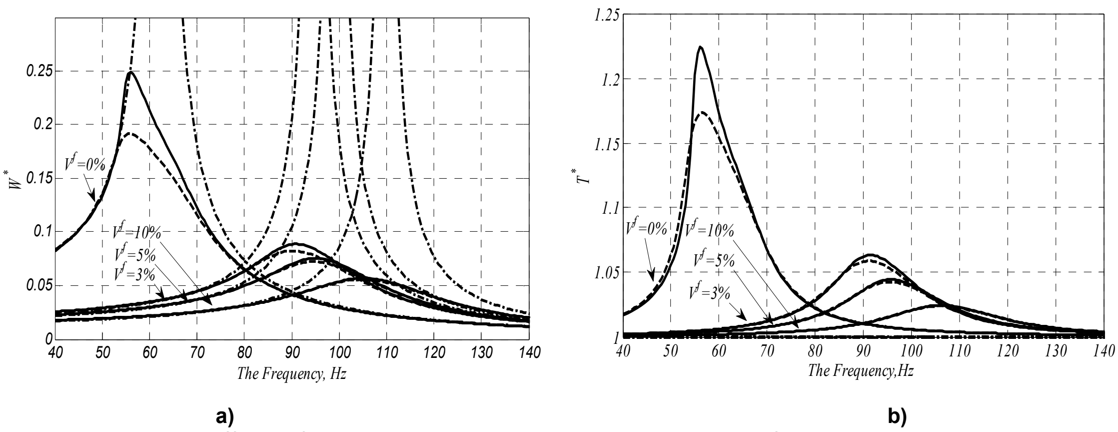
a) b)

Fig. 1. The effects of geometrical nonlinearity on (a) amplitude-frequency characteristics and (b) temperature-frequency characteristics (comparison of the problem *1* and *3*)

This figure shows the increase of volume fraction leads to shift of the region of the first resonance toward higher values of frequency, and also it is evident that, the influence of geometric nonlinearity on the amplitude and temperature-frequency characteristics becomes pronounced, with a decrease in the volume fraction of CNTs fiber. Also, the beam displays soft nonlinear behavior. Note that, the maximal deflection amplitudes and temperatures of the resonance frequencies for linear viscoelastic problem (solid lines) are somewhat lower than similar quantities of the resonance frequencies of the nonlinear viscoelastic problem (dashed lines). The effects of geometric nonlinearity are manifest to a greater extent with a reduction in the level of viscosity and an increase in the level of elasticity with increasing in volume fraction. Therefore, the low value of the maximum deflection amplitudes, w^* , can be limited to the linear formulation.

The effect of physically nonlinearity. In this investigation, to study the effects of thermomechanical coupling (*TMC*) and volume fraction on the amplitude of deflection and temperature frequency characteristics, third and fourth problems are considered. For this reason, the both of problems are solved at the different volume fraction of CNTs fibers and constant amplitude of cyclic loading. In Figs. 2 (a) and (b), the frequency dependencies of the maximum value of the dimensionless deflection, w^* , and the steady-state temperature, T^*, corresponding to the both of nonlinear viscoelastic problems(*3* and *4*) at the $q'=70$ kPa are presented. In this figure, the dot-dash lines show the results of solution of the completely linear elastic system (problem *1* with the physically linear elastic responses), also the solid and dashed lines correspond to the results of the nonlinear viscoelastic problem, in which TMC is not considered (problem *3*) and the completely nonlinear viscoelastic problem (problem *4*), respectively. A comparison of the curves of variation in the region of natural frequency for completely linear elastic problem (dot-dosh lines) and nonlinear viscoelastic problem (solid and dash lines) reveals that, the effects of viscoelastic response of materials on the temperature and deflection–frequency responses become more profound for all amounts of volume fraction. In Fig. 2 (b), the presented results show with taking into account viscoelastic responses the stationary dimensionless temperatures, T^*, increase in the region of first resonance, while the temperatures of dissipative heating in elastic problems are equal zero. However, Fig. 2 (a) shows the amplitudes of dimensionless deflection, w^*, strongly decrease with considering viscoelastic responses.

From Figs. 2 (a) and (b) it follows that, the deflection amplitude and dissipative-heating temperature are maximum in neat polymer beam (V^f=0 %) for both considered nonlinear viscoelastic problems (*3* and *4*), while they decrease with increasing volume fraction. Also, analysis of the curves in these figures show that, the effect of thermomechanicl coupling is more significant in lower volume fraction of CNTs fiber.. It is evident that, with taking into account physically nonlinearity (problem *4*) leads to a decrease in the main amplitude of deflection in the region of first resonance frequency of the beam and formation of the amplitude-frequency characteristics and temperature-frequency characteristics of the soft type. However, with increase in volume fraction, this effect of physically nonlinearity and dependence of nanocomposite on temperature decrease. This is due to reducing of inelastic response and imaginary part of complex moduli with increasing volume fraction under constant amplitude loading.



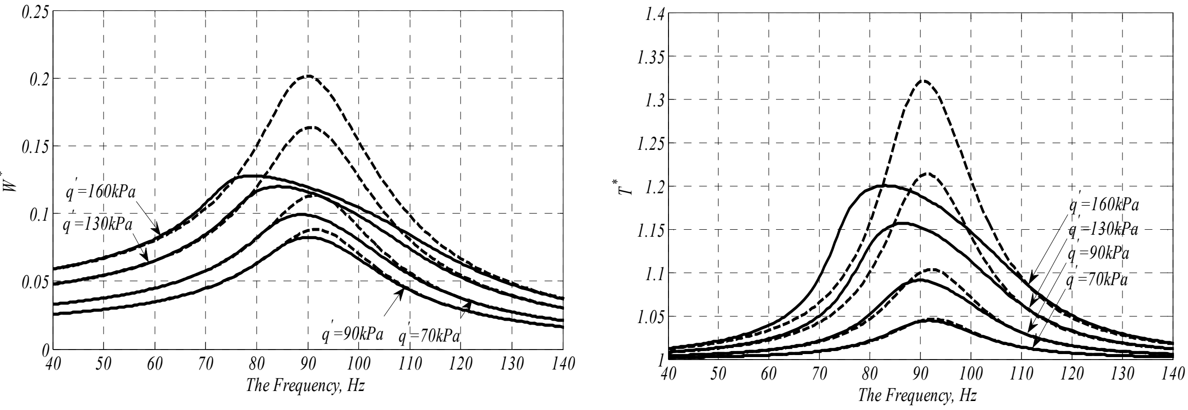
a) b)

Fig. 2. The effects of thermomechanical coupling on (a) amplitude-frequency characteristics and (b) temperature-frequency characteristics (comparison of the completely linear elastic problem, problem *3* and *4*)

The comparison the curves of Figs. 2 (a) and (b) for problems *3* and *4* and analysis of the interaction of two types of nonlinearity show the geometric nonlinearity becomes the more significant determining factor in the low volume fraction of CNTs fibers. It reveals that, with reduce inelastic behavior and increase strength of material, the interaction of two types of nonlinearity decreases. Therefore, the low value of the maximum deflection amplitudes corresponding to the high volume fraction may be limited to the linear formulation.

The effect of amplitude of loading. The frequency dependences of relative maximal amplitudes and temperatures of dissipative heating calculated in the vicinity of the main resonance frequency of vibrations for the different amplitudes of harmonic transverse pressure, q'=70, 90, 130 and 160 kPa, are presented in Figs. 3 (a) and (b) for unidirectionaly oriented nanocomposite beam with 3 % volume fraction of CNTs fiber. In this figure, the solid lines show the results of solution of the completely nonlinear viscoelastic problem (problem *4*), in which the properties are considered to be dependent of temperature, and dashed lines correspond to the results of solution of the linear viscoelastic problem (problem *1*), in which TMC is not considered. Analysis of the curves in these figures for complete nonlinear viscoelastic problem (solid lines) show the great influence of two nonlinearity factors on the frequency characteristics at the different amplitude of loading in the forced vibrations of the considered beam. It is necessary to mention that, the curves with dashed lines show the influence of viscoelastic response of material on frequency characteristics without considering the physically and geometrical nonlinearity effect in isothermal process. Also, the presented results in Figs. 3 (a) and (b) show the importance of studying the interaction of two types of nonlinearity to known the first resonance region and the type of nonlinearity behavior. According to the curves of amplitude and temperature frequency characteristics, the nonlinearity at resonance is of soft type under different amplitudes loading for nanocomposite beam with 3 % volume fraction of CNTs fibers. A comparison of the results presented in Figs. 3 (a) and (b) demonstrates that, with increasing amplitude of harmonic loading the effects of physically nonlinear viscoelastic responses will be pronounced and the level of viscosity and the amplitude of temperature of dissipative heating increase. In this situation, the role of thermomechanical coupling increases due to an increase in deflection of the beam. As mentioned earlier, the complex moduli or viscoelastic response of material depend on amplitude of stress or loading. As result, with increasing amplitude of loading, the leftward shift of the first resonance region is connected with thermal softening of the material and increasing viscoelastic response, so that, it indicates the predominant effect of TMC nonlinearity.

The effect of volume fraction on the stationary temperatures at critical loading. It is well known that, the certain cyclic-loading and heat-transfer conditions may lead to thermal fatigue failure due to material softening or even melting. This problem is important for many fields of engineering and technology which use nanocomposite materials.

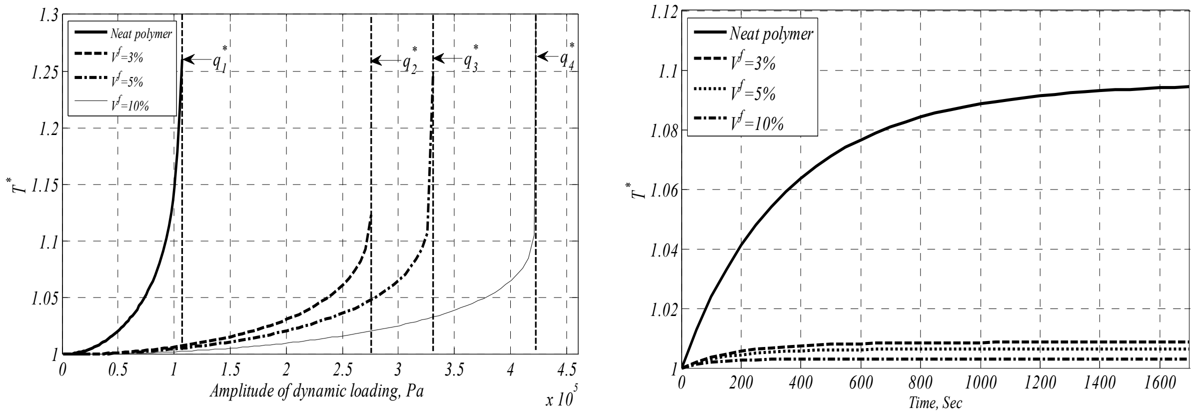


a) **b)**

Fig. 3. The effects of amplitude of cyclic loading on (a) amplitude-frequency characteristics and (b) temperature-frequency characteristics under different amplitudes loading, q'=70, 90, 130 and 160 kPa, for nanocomposite beam with 3 % volume fraction (comparison of problem *1* and *4*)

To study critical value of amplitude of harmonic loading, *q**, we consider the nanocomposite beam with small deflection under small static loading, q^o=10 Pa and different harmonic loading. Fig. 4 (a) shows curves depicting the change in the maximum values of the stationary dimensionless temperatures of dissipative heating in relation to considered conditions for completely nonlinear viscoelastic problem (problem *4*) at different volume fraction and constant frequency.

In this figure, q_1^*, q_2^*, q_3^* and q_4^* are the critical value of amplitude of harmonic loading which correspond to the neat polymer and nanocomposite beam with, 3, 5, and 10 % volume fraction of CNTs fibers at $f = 47.3$ Hz, which lie to the left of the first resonance frequency. In Fig. 4 (a), it is clearly evident that, with an increase in volume fraction at the constant frequency, the thermal instability occurs in high level of harmonic loading, $q_1^* < q_2^* < q_3^* < q_4^*$. It shows the physically nonlinear response of material plays a significant role, so that the level of viscosity decreases with an increase in volume fraction and also the thermal conductivity of the beam will be improved.



a) b)

Fig. 4. The variation of the stationary temperature in complete nonlinear problem at different volume fraction and f = 47.3 Hz (a), the evolutions of the temperature over time at f = 46 Hz (b)

Nonsteady-state behavior (thermal instability). To solve the non-steady problem, the derivatives with respect to time in (7) is replaced by a difference approximation as $T(t+\Delta t) - T(t) / \Delta t$. Accordingly, in this investigation, we used an implicit scheme to solve the system of (7) and (12)–(15). The nonlinear boundary-value problem which arises at each time step is solved by the method of quasi-linearization with numerical approach. This is realized by the numerical method with a small incremental time step in the first stage of the process would have required a very small step with respect to the coordinate x. In this section, we restrict ourselves to examine non-steady state behavior of nanocomposite beam in the framework of completely nonlinear viscoelastic problem (problem *4*).

Fig. 4 (b) shows curves of the evolutions of temperature over time for considered nanocomposite beam with different volume fraction of CNTs fiber under constant amplitude harmonic loading, q'=100 kPa, at f = 46 Hz which lie to the left side and vicinity of first resonance frequency for neat polymer beam. According to these results, it is observed that, the self-heating temperature evolution of each polymeric nanocomposite beam grows until reaching the steady state. It is worth to emphasize that in all curves demonstrate the saturation type behavior of the temperature.

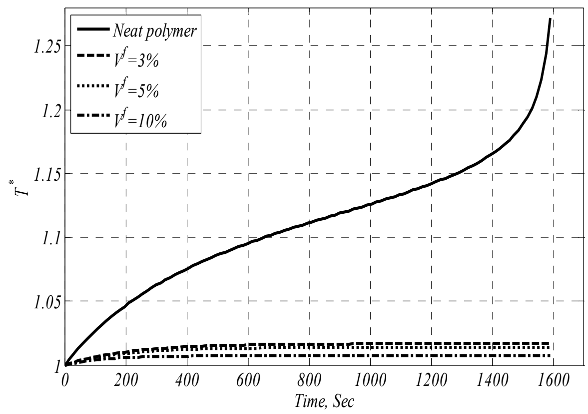


Fig. 5. The evolutions of the dimensionless temperature over time for, q'=155 kPa

The temperature evolution for considered nanocomposite beam with various volume fractions at the constant high level amplitude of harmonic loading, q'=155 kPa at f = 37 Hz are presented in Fig. 5. In this situation, the curves corresponding to the nanocomposite beam with neat polymer shows the self-heating temperature grows exponentially and finally rapid temperature growth occurs until the breakdown of the beam and the thermal failure occurs. It is important to note that, with an increase in volume fraction the thermal instability happens at the higher frequency. Also, it is apparent from this figure, the nanocopmosite beam with V^f=3, 5 and 10 % under considered conditions demonstrates the thermal equilibrium. It is worth to be mentioned, the fatigue process for neat polymer and nanocomposite beams, V^f=3, 5 and 10 % may be presented by the evolution temperature in Fig. 5. In general, each curve with thermal instability can be separated into three phases, which will be explained as follow. In the first part a typical temperature growth is observed, which shows the changes in components of complex moduli and accompanies with the decrease of a storage modulus and increase of a loss modulus.

It is clearly evident in curves of Fig. 5 that, in this phase, with increasing volume fraction and decreasing deflection of beam (reduction of viscoelastic response), the temperature growth occurs slowly. In the second part, at the critical values of deformation conditions, after equalizing of amounts of dissipated and convection energy at the beginning of this phase the slight temperature growth may be observed. As mentioned earlier, the trend of temperature growth in this phase depend on deformation conditions such as amplitude of loading, frequency, geometric of beam and ability of element to transfer dissipative heating to surround media. According to the presented results in Fig. 5, for neat polymer, the trend of temperature growth shows the beam isn't able to transfer dissipative heating to surround media, so that the temperature grows in the beam. Accordingly, in this phase both mechanical and thermal destruction occurs for neat polymer while the results of solution for nanocomposite beams with adding

3, 5 and 10 % volume fraction of CNTs fibers into polymer show after equalizing of amounts of dissipated and convection energy at the beginning of the second phase the slight temperature growth is observed and ultimately the saturation type behavior of the temperature is demonstrated. Finally, in the third phase the self-heating temperature grows rapidly in a short time period until breakdown. It is evident from curves for beam with neat polymer. Research has shown that, the third phase started due to the initiation of cracks in the area of stress concentration and highest temperature [16].

Conclusion. We have presented the approximate formulation of the problem of forced resonance vibrations and dissipative heating of a nanocomposite beam with unidirectionally oriented CNTs fibers with regard for geometric and physically nonlinearity. We have studied their influence on the dynamic characteristics and temperature of dissipative heating of the nanocomposite beam in the case of static and cyclic loads. We have shown that the effect of geometric nonlinearity on dynamic characteristics and temperature of dissipative heating reduce with increasing volume fraction. We have investigated the influence of volume fraction of nanofibers on the critical value of load amplitude, under which the temperature of vibration heating reaches the thermal instability point.

References

1. *Karnaukhov V. G.* Coupled thermo-viscoelastic problems . *V. G. Karnaukhov.* – Kiev : Naukova Dumka, [in Russian], 1982.
2. *Chang T. P.* Nonlinear vibration analysis of geometrically nonlinear shell structures / T. P. Chang, H. C. Chang // Mech. Research Commun. – 2000. – Vol. 27. – No. 2. – P. 173–180.
3. Coupled thermo-viscoplastic behaviour of bodies under harmonic load / I. K. Senchenkov, Y. A. Zhuk, G. A. Tabieva, O. P. Chervinko // International Applied Mechanics. – 1997. – Vol. 33. – No. 9. – P. 698 –706.
3. Constable I., Williams J. G., Bums D.J. Fatigue and thermal softening of thermoplastics // J. Mech. Eng. Sci. – 1970. – Vol.12. – P. 20-29.
4. *Senchenkov I. K.* Thermal fatigue failure of a rectangular prism with a cut / I. K. Senchenkov, N. N Yakimenko, and O. P. Chervinko // Numerical simulation and experiment," Teor. Prikl. Mekh. – 2004. – Vol. 39. – P. 42–47.
5. *Merzlyakov V. A.* Elastoplastic deformation of shells of revolution under nonaxisymmetric loading (review) / V. A. Merzlyakov, Yu. N. Shevchenko // Int. Appl. Mech. – 1999. – Vol. 35. – No. 5. – P. 431– 461.
6. *Sansour C.* Large viscoplastic deformations of shells. Theory and finite element formulation / C. Sansour, F .Kollmann // Comput. Mech. – 1998. – Vol. 21. – No. 21. – P. 512–525.
7. *Senchenkov I. K.* Modelling the Thermomechanical Behavior of Physically Nonlinear Materials Under Monoharmonic Loading / I. K. Senchenkov, Y. A.. Zhuk, V. G. Karnaukhov // Int. Appl. Mech. – 2004. – Vol. 40, – No. 9. – P. 30–34.
8. *Mori T.* Average stress in matrix, average elastic energy of materials with misfitting inclusions / T. Mori, K. Танака // Acta Metall. – 1973. – Vol. 21. – P. 571–574.
9. *Karnaukhov V. G.* Coupled Problems of the Theory of Viscoelastic Plates and Shells / V. G . Karnaukhov, I. F. Kirichok. – Kiev : Naukova Dumka, [in Russian],1986.
10. *Senchenkov I. K.* Forced nonlinear vibrations and dissipative heating of a viscoelastic beam / I. K. Senchenkov, I. F. Kirichok // Int. Appl. Mech. – 1987. – Vol. 23. – No. 1. – P. 80–86.
11. *Zhuk Y. A.* Frequency and amplitude dependence of complex moduli of composite material reinforced with nanofibers / Y. A.. Zhuk, // Journal of Phy. Math. Modeling & Inf. Tech. – 2016. – Vol. 23. – P. 92–107.
12. *Hashemi M.* Influence of frequency and amplitude of harmonic loading on complex moduli for polymer materials / M. Hashemi, Y. A.. Zhuk // Bulletin of KNU. – 2016. – Vol. 35. – P. 53–57.
13. *Hashemi M.* A procedure for complex moduli determination for polycarbonate plastic under harmonic loading / M. Hashemi, Y. A.. Zhuk // Bulletin of KNU. Phy. & Math. Sci. – 2015. – Vol. 4. – P. 67–73.
14. *Zimme M.* Through-Thickness Thermal Conductivity Prediction Study on Nanocomposites and Multiscale Composites / M. Zimme, X. Fan, J. Bao // Mater. Sci. & Applications. – 2012. – Vol. 3. – P. 131–138.
15. *Budiansky B.* Thermal and thermoelastic properties of isotropic composites / B. Budiansky // J. Compos Mater. – 1970. – Vol. 4. – P. 286–295.
16. *Shen H. S.* Thermal buckling and post buckling behavior of functionally graded carbon nanotube-reinforced composite cylindrical shells / H. S. Shen // Composites: Part B. – 2012. – Vol. 43. – P. 1030 –1038.
17. *Yang D. J.* Thermal conductivity of multiwalled carbon nanotubes / D. J. Yang, Q Zhang, G. Chen // Phys. Rev. – B 66, 165440.
18. *Grigorenko Y. M.* Numerical and Analytical Solution of the Problems of Shell Mechanics Based on Different Models [in Russian] / Y. M. Grigorenko, G. G. Vlaikov, A. Y. Grigorenko, – Akademperiodika, Kiev, 2006.
19. *Stevens K. K.* Transverse vibration of a viscoelastic column with initial curvature under periodic axial load / K. K. Stevens // Trudy Am. Obshch. Inzh. Mekh., Ser. E: Prikl. Mekh. – 1969. – Vol. 4. – P. 168–173.

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ЗГИННІ КОЛИВАННЯ ТА ДИСИПАТИВНИЙ РОЗІГРІВ НАНОКОМПОЗИТНОЇ БАЛКИ ПРИ СТАТИЧНОМУ І ГАРМОНІЧНОМУ НАВАНТАЖЕННЯХ

Досліджуються вимушені резонансні коливання і дисипативний розігрів в'язкопружної балки з композитного полімерного матеріалу, армованого однонаправленими нановолокнами, виготовленими з одношарових карбонових нанотрубок. Враховується геометрична нелінійність конструкції (квадрати кутів повороту перерізів) та температурна залежність комплексних модулів нанокомпозитного матеріалу. Для розв'язання зв'язаної нелінійної задачі термов'язкопружності при циклічному навантаженні використовується методика еквівалентної лінеаризації (для визначення комплексних модулів) у поєднанні з методом дискретної ортогоналізації із застосуванням ітераційної процедури. На кожній ітерації методом ортогональної дискретизації розв'язуються комплексні аналоги рівнянь коливань балки. Для розв'язання задачі теплопровідності використовується явна схема методу скінченних різниць. Досліджено вплив дисипативного розігріву, фізичної та геометричної нелінійності на динамічні характеристики коливань, температуру вібророзігріву та демпфірування вимушених коливань нанокомпозитної балки для різних значень об'ємного вмісту нановолокон при поперечному комбінованому статичному та гармонічному навантаженнях.

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ИЗГИБНЫЕ КОЛЕБАНИЯ И ДИССИПАТИВНЫЙ РАЗОГРЕВ НАНОКОМПОЗИТНОЙ БАЛКИ ПРИ СТАТИЧЕСКОЙ И ГАРМОНИЧЕСКОЙ НАГРУЗКАХ

Исследуются вынужденные резонансные колебания и диссипативный разогрев вязкоупругой балки из композитного полимерного материала, армированного однонаправленными нановолокнами, изготовленными из однослойных углеродных нанотрубок. Учитывается геометрическая нелинейность конструкции (квадраты углов поворота сечения) и температурная зависимость комплексных модулей нанокомпозитного материала. Для решения связанной нелинейной задачи термовязкоупругости при циклической нагрузке используется методика эквивалентной линеаризации (для определения комплексных модулей) в сочетании с методом дискретной ортогонализации с использованием итерационной процедуры. На каждой итерации методом ортогональной дискретизации решаются комплексные аналоги уравнений колебаний балки. Для решения задачи теплопроводности используется явная схема метода конечных разностей. Исследовано влияние диссипативного разогрева, физической и геометрической нелинейности на динамические характеристики колебаний, температуру виброразогрева и демпфирование вынужденных колебаний нанокомпозитной балки для разных значений объемного содержания нановолокон при поперечной комбинированной статической и гармонической нагрузках.