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ЛАЗЕРНА АБСОРБЦІЙНА СПЕКТРОСКОПІЯ ПЛАЗМИ ЕЛЕКТРОДУГОВОГО РОЗРЯДУ З ДОМІШКАМИ МІДІ

Для діагностики плазми електродугового розряду між композитними Cu-Mo електродами застосовано методику лінійної лазерної абсорбційної спектроскопії. Реалізовано експериментальну схему реєстрації просторових розподілів інтенсивності лазерного випромінювання за допомогою ПЗЗ-матриці. Розроблено програмний інтерфейс користувача для обробки експериментальних даних, визначено ймовірну експериментальну похибку. Отримані просторові розподіли заселеності $^5D_{5/2}$ рівня атомів міді використано для розрахунку складу плазми у припущенні локальної термодинамічної рівноваги.

Ключові слова: лазерна абсорбційна спектроскопія, плазма електродугового розряду.

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ЛАЗЕРНАЯ АБСОРБЦИОННАЯ СПЕКТРОСКОПИЯ ПЛАЗМЫ ЭЛЕКТРОДУГОВОГО РАЗРЯДА С ПРИМЕСЬЮ МЕДИ

Для диагностики плазмы электродугового разряда между композитными Cu-Mo электродами применена методика лазерной абсорбционной спектроскопии. Реализована экспериментальная схема регистрации пространственных распределений интенсивности лазерного излучения с помощью ПЗС-матрицы. Разработан программный интерфейс пользователя для обработки экспериментальных данных и определена ожидаемая ошибка эксперимента. Полученные пространственные распределения заселенности энергетического уровня $^5D_{5/2}$ атомов меди использованы для расчета компонентного состава плазмы в предположении локального термодинамического равновесия.

Ключевые слова: лазерная абсорбционная спектроскопия, плазма электродугового разряда.

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PROPAGATION OF TEMPERATURE WAVES IN MEDIUM WITH INTERNAL THERMAL RELAXATION

In this article the wave solutions of heat transfer equations, for classical model and for two alternative non-stationary models (hyperbolic and model with a time delay) have been studied. Modeling of temperature impulse propagation was performed according to dispersion relations obtained from heat transfer equations. It was shown that propagation of temperature impulse in non-stationary models could significantly differ from temperature diffusion in classical model. This can be used for signal transmission and that temperature waves could be used for delay line construction.

Keywords: wave heat transfer, non-stationer models of heat transfer, temperature impulse propagation, undumped temperature waves, dispersion of temperature waves.

Introduction. For the long time the heat transfer problems were regarded using only a classical Fourier's hypothesis that heat flux is proportional to a module of temperature gradient and have opposite direction. In the 50^{th} of the last century first attempt to regard non-stationary heat transfer processes were made by Cattaneo and Vernotte, what leads to a hyperbolic equation for temperature field [2, 12]. Since then the hyperbolic and non-linear parabolic heat transfer models were intensively studied and a big amount of new heat transfer regimes were established, such as traveling waves, blow-up regimes and some others [7–11]. Hypothesis of finite velocity of thermal signal propagation became especially popular in last two decades. Also the non-stationary solutions, such as the temperature waves, started to attract attention of researchers, especially for application in the scanning thermo wave microscopy (STWM), which is one of the method to investigate the under layers of surface for the purpose to determine heat conductivity [1, 5, 6]. In present work we regard thermal wave solutions not only for a classical heat transfer model but also for non-stationary models: hyperbolic and with time delay. Obtained relations of dispersion were used for modeling of temperature impulse propagation, regarding the possibility to use temperature field for signals transmission.

Wave solutions of heat transfer equations. General form of classical heat transfer equation is

$$\rho c\frac{\partial T(x,t)}{\partial t}=\lambda\Delta T(x,t) \qquad (1)$$

where ρ is density, c is heat capacity and λ is heat conductivity. Suppose that:

$$T=T_0e^{i(\omega t-kx)} \qquad (2)$$

From (1) and (2) and regarding that k is a complex value $k=k_1+ik_2$ dispersion relation could be obtained:

$$i\rho c\omega=-\lambda\left(k_1^2-k_2^2\right)-i2\lambda k_1k_2 \qquad (3)$$

It gives the system of equations:

$$\begin{cases}\lambda\left(k_1^2-k_2^2\right)=0\\ \rho c\omega+2\lambda k_1k_2=0\end{cases} \qquad (4)$$

In a second half of twenty century the hyperbolic heat transfer equation became more common, especially in the problems of laser impulse interaction with matter. Since the duration of laser impulses could be very short, it yields to non-stationary problems of heat transfer [6]. Hyperbolic heat transfer equation is as follows:

$$\rho c\frac{\partial T(x,t)}{\partial t}+\tau\rho c\frac{\partial^2T(x,t)}{\partial t^2}=\lambda\Delta T(x,t) \qquad (8)$$

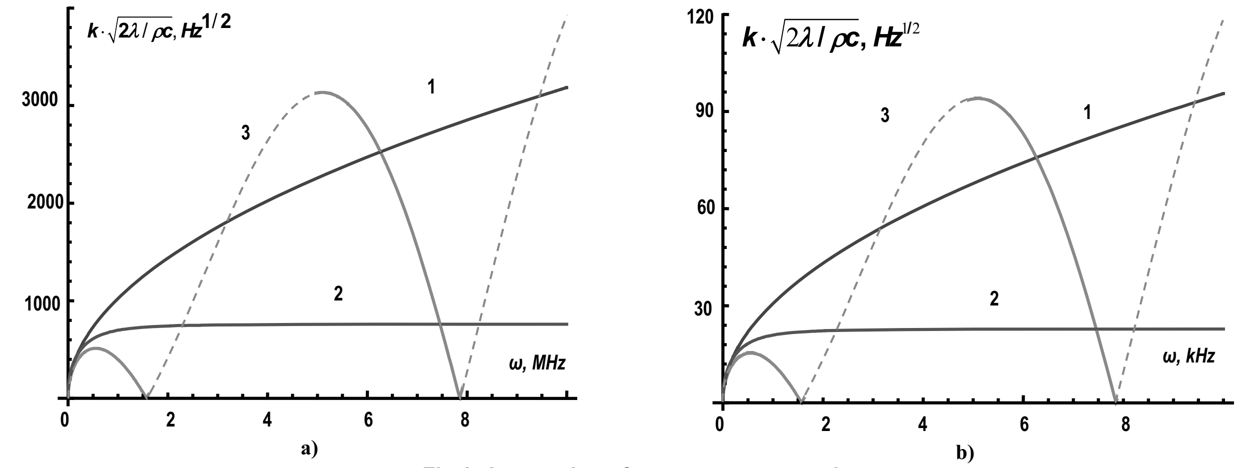

Fig.1. Attenuation of temperature waves in
1 – classical model, 2 – hyperbolic model, 3 – model with delay.
The dashed line corresponds to areas of frequencies in which $\cos\omega\tau < 0$ and thermal waves are not generated

Here τ is a relaxation time, parameter that describes the relaxation of local macroscopic subsystems to equilibrium thermal distribution. By the same way as it was done for the classical model, the dispersion relations for the hyperbolic model could be found as follows:

$$\begin{cases} \rho c\omega + 2k_1k_2\lambda = 0 \\ \lambda\left(k_1^2 - k_2^2\right) - \tau\rho c\omega^2 = 0 \end{cases} \quad (9)$$

For such model the temperature waves becomes as:

$$\begin{cases} T = T_0 e^{\pm(k/\beta_+)x} e^{i\left(\omega t \pm k\beta_+ x\right)} \\ T = T_0 e^{\pm k\beta_- x} e^{i\left(\omega t \pm (k/\beta_-)x\right)} \end{cases} \quad (10)$$

Where

$$\beta_\pm = \sqrt{\sqrt{\omega^2\tau^2 + 1} \pm \omega\tau} \quad (11)$$

From (10) it can be seen that for the hyperbolic model were obtained two different possible situations for the same conditions. It has also be notices that in both cases attenuation do not increase to infinity with frequency.

In last two decades gained popularity single- and dual-phase-lag heat conduction models [3,4]. Here we consider only single-phase-lag equation, also called as equation with time delay. It has the next from:

$$\rho c \frac{\partial T(x, t+\tau)}{\partial t} = \lambda \Delta T(x,t) \quad (12)$$

Analysis of dispersion relations for this model:

$$\begin{cases} \rho c\omega \cos\omega\tau + 2k_1k_2\lambda = 0 \\ \lambda\left(k_1^2 - k_2^2\right) - \tau\rho c\omega \sin\omega\tau = 0 \end{cases} \quad (13)$$

lead to the next temperature waves:

$$T = T_0 e^{\pm k\frac{\cos\omega\tau}{\sqrt{1+\sin\omega\tau}}x} e^{i\left(\omega t \pm k\sqrt{1+\sin\omega\tau}\,x\right)} = T_0 e^{\pm k\sqrt{1-\sin\omega\tau}\,x} e^{i\left(\omega t \pm k\frac{\cos\omega\tau}{\sqrt{1-\sin\omega\tau}}x\right)} \quad (14)$$

This model with delay predicts not only the possibility of attenuation decreasing with frequency at $\cos\omega\tau > 0$, but also predicts the existence of undamped thermal waves at $\omega\tau = 2n\pi + \pi/2,\ n = 0,1,2...$. The excitation of thermal waves is impossible at $\cos\omega\tau < 0$, because of unlimited increasing amplitude of such waves when $x \to \infty$. The frequency dependence of attenuation absolute value is shown on fig. 1 a) and b) for relaxation time equal to microseconds and milliseconds respectively.

Modeling of temperature impulse evolution. The modeling of single temperature impulse evolution was performed for normalized temperature. The initial impulse is given with a formula:

$$T_0(x=0,t) = \frac{T(x=0,t) - T_{\min}}{T_{\max} - T_{\min}} = h\left(t + \frac{P}{2}\right) - h\left(t - \frac{P}{2}\right) \quad (15)$$

Where P – duration of impulse, $h(t)$ - Heaviside function.

In what follows F[.] means Fourier transform operator. Fourier transform of initial impulse is:

$$T_0(\omega) = F[T_0(x=0,t)] = \frac{P}{\sqrt{2\pi}} Sinc\left(\frac{\omega P}{2}\right) \quad (16)$$

Evolution of thermal impulse will be given with:

$$T_0(x,t) = F^{-1}[T_0(\omega)\cdot e^{i(\omega t - k(\omega)x)}] \quad (17)$$

On fig. 2 results of modeling are shown. For the calculations the next parameters were used: $G = \lambda / c\rho = 3.10^{-3}\, cm^2/s$, $\tau = 10\, s$. On fig. 2 a),b) is shown evolution of temperature impulse according to a classical equation, it is obviously that according to this model propagation of thermal impulse is impossible because of dispersion initial impulse loose it's form, and in a distant spot it is impossible to detect beginning and end of thermal impulse. Opposite situations take a place in Fig. 2 c), d). In hyperbolic heat transfer model traveling impulse solution is possible, but also deformation of impulse form takes a place due to the dispersion. Nevertheless, it is possible to detect beginning and end of impulse. It makes possible to use temperature field to transfer information, but it should be mentioned that propagation of information in thermal field is very slow comparing to electromagnetic field and acoustics waves. It means that temperature field could be used for information transfer only when there is a need to create delay between two signals. But still as it is shown on fig. 2 c), d) amplitude of temperature impulse decrease very fast with distance from initial source. Situation on fig. 2 e), f) differ a lot from previous. Also traveling wave solution take a place but contour of wave differs from the one of initial impulse. On Fig. 3 the thermal waves appears as a response of medium on single impulse heating. All those results could be important in problems of microchips cooling, because many modern microchips works at frequencies comparable with $\frac{1}{\tau}$.

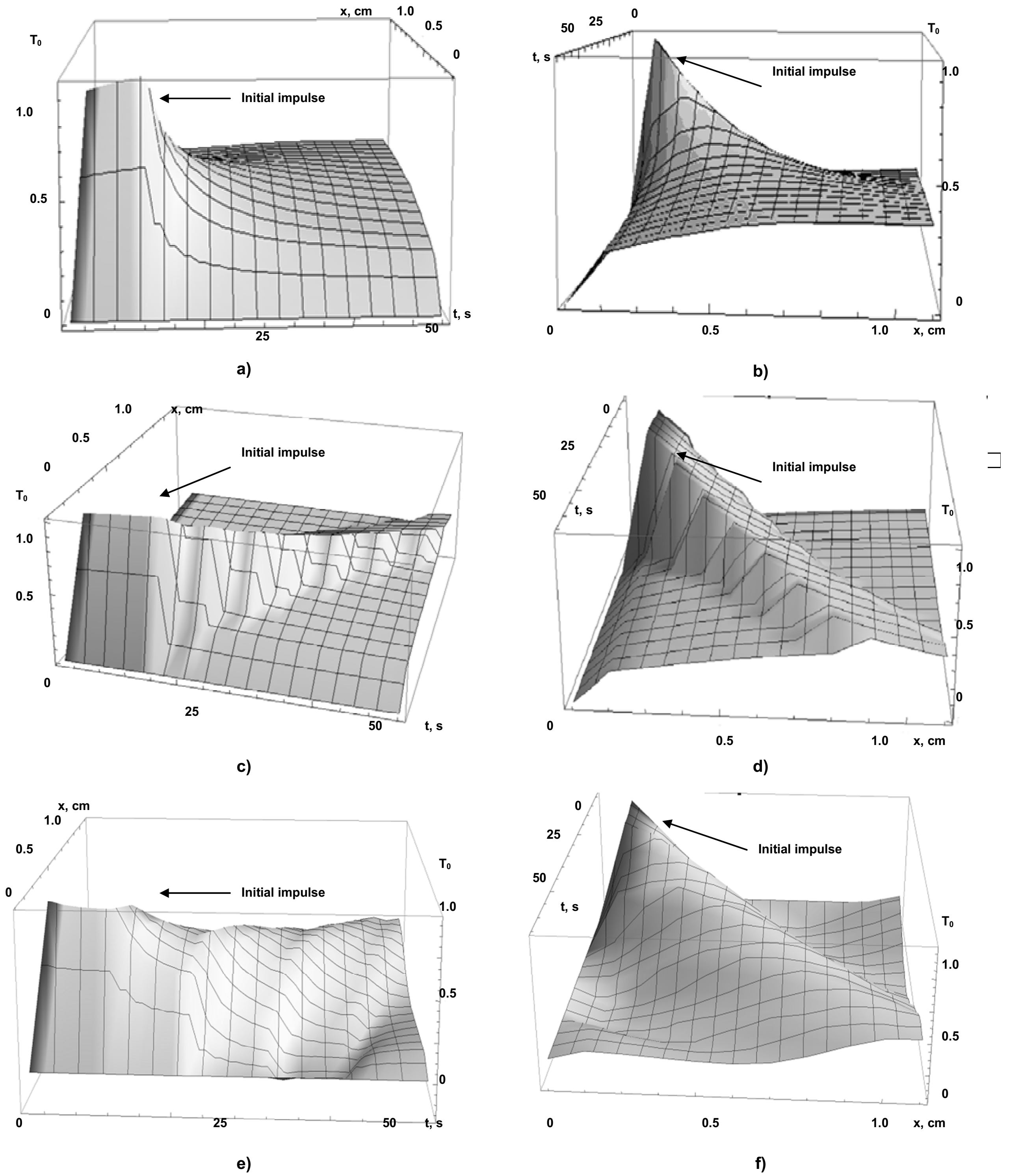

Fig. 2. Numerical evaluation of thermal impulse propagation
a),b) – classic model; c),d) – hyperbolic model; e),f) – delay model with denied zones in specter

Conclusions. It was shown that for the non-stationary models of heat transfer with hypothesis of the relaxation time propagation of traveling temperature the impulse is possible. What theoretically creates the possibility to use temperature field for the purpose of slow signal transfer and constructing of the delay lines.

Performed modeling reveals a possibility to use the temperature field as a filter, what follows from modeling shown on figure 3. It should be mentioned that the model with time delay also is a filter, because there are zones in the specter where amplitude of waves should increase. Even the fact that temperature waves can't transfer energy can't explain the possibility for existing of such kind of waves. This kind of system will be not stable and any small fluctuation will lead to a macroscopic periodic processes. However, it is more natural to regard it as a limitation of model with delay. Thus in modeling amplitudes of waves in denied zones were forcefully made to be zero. In the Fig. 3 was shown a transformation of initial thermal impulse into temperature waves due to the different value of attenuation coefficient at different frequencies.

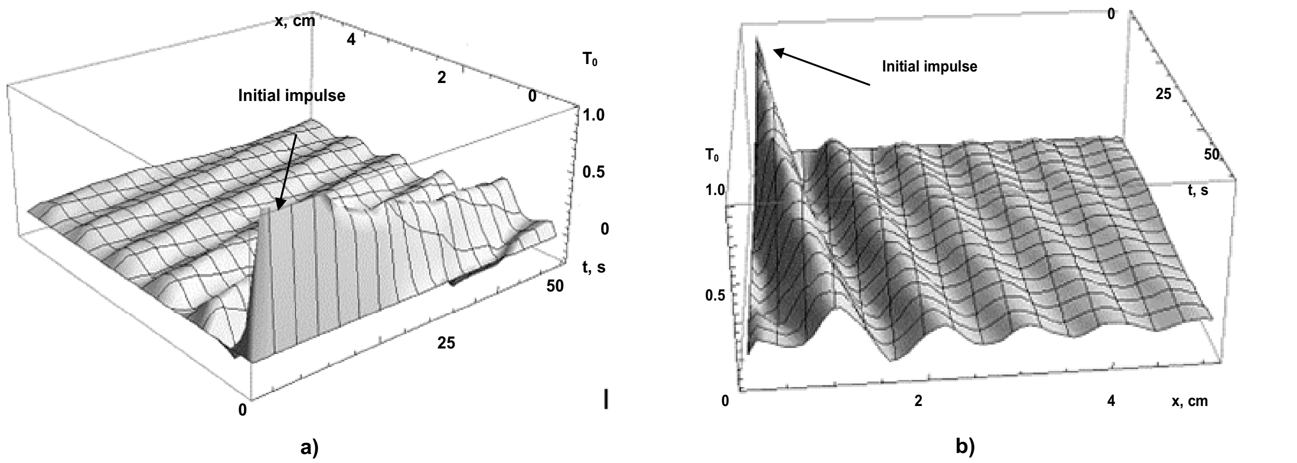

Fig.3. Numerical evaluation of thermal impulse propagation
a), b) delay model with denied zones in specter transformation of thermal impulse into thermal waves

Also should be taken into consideration the fact that both, hyperbolic heat transfer equation and equation with time delay, are not precisely from kinetic equation, since in both cases some simplifications were made. However these models are more accurate for non-stationary problems than for classical equation. In order to obtain the last one a time derivative of distribution function is suppose to be equal to 0, while non-stationary models are obtained in assumption that time derivative is not zero but a small value and, approximately, could be regarded as a difference. Of course for the models when high harmonics have significant amplitudes the assumptions on what the non-stationary models with relaxation time hypothesis are based becomes not appropriate.

Undamped slow thermal waves with velocity

$$v_n = \sqrt{2G\omega_n},\ \ \omega_n = (2n\pi + \pi / 2)/\tau,\ \ n = 0,1,2... \qquad (18)$$

can be used for signal transmission and at delay line construction. Such slow waves also can be used in thermo-optic deflectors (analog of acoustic-optic deflectors) for angular control and modulation of weak laser radiation.

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РОЗПОВСЮДЖЕННЯ ТЕМПЕРАТУРНИХ ХВИЛЬ В СЕРЕДОВИЩАХ З ТЕПЛОВОЮ РЕЛАКСАЦІЄЮ

У роботі розглянуто хвилеві розв'язки рівнянь теплопровідності для класичної моделі, а також для двох альтернативних нестаціонарних моделей. Представленні результати моделювання поширення температурних імпульсів з урахуванням дисперсійних співвідношень отриманих на основі класичної моделі теплопровідності, гіперболічної моделі теплопровідності та моделі з часом затримки. Показано, що при умові співрозмірності тривалості температурних імпульсів із часом релаксації середовища в нестаціонарних моделях має місце поширення температурних фронтів, що за своїм характером суттєво відрізняється від дифузії тепла в класичній моделі і може бути використано при передачі інформації, зокрема в лініях затримки.

Ключові слова: хвилева теплопровідність, нестаціонарні моделі теплопровідності, розповсюдження температурного імпульсу, незатухаючі температурні хвилі, дисперсія температурних хвиль.

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РАСПРОСТРАНЕНИЕ ТЕМПЕРАТУРНЫХ ВОЛН В СРЕДАХ С ТЕПЛОВОЙ РЕЛАКСАЦИЕЙ

В статье рассмотрены волновые решения уравнений теплопроводности для классической модели, а также для двух альтернативных нестационарных моделей. Представлены результаты моделирования распространения температурных импульсов, с учетом дисперсионных соотношений полученных основываясь на классической модели теплопроводности, гиперболической модели теплопроводности и модели со временем задержки. Показано, что при условии соизмеримости длительности температурных импульсов со временем релаксации среды в нестационарных моделях возможно распространение температурных фронтов, что по своему характеру существенно отличается от диффузии тепла в классической модели и может быть использовано для передачи информации, в частности в линиях задержки.

Ключевые слова: волновая теплопроводность, нестационарные модели теплопроводности, распространение температурного импульса, незатухающие температурные волны, дисперсия температурных волн.