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ПЛАЗМОВО-РІДИННА СИСТЕМА З ОБЕРТАЛЬНОЮ КОВЗНОЮ ДУГОЮ ТА РІДКИМ ЕЛЕКТРОДОМ

У роботі представлені результати дослідження обертальної ковзної дуги з рідким електродом. Досліджено спектри випромінювання плазми обертальної ковзної дуги з рідким електродом. Виміряні вольт-амперні характеристики обертальної ковзної дуги в діапазоні потоків повітря 0–220 см³/с. Визначено температури заселення збуджених електронних T_e^*, коливних T_v^* та обертових T_r^* рівнів. Досліджено розподіл цих температур вздовж плазмового факелу.

Ключові слова: плазма, обертальна ковзна дуга, рідкий електрод, плазмово-рідинна система, електричний розряд.

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ПЛАЗМЕННО-ЖИДКОСТНАЯ СИСТЕМА С ВРАЩАТЕЛЬНОЙ СКОЛЬЗЯЩЕЙ ДУГОЙ И ЖИДКИМ ЭЛЕКТРОДОМ

В работе представлены результаты исследования вращательной скользящей дуги с жидким электродом. Исследованы спектры излучения плазмы вращательной скользящей дуги с жидким электродом. Измеренные вольтамперные характеристики вращательной скользящей дуги в диапазоне потоков воздуха 0–220 см3/с. Определены температуры заселения возбужденных электронных T_e^*, колебательных T_v^* и вращательных T_r^* уровней. Исследовано распределение этих температур вдоль плазменного факела.

Ключевые слова: плазма, вращающательная скользящая дуга, жидкий электрод, плазменно-жидкостная система, электрический разряд.

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DYNAMICS OF GENERALIZED PHASES IN A SYSTEM OF TWO WEAKLY-COUPLED SPIN-TORQUE NANO-OSCILLATORS WITH RANDOM EIGEN FREQUENCIES: THE CASE OF GLOBAL COUPLING

Dynamics of generalized phases in a system of two weakly-coupled spin-torque nano-oscillators (STNOs) with random eigen frequencies (Gaussian distribution) is analyzed. It is shown that the system dynamics is conveniently described by a complex order parameter in the scope of global coupling model. The numerical analysis of time dynamics of the modulus of complex order parameter was performed. It is shown that the synchronization of two STNOs is most effective when the amplitude of coupling Λ is big and the phase of coupling β is multiple of π.

Key words: spin-torque nano-oscillator, synchronization, coupled oscillations, random eigen frequency.

Introduction. The spin-transfer torque (STT) [1, 3, 15, 20–21] carried by a spin-polarized electric current can give rise to several types of magnetization dynamics (magnetization auto-oscillations [5–6, 8–11, 16–17] and reversal [7, 22]) and, therefore, allows one to manipulate magnetization of a nano-scale magnetic object [17].

The STT effect opens a possibility for the development of a novel type of nano-scale microwave devices – spin-torque nano-oscillators (STNOs). The practical application of STNOs faces four main problems:

- low enough operation frequencies of devices based on STNOs (typically, 1–15 GHz);
- low output microwave power (or DC power if a STNO is used as a microwave detector);
- large generation linewidth;
- imperfect manufacturing technology of STNOs.

The last three problems can be solved using the mutual phase-locking of several STNOs [6, 11, 14, 18–19]. For instance, using numerical simulations it has been demonstrated [4, 12] that the finite delay time of the coupling signal can lead to a substantial (~ 100 times) increase in the frequency band of phase-locking.

In this work we perform numerical simulations of phase-locking of two STNOs with random eigen frequencies (which are caused by the parameters spread of the STNOs due to the imperfect manufacturing technology) with account of a delay of the coupling signal. We consider the general case of two coupled nano-contact STNOs without account of the exact type of coupling. Thus, our results are valid for different types of coupling.

Theoretical model. The dynamics of the two weakly-coupled STNO can be described by the system of coupled nonlinear equations for the complex amplitudes $c_j(t)$ of spin wave modes, excited in *j*-th nano-contact [2, 14, 19]:

$$\frac{dc_1}{dt} + i\,\omega_1\left(\left|c_1\right|^2\right)c_1 + \Gamma_{eff,1}\left(\left|c_1\right|^2\right)c_1 = \Omega_{12}c_2\,e^{i\beta_{1,2}}\,,$$
$$\frac{dc_2}{dt} + i\,\omega_2\left(\left|c_2\right|^2\right)c_2 + \Gamma_{eff,2}\left(\left|c_2\right|^2\right)c_2 = \Omega_{21}c_1\,e^{i\beta_{2,1}}\,, \qquad (1)$$

where $j,k=\{1,2\}$, $\omega_j\left(\left|c_j\right|^2\right)$ and $\Gamma_{eff,j}\left(\left|c_j\right|^2\right)$ are the frequency and effective damping rate (that includes contribution from the positive natural damping and current-induced negative damping) of *j*-th mode, coupling frequencies $\Omega_{j,k}$ are defined by Eq. (6) in [19], and $\beta_{j,k}$ is the phase shift, for example, the phase shift of the spin wave (radiated by the *k*-th nano-contact) acquired during its propagation to the *j*-th nano-contact.

The system (1) without time delay ($\beta_{j,k}=0$) was derived and analyzed in [19]. The analysis of the system (1) was carried out in [2, 14] for the case of fixed (not random) frequencies and with account of time delay ($\beta_{j,k} \neq 0$).

In the absence of coupling ($\Omega_{j,k} = 0$) each of Eqs. (1) has a free-running solution [2, 14]

$$c_j(t) = \sqrt{p_j(t)}\, e^{i\omega_j(p_j(t))t}, \qquad (2)$$

where the power $p_j(t) = |c_j(t)|^2$ is determined by condition of the vanishing of total damping $\Gamma_{eff,j}(p_j) = 0$. For a weak coupling ($\Omega_{j,k} << \omega_j, \omega_k$) it is possible to perform a perturbative analysis of Eqs. (1), and obtain criteria of phase-locking in a closed analytical form [2].

Analysis of mutual phase-locking of two oscillators with an account of phase of the interaction (or, equivalently, delay of the coupling signal) showed that each oscillator can be described by one dynamical variable – "generalized phase" $\varphi_j(t)$ .It can be introduced as in that case the system (1) transforms to the equations of motion for the phases $\varphi_j(t)$, which can be written in the form

$$\frac{d\varphi_1}{dt} - \omega_1 = \Lambda_{1,2}\sin(\varphi_2 - \varphi_1 + \beta_{1,2}),$$
$$\frac{d\varphi_2}{dt} - \omega_2 = \Lambda_{2,1}\sin(\varphi_1 - \varphi_2 + \beta_{2,1}). \qquad (3)$$

Here ω_j is the natural (free-running) frequency of *j*-th oscillator, $\Lambda_{j,k}$ is the amplitude of the coupling between *j*-th and *k*-th oscillators, and $\beta_{j,k}$ is the coupling phase. Both amplitude $\Lambda_{j,k}$ and phase $\beta_{j,k}$ of the coupling are renormalized by the nonlinearity of the oscillators.

Eqs. (3), were derived from a simple auto-oscillator model [14, 18, 19]. It can be shown, however, that the model (3) is rather general and can be applied to the description of phase-locking of oscillators of any nature [14, 18, 19]. This model will serve as a basis for numerical and analytical study of phase-locking phenomenon in large arrays of spin-torque nano-oscillators.

In experiments, frequencies ω_j of the oscillators in an array differ due to uncertainties in technological process, presence of impurities, etc. Therefore, the free-running frequencies ω_j in system (3) should be considered as random quantities having certain probability distribution $P(\omega_j)$ (we assume that probability distribution is the same for each oscillator). Without loss of generality, we assume that the average frequency $\omega_{j,av}$ of oscillators is zero (otherwise, one can perform transformation $\varphi_j(t) \to \tilde{\varphi}_j(t) = \varphi_j(t) - \omega_{j,av}t$, that does not change form of Eqs. (3)). For simplicity, we consider only the case, where the frequency distribution $P_G(\omega_j)$ is the Gaussian distribution:

$$P_G(\omega_j) = \frac{1}{\sqrt{2\pi}\Delta\omega}\exp\left[-\frac{1}{2}\left(\frac{\omega_j}{\Delta\omega}\right)^2\right]. \qquad (4)$$

Here $\Delta\omega$ is a characteristic width of the frequency distribution (we assume it is the same for all oscillators).

The coupling amplitudes $\Lambda_{j,k}$ and phases $\beta_{j,k}$ depend on the coupling mechanism between oscillators and on their properties (nonlinearity). Here we analyze the simplest case of practical interest – the case of global coupling. This is the typical case of coupling by the common current bias current, when coupling between all oscillators is the same:

$$\Lambda_{j,k} = \Lambda = \text{const},\ \beta_{j,k} = \beta = \text{const}. \qquad (5)$$

In the following we will study global coupling of oscillators using model (3) with coupling (5), when the eigen frequencies of oscillators are characterized by the distribution (4). The main task is to analyze dependence of the locking process on the amplitude Λ and phase β of the coupling.

The convenient parameter for the phase-locking characterization is a complex order parameter [13]

$$r = R\,e^{i\psi} = \frac{1}{2}\left(e^{i\varphi_1} + e^{i\varphi_2}\right). \qquad (6)$$

The amplitude R of the order parameter characterizes number of phase-locked oscillators, while the rate of change of the phase $\varpi = d\psi/dt$ gives the frequency of phase-locked oscillations.

Numerical model. Our numerical model is based on the numerical solution of Eqs. (3) for some particular values of $\Lambda_{1,2} = \Lambda_{2,1} = \Lambda$, $\beta_{1,2} = \beta_{2,1} = \beta$, ω_1 , ω_2 and known initial conditions $\varphi_1(0)$, $\varphi_2(0)$. During the numerical analysis we assume for simplicity that the average frequency of the STNOs is zero and chose the characteristic width of the frequency distribution $\Delta\omega$ as $\Delta\omega = 0.1$ (the case of small frequency deviations, i.e. the case of small uncertainties in technological process of STNO fabrication). We also assume that the initial phases of STNOs $\varphi_1(0)$, $\varphi_2(0)$ are random values with rectangular distribution:

$$P_R(\varphi_{1,2}(0)) = \begin{cases} 1/2\pi, & 0 \le \varphi_{1,2}(0) \le 2\pi \\ 0, & \text{otherwise} \end{cases}. \qquad (7)$$

Since the frequencies ω_1 and ω_2 are random values described by frequency distribution law (4) and initial phases $\varphi_1(0)$, $\varphi_2(0)$ are random values described by distribution law (7), it is clear that the dynamics of general phases $\varphi_1(t)$, $\varphi_2(t)$ will depend on ω_1 , ω_2 and $\varphi_1(0)$, $\varphi_2(0)$ and, therefore, may also be random (at least partially). To eliminate this influence of randomness on the system dynamics and to obtain statistically correct results we performed multi-pass numerical analysis of the Eqs. (3). The used algorithm may be described as follows:

1) We chose some particular values of the amplitude of coupling Λ and the phase of coupling β .

2) We generate the random vector of initial phases $\vec{\varphi}(0) = \{\varphi_1(0), \varphi_2(0)\}$ using the distribution law (7) and the random vector of oscillator eigen frequencies $\vec{\omega} = \{\omega_1, \omega_2\}$ using the distribution law (4) for the case of $\Delta\omega = 0.1$.

3) We numerically solve the equations (3) for the case of some particular values of equation's parameters Λ , β , $\vec{\omega}$, $\vec{\varphi}(0)$ defined above. We calculate the modulus of the complex order parameter as a function of time $R(t)$ using numerically calculated phase vector $\vec{\varphi}(t) = \{\varphi_1(t), \varphi_2(t)\}$.

4) We repeat the stages 2 and 3 N times, where N is the number of passes. After that we do the averaging of the obtained data of $R(t)$ for all realizations and obtain an averaged dependence

$$\langle R(t)\rangle_N = \frac{1}{N}\sum_{k=1}^{N} R_k(t), \qquad (8)$$

where $R_k(t)$ is a dependence of the modulus of the complex order parameter on time obtained at k -th pass.

We believe that for a large enough value of N the influence of randomness on the system dynamics is minimal. The minimal number of passes N, which correspond to that case might be found numerically.

Results and discussion. Using the algorithm stages 1–3 described in the previous section we calculated the time dependencies of general phases $\varphi_1(t)$ and $\varphi_2(t)$ of two weakly-coupled STNOs. The typical obtained results (after renormalization – division by 2π) are presented in Fig. 1. The corresponding dependence of $R(t)$ is shown in Fig. 2.

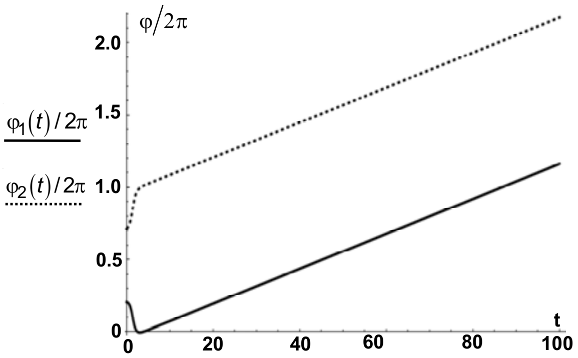

Fig. 1. The typical time dependencies of normalized general phases $\varphi_1(t)/2\pi$ (solid line) and $\varphi_2(t)/2\pi$ (dashed line) of two weakly-coupled STNOs for the case of model parameters: λ = 1, β = 0, ω={-0.0016, 0.1500}, φ(0)={1.3, 4.5}

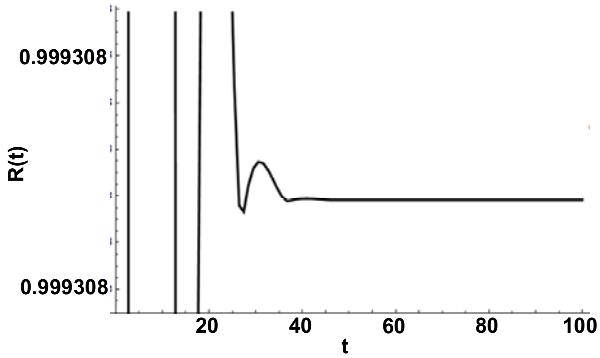

Fig. 2. The typical time dependencies of the modulus of complex order parameter $R(t)$ for a system of two weakly-coupled STNOs for the case of model parameters: λ =1, β = 0, ω={0.0016, 0.1500}, φ(0)={ 1.3, 4.5}.

We also analyzed the dependence of $\langle R(t)\rangle_N$ on N (see Fig. 3). We show that the dependence $\langle R(t)\rangle_N$ is statistically stable if N > 100 for the case λ = 1, β = 0. The obtained results depend on Λ, β, but it seems that N is almost the same for any values of Λ, β, if Λ is big enough and β is far from $\pi/2$ or $3\pi/2$.

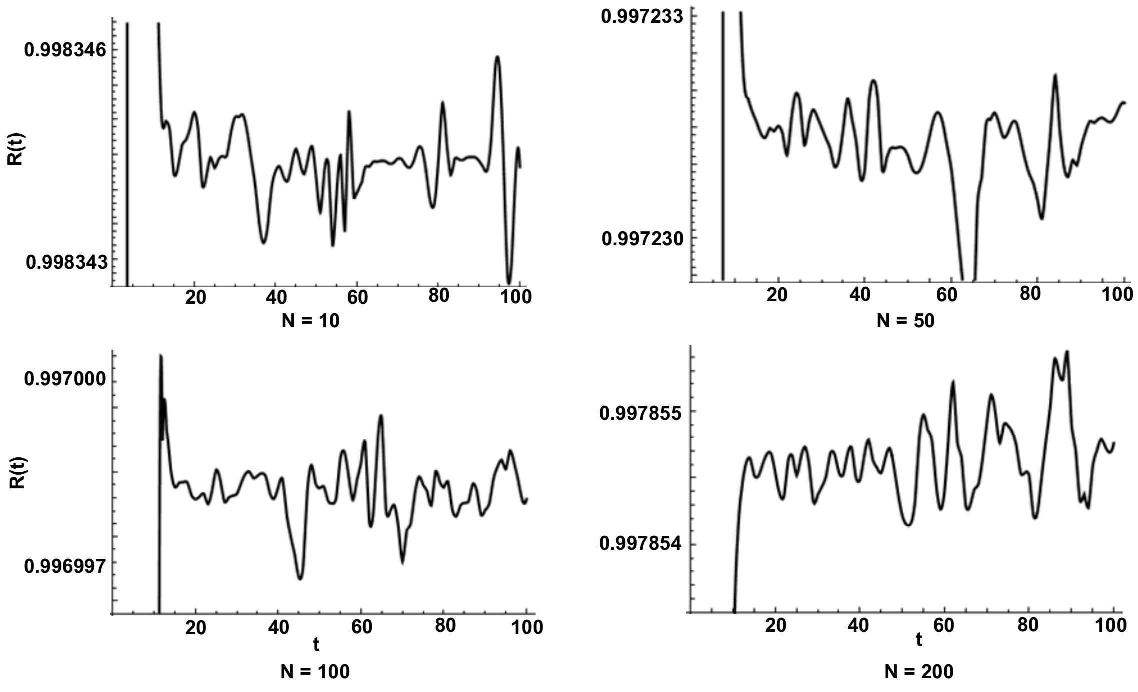

Fig. 3. The dependencies of the averaged modulus of complex order parameter $\langle R(t)\rangle_N$ on time for different number of passes N. λ =1, β = 0

The following analysis was carried out with number of passes N = 200. First, we fix the phase of coupling β (β=0) and analyzed the dependence of $\langle R(t)\rangle_N$ for different amplitude of coupling Λ (Fig. 4). One can see, decrease of Λ leads to a great perturbation of the system dynamics, thus, it seems the efficiency of synchronization decreases in that case. Second, we fix the amplitude of coupling Λ ($\Lambda = 1$) and analyze the dependence of $\langle R(t)\rangle_N$ on phase of coupling β (Fig. 5).

One can see, the efficiency of synchronization $\langle R(t)\rangle_N$ rapidly increases if β is close to 0 or π. It is obvious, that in real in Fig. 4 and Fig. 5 there are plots with almost linear dependence of averaged modulus of complex order parameter $\langle R(t)\rangle_N$ on time. The visibility of function oscillation defined only by choosing the appropriate graph scale. The efficiency of synchronization for different amplitudes and phases of coupling is clearly shown in Fig. 6(b). One can see, that it is most effective for big amplitudes of coupling when phase of coupling multiple of π. This means that the model of global coupling may be applicable only in that case.

The final state of the system (value of $\langle R(t)\rangle_N$ at the moment $t = 100$) is analyzed on Fig.6(a).

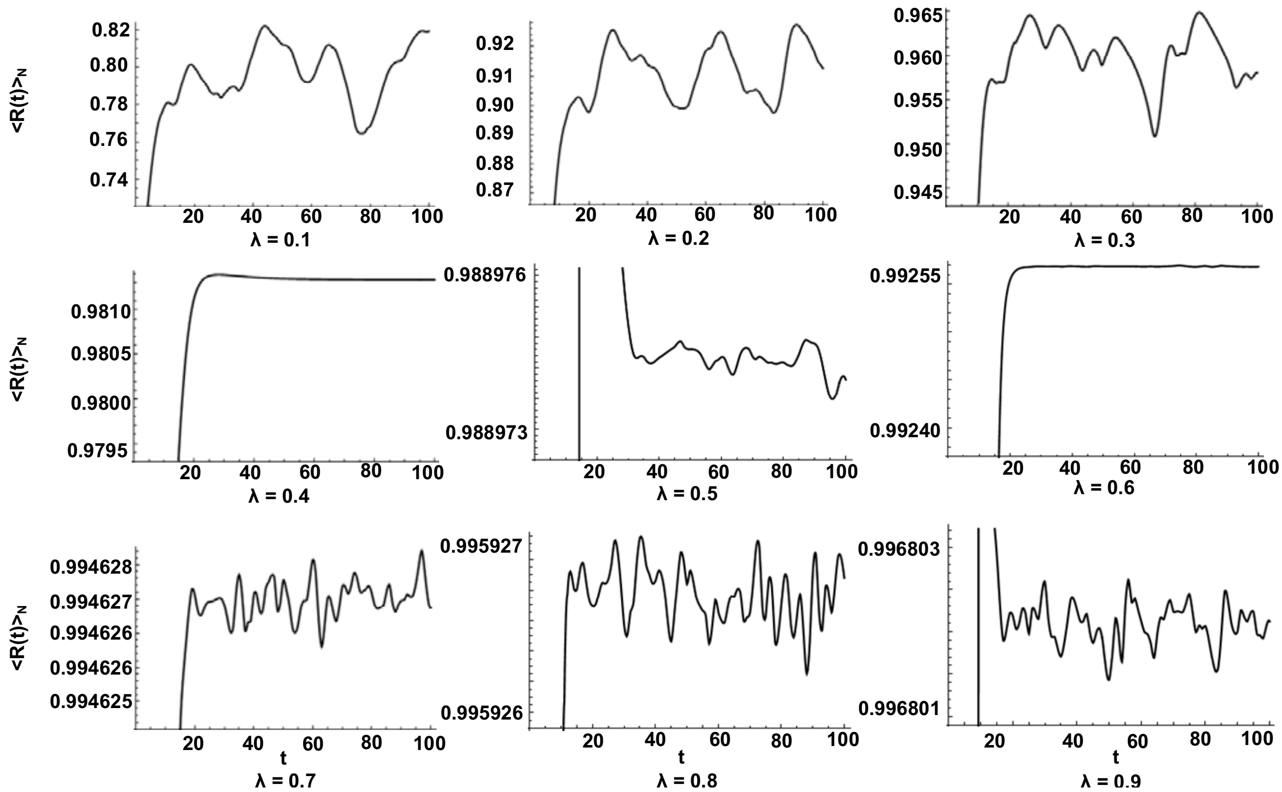

Fig. 4. The dependencies of the averaged modulus of complex order parameter $\langle R(t)\rangle_N$ on time for different amplitudes of coupling Λ. N = 200, β = 0

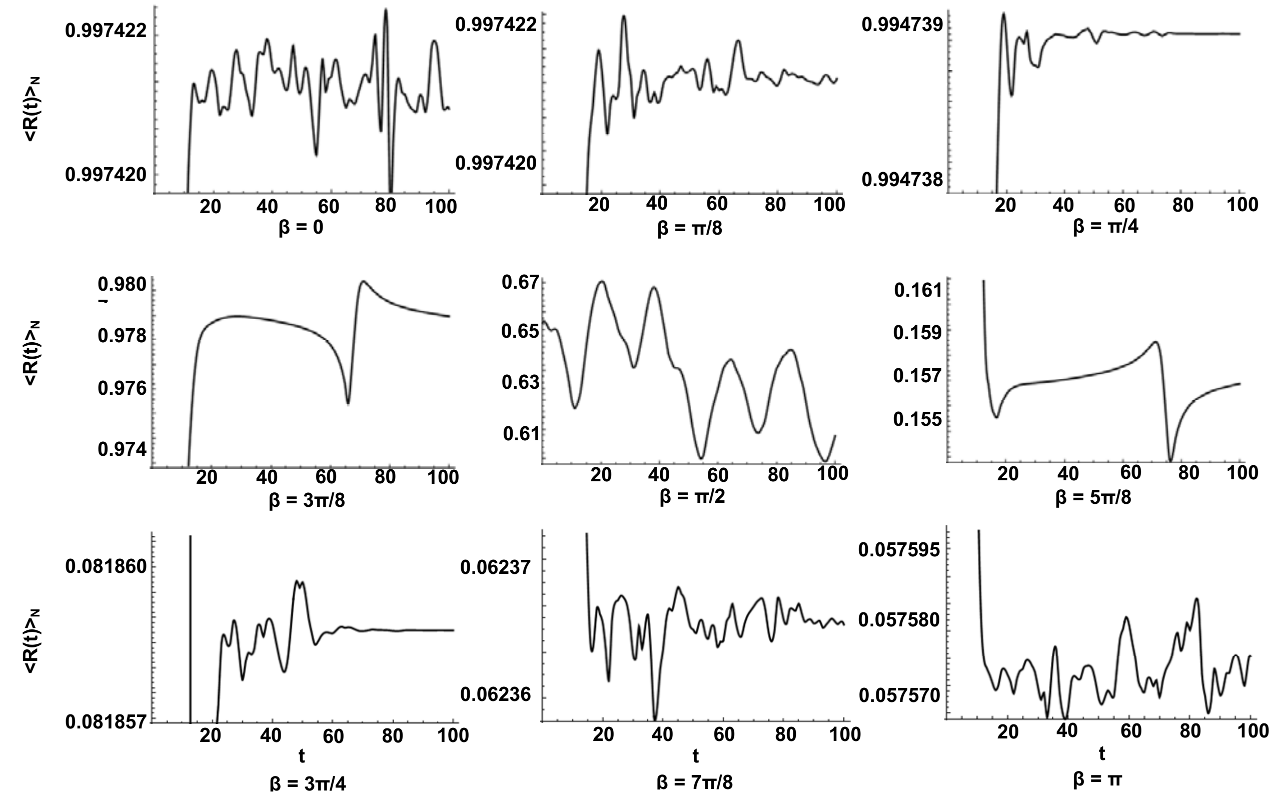

Fig. 5. The dependencies of the averaged modulus of complex order parameter $\langle R(t)\rangle_N$ on time for different phases of coupling β. N = 200, λ = 1

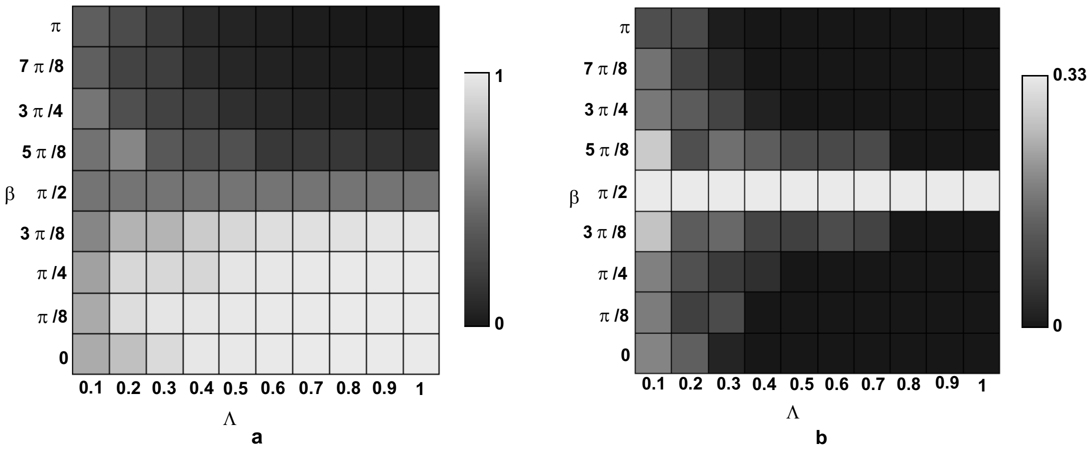

Fig. 6. Density plots of the averaged modulus of complex order parameter at the moment t=100 (a) and difference between maximum and minimum values of averaged modulus of complex order parameter on time segment [80,100] (b) for different phases of coupling β and amplitudes of coupling Λ

Conclusion. We have demonstrated that the dynamics of two weakly-coupled STNOs is conveniently described by a complex order parameter in the scope of global coupling model. Using numerical analysis we show that the synchronization of two STNOs is most effective when the amplitude of coupling Λ is big and the phase of coupling β is far lesser or larger than $\pi/2$.

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ДИНАМІКА УЗАГАЛЬНЕНИХ ФАЗ У СИСТЕМІ ДВОХ СЛАБКО ВЗАЄМОДІЮЧИХ МАГНІТНИХ НАНООСЦИЛЯТОРІВ З ВИПАДКОВИМИ ВЛАСНИМИ ЧАСТОТАМИ: ВИПАДОК ГЛОБАЛЬНОГО ЗВ'ЯЗКУ

Проаналізовано динаміку узагальнених фаз в системі двох слабко зв'язаних магнітних наноосциляторів з випадковими власними частотами (розподіленими за нормальним законом). Показано, що у наближенні глобального зв'язку динаміку такої системи зручно описувати за допомогою комплексного параметра порядку. Методами числового аналізу досліджено часові зміни модуля комплексного параметра порядку. Показано, що синхронізація двох магнітних наноосциляторів відбувається найбільш ефективно при великих значеннях амплітуди коефіцієнта зв'язку Λ та фазі коефіцієнта зв'язку β кратній π.

Ключові слова: магнітний наноосцилятор, синхронізація, зв'язані коливання, випадкова власна частота.

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ДИНАМИКА ОБОБЩЕННЫХ ФАЗ В СИСТЕМЕ ДВУХ СЛАБО ВЗАИМОДЕЙСТВУЮЩИХ МАГНИТНЫХ НАНООСЦИЛЛЯТОРОВ СО СЛУЧАЙНЫМИ СОБСТВЕННЫМИ ЧАСТОТАМИ: СЛУЧАЙ ГЛОБАЛЬНОЙ СВЯЗИ

Проанализирована динамика обобщенных фаз в системе двух слабо связанных наноосцилляторов со случайными собственными частотами (распределенными по нормальному закону). Показано, что в приближении глобальной связи динамику такой системы удобно описывать с помощью комплексного параметра порядка. Методами численного анализа исследована зависимость изменений модуля комплексного параметра порядка от времени. Показано, что синхронизация двух магнитных наноосцилляторов происходит наиболее эффективно при больших значениях амплитуды коэффициента связи Λ и фазе коэффициента связи β кратной π.

Ключевые слова: магнитный наноосциллятор, синхронизация, связанные колебания, случайная собственная частота.