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THE EVOLUTION OF POLARIZATION STATE IN LONGITUDINALLY INHOMOGENEOUS MEDIUM WITH LINEAR BIREFRINGENCE

The Features of formation of anisotropy in longitudinally inhomogeneous media with linear birefringence were considered and studied. Equivalent model of representation of this class of media as a series of identical action of phase plates rotated relative to each other by the same angle was used. The method of differential Mueller matrices to describe the polarization properties of the medium was used. Evolution of the Stokes vector on the basis of differential Mueller matrix and vector transport equation was derived. Evolution of azimuth and angle of ellipticity in the direction of light propagation in this class of media was obtained and studied. Period of changes of polarization ellipse parameters were determined. Comparison of the features of evolution of polarization state with relevant features in the homogeneous media with elliptical birefringence was made. Effective value of optical activity for any state of polarization of light propagating in the medium was found. Features of rotation of the polarization plane of privileged waves were considered.

Keywords: *Mueller matrix, birefringence, azimuth, angle of ellipticity, longitudinally inhomogeneous media, polarization state.*

Introduction. Development of liquid crystal technology has led to the fact that one of the actually problems of modern polarimetry is to study the polarization properties of longitudinally inhomogeneous medium with linear birefringence. In particular, such media include twisted nematic and cholesteric, which used in the creation of liquid crystal displays [6, 11]. In the works [7–9] were considered problems related to the features of eigenwaves which propagating in such media and ortohonalization properties of such media. Another important problem related to study of anisotropic properties of longitudinally inhomogeneous medium with linear birefringence are features of the evolution of polarization state of light [4, 6]. The study of with problem associated with the fact that this class of media against a background of linear birefringence and due to the special structure arises effective circular birefringence (optical activity) [5].

In view of the problems, the aim of this paper is to investigate the features of formation anisotropy in longitudinally inhomogeneous media with linear birefringence and its effect on the evolution of polarization state at the light propagating in the direction of the helical axis.

The matrix model of the medium. Nondepolarizing longitudinal inhomogeneous medium with linear birefringence can be equivalent representation as a sequence of molecular planes which consist of elongated molecules, oriented parallel to each other [3]. Each of such molecular planes can be present as a thin phase plate with linear birefringence [4]. Fast (slow) axis of the plate is parallel (perpendicular) to the direction along which direct the plane of the molecules what is considered, and lies entirely in this plane. Considering the totally polarized light propagating along the axis perpendicular to the molecular plane (helical axis), we assume that the direction of this axis coincides with the direction of the axis z of some Cartesian rectangular coordinate system. Axes x and y coincide with the main (fast and slow) axes of birefringence of input molecular plane (Fig. 1).

In such media, the motion in the direction perpendicular to the molecular planes the direction of anisotropy axis of each molecular layer returns (twisted) compared to the previous by some angle α_0 [3, 4]:

$$\alpha_0 = \frac{2\pi}{p} \tag{1}$$

where p – a step of helical structure of medium (the smallest distance between the planes with the same orientation of molecules). Then the molecular orientation of the plane in distance *z* from the input can be defined as:

$$\alpha = \alpha_0 z \tag{2}$$

Anisotropic properties of one molecular layer of such medium are described by the differential Mueller matrix [1]:

$$m = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\delta_0 \sin(2\alpha_0 z) \\ 0 & 0 & 0 & \delta_0 \cos(2\alpha_0 z) \\ 0 & \delta_0 \sin(2\alpha_0 z) & -\delta_0 \cos(2\alpha_0 z) & 0 \end{bmatrix} \tag{3}$$

This matrix is similar to the corresponding differential Mueller matrix for a homogeneous medium with linear birefringence [1]. The only difference between the matrices is (2), i. e. the angle α is a function of the coordinates *z*.

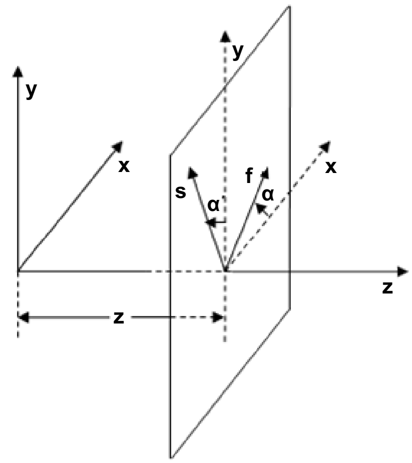

Fig. 1. Molecular plane of longitudinally inhomogeneous medium at a distance z from the input plane. f and s – directions of fast and slow axes of birefringence, which for incoming planes coincide with axes x and y coordinate systems

The evolution of Stokes parameters. Now we will study the evolution of the polarization state of waves propagating in a given class of media. As the example will choose a linearly polarized light, as it is the simplest and yet the most common case of using in polarimetry as a tool to study the anisotropy of the medium [12]. The Stokes vector of light in the case of linear polarization with azimuth of orientation of the electric field θ_{Inp} is following [4]:

$$S_{inp} = \begin{pmatrix} 1 & \cos 2\theta_{Inp} & \sin 2\theta_{Inp} & 0 \end{pmatrix}^T \tag{4}$$

The main equations describing the evolution of the Stokes vector (and hence the corresponding state of polarization) is the transport equation of light through anisotropic medium [1]:

$$\frac{dS}{dz} = mS \tag{5}$$

where S is a Stokes vector which describes the polarization state of the light, m is a differential Mueller matrix describing the anisotropic properties of medium (3). Vector differential equation (5) can be represented as a system of four scalar differential equations for the individual Stokes parameters S_1, S_2, S_3, S_4. Solving this system of four differential equations with initial conditions (4), the result can be represented as the ratio:

$$S_{1,2,3,4}(z) = f(m_{ij}, S_{1,2,3,4}(0), z); \overline{i,j} = 1,4 \tag{6}$$

where $S_{1,2,3,4}(z)$ – Stokes parameters of light at the output layer anisotropic medium with thickness z, $S_{1,2,3,4}(0)$ – Stokes parameters of light at the input layer of anisotropic medium (initial conditions (4)), m_{ij} – elements of the differential Mueller matrix.

The evolution of polarization state. Solutions are presented in the form (6) describes the evolution of the polarization state with coordinate z in the direction of light propagation in a medium whose anisotropy is given a particular view of the differential matrix m. At propagation of the light in a medium is very informative the evolution of parameters of the polarization ellipse as a function of z, i.e. azimuth θ_0 and angle of ellipticity e_O which are defined as follows [4]:

$$\begin{aligned} \theta_O &= \frac{1}{2} arctg\left[\frac{S_{out(3)}}{S_{out(2)}}\right], \\ e_O &= \frac{1}{2} arctg\left[\frac{S_{out(4)}}{\sqrt{S_{out(2)}^{\;2} + S_{out(3)}^{\;2}}}\right] \end{aligned} \tag{7}$$

Graphical interpretation of equation (7), describing the evolution of the azimuth and angle of ellipticity of the input polarization (4) are shown in Figure 2.

Analyzing Fig. 2 we can see that the azimuth is non-linear and non-periodic function of the coordinate z. Changing the azimuth at the minimum to the maximum value corresponds to the mathematical features of functions $\arctan$. The angle of ellipticity changing with coordinate z periodic and the period does not coincide with the period of helical structure of cholesteric p and equal:

$$Z = \frac{2\pi}{\sqrt{4\alpha_0^2 + \delta_0^2}} \tag{8}$$

in the case considered in Fig. 2 Z = 4,62 (μm). Moreover, the relation (8) can be obtained from the condition $\delta = \arccos\left(M_{44}\right) = 0$ [8].

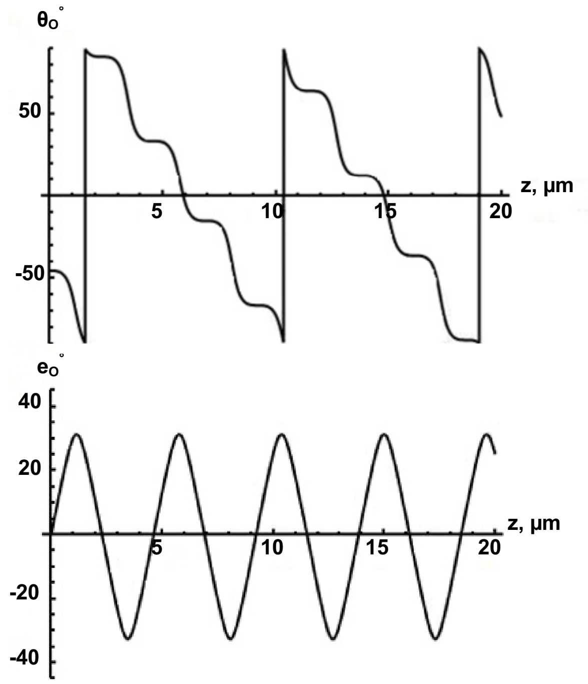

Fig. 2. Evolution of azimuth θ_O and angle ellipticity e_O of light with azimuth θ_{inp} = -0,8(rad) with z coordinates in longitudinally inhomogeneous medium with linear birefringence and parameters: δ_0 = 1,22 (rad/μm), α_0 = 0,3(rad/ μm)

The periodic changing of angle of ellipticity with coordinate's z can be explained by the fact that this class of media with changing z may exhibit properties of linear, circular or elliptical phase anisotropy. However, as is well known [2, 10] with the change of angle of ellipticity caused only a linear phase anisotropy. That is, the changing of the angle of ellipticity in this case is such as in the media with linear phase anisotropy. It should also be noted that both parameters changed similarly as in a homogeneous medium with elliptical birefringence in the case commensurate with parameters of linear and circular birefringence [10]. So it should be noted that this class of longitudinally inhomogeneous medium characterized by effective circular birefringence which can be determined by applying solutions of inverse problems to the corresponding integral Mueller matrix [1] (3):

$$\varphi^{eff} = \frac{ArcCos\left[-1 + 2Cos\left[z\alpha\right]^2 Cos\left[\frac{1}{2}z\sqrt{A}\right]^2 + \frac{8\alpha^2 Sin\left[z\alpha\right]^2 Sin\left[\frac{1}{2}z\sqrt{A}\right]^2}{A} + \frac{2\alpha Sin\left[2z\alpha\right] Sin\left[z\sqrt{A}\right]}{\sqrt{A}}\right]\left(\pm 2\alpha\sqrt{A}Cot\left[za\right] + ACot\left[\frac{1}{2}z\sqrt{A}\right]\right)}{2z\delta\sqrt{A}\sqrt{\frac{4\alpha^2 Cot\left[z\alpha\right]^2 - 4\alpha\sqrt{A}Cot\left[z\alpha\right]Cot\left[\frac{1}{2}z\sqrt{A}\right] + ACot\left[\frac{1}{2}z\sqrt{A}\right]^2 + \delta^2 Csc\left[2\alpha\right]^2}{\delta^2}}} \tag{9}$$

where $A = 4\alpha^2 + \delta^2$.

and privileged waves propagating in a given class of media as a homogeneous optically active medium "not feeling" linear birefringence, and the ratio (9) is simplified to the form:

$$\varphi^{eff} = 2\alpha_0 z \tag{10}$$

where φ^{eff} – effective circular birefringence.

Conclusions. It is shown that the changing of the polarization state of the wave propagating in longitudinally inhomogeneous medium with linear birefringence is similar to the corresponding law, for the case of a homogeneous medium with elliptical birefringence. The latter can be described as the emergence of an effective circular birefringence (optical activity) for the state of polarization of the incident wave. In the case of privileged waves propagation this class of media is equivalent to a homogeneous optically active medium with the appropriate size of the circular birefringence.

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ЕВОЛЮЦІЯ СТАНУ ПОЛЯРИЗАЦІЇ В ПОЗДОВЖНЬО НЕОДНОРІДНОМУ СЕРЕДОВИЩІ З ЛІНІЙНИМ ДВОПРОМЕНЕЗАЛОМЛЕННЯМ

Розглянуті та проаналізовані особливості формування анізотропії в поздовжньо неоднорідному середовищі з лінійним двопроменезаломленням. Була застосована еквівалентна модель представлення даного класу середовищ послідовною дією однакових фазових пластинок кожна з яких повернута на однаковий кут по відношенню до попередньої. Для представлення поляризаційних властивостей середовища був використаний диференційний матричний метод Мюллера. Було отримано еволюцію вектора Стокса на основі диференційної матриці Мюллера та векторного рівняння переносу. Отримано та вивчено еволюцію азимута та кута еліптичності в напрямку розповсюдження світла в даному класі середовищ. Визначено період зміни параметрів еліпса поляризації. Було проведено порівняння особливостей еволюції стану поляризації з відповідними особливостями в поздовжньо однорідному середовищі з еліптичним двопроменезаломленням. Визначено ефективне значення оптичної активності для довільного стану поляризації світла, що розповсюджується в середовищі. Були розглянуті особливості обертання площини поляризації превалюючи хвиль.

Ключові слова: *матриця Мюллера, двопроменезаломлення, азимут, кут еліптичності, поздовжньо неоднорідне середовище, стан поляризації.*

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ЭВОЛЮЦИЯ СОСТОЯНИЯ ПОЛЯРИЗАЦИИ В ПРОДОЛЬНО НЕОДНОРОДНОЙ СРЕДЕ С ЛИНЕЙНЫМ ДВУЛУЧЕПРЕЛОМЛЕНИЕМ

Рассмотрено и проанализировано особенности формирования анизотропии в продольно неоднородной среде с линейным двулучепреломлением. Была применена Эквивалентная модель представления сред данного класса последовательным действием фазовых пластинок каждая, из которых повернута относительно предыдущей на один и тот же угол. Для представления поляризационных свойств сред был использован дифференциальный матричный метод Мюллера. Было получено эволюцию вектора Стокса на основе дифференциальной матрицы Мюллера и векторного уравнения переноса. Получено и изучено эволюцию азимута и угла эллиптичности в направлении распространения света в данном классе сред. Определен период изменения параметров эллипса поляризации. Было осуществлено сравнения особенностей эволюции состояния поляризации с соответственными особенностями в продольно однородной среде с эллиптическим двулучепреломлением. Определено эффективное значения оптической активности для произвольного состояния поляризация света распространяющегося в среде. Были рассмотрены особенности вращения плоскости поляризации превалирующих волн.

Ключевые слова: *матрица Мюллера, двулучепреломление, азимут, угол эллиптичности, продольно неоднородные среды, состояния поляризации.*