Attention is drawn to the fact that in equiaxed nanosilicon films (film thickness < 70 nm) joint lines lie along different directions, in particular, [211], [321], [431], [110]. This correlates with the absence of texture in these films. At the same time, in fibrous films (thickness ≥ 70 nm) joint lines always coincide with the direction [110], which correlates with the presence of preferred orientation [110] in these films [7].

Conclusions

1. Triple and multiple joints of grain boundaries are observed in nanosilicon films. Crystallographic classification of multiple joints carried out.

2. As triple and multiple grain boundaries joints are divided in joints of general type and special joints.

3. There are several types of special grain boundaries joints, which differ in the number and mutual arrangement of special boundaries $\Sigma = 3^n$.

4. In films with equiaxed structure (thickness < 70 nm) joint lines lie along different directions, in particular, [211] [321] [431] and [110] that corresponds to a disordered film structure.

5. In films with fibrous structure (thickness ≥ 70 nm) joint lines coincide that correlates with the presence of texture [110] in these films.

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СПЕЦІАЛЬНІ СТИКИ ГРАНИЦЬ ЗЕРЕН У НАНОКРЕМНІЄВИХ ПЛІВКАХ З РІВНООСЬОВОЮ ТА ВОЛОКНИСТОЮ СТРУКТУРОЮ

Методами атомної силової мікроскопії досліджено с пеціальні стики границь зерен в нанокремнієвих плівках. Показано, що в плівках з рівноосьовою та волокнистою структурою стики відрізняються кількістю та взаємним розташуванням спеціальних границь $\Sigma = 3^n$ та границь загального типу.

Ключові слова: *нанокремнієві плівки; структура; стики границь зерен*

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СПЕЦИАЛЬНЫЕ СТЫКИ ГРАНИЦ ЗЕРЕН В НАНОКРЕМНИЕВЫХ ПЛЕНКАХ С РАВНООСНОЙ И ВОЛОКНИСТОЙ СТРУКТУРОЙ

Методами атомной силовой микроскопии исследованы стыки границ зерен в нанокремниевых пленках. Показано, что в пленках с равноосной и волокнистой структурой стыки отличаются количеством и взаимным расположением специальных границ $\Sigma = 3^n$ и границ общего типа.

Ключевые слова: *нанокремниевые пленки; структура; стыки границ зерен*

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THE MUELLER MATRIX STRUCTURE OF MEDIA WITH ORTHOGONAL EIGENPOLARIZATIONS

The Mueller matrix structure and relationships between its elements for media with orthogonal eigenpolarizations were studied. Relations has been derived were verified on basic types of anisotropy and mixtures of ones. It was shown that Mueller matrix of medium with orthogonal eigenpolarizations has less than twelve independent elements. In addition the conditions which determine the values of anisotropy parameters for the eigenpolarizations to be orthogonal have been derived and examined for several characteristic mixtures of anisotropies. In particular it was founded that the orthogonality of eigenpolarization is always possible in mixtures of four basic types of anisotropy by properly fitted birefringent part. Finally the symmetry of the Mueller matrix, resulted from eigenpolarizations orthogonality, was established and analyzed for optimal measurement.

Keywords: *Mueller matrix, eigenpolarizations orthogonality, parameters of anisotropy.*

Introduction. It is well known that in optics and electrodynamics the crystalline medium are characterized by the types of eigenpolarizations that this medium possesses. Eigenpolarizations are those polarization states of light that do not change when passing through a medium. The amplitude and the overall phase of the beam of light with an eigenpolarization do, however, change. These changes are described by the corresponding eigenvalues. In scope of this research we intend to study the conditions, under which homogeneous anisotropic medium has in general case the orthogonal eigenpolarizations (eigenmodes). It is known, that all four basic types of anisotropy, circular and linear birefringence and circular and linear dichroism, each taken separately, possess orthogonal eigenpolarizations. Generalized birefringence, i.e. the case of medium exhibiting linear and circular birefringence simultaneously, is characterized by unitary matrix model and has orthogonal eigenpolarizations. At the same time, simultaneous presence of dichroism and birefringence in a medium may lead to nonorthogonal eigenpolarizations. However, to the best of our knowledge, so far there has been no systematic study of conditions under which such medium possesses orthogonal eigenpolarizations. Ascertainment of generalized conditions for orthogonality of medium's eigenpolarizations allows determining the structure and symmetry of matrix model for such class of media.

Given input and scattering directions and wavelength of input radiation, medium anisotropy properties are completely described by 4x4 real Mueller matrix. The measurement of complete Mueller matrix oriented to the case when all 16 elements of the Mueller matrix are independent. However, in practice, all 16 elements may very frequently not be independent. Some are zero and some are identical to others, depending on the symmetry and certain properties of the studied medium. A typical example of such situation is the deterministic class of crystalline media [1, 3, 6, 7] with Mueller matrices consisting in general of 7 independent elements – degrees of freedom. Measuring all 16 elements of the Mueller matrix for this class of media one consequently makes more than 50% ''uninformative'' measurements. In practice the part of uninformative measurements is even higher. This means that determination of the symmetry of inner structure of Mueller matrix model of medium, that is, the determination of the number and location of independent, informative elements in matrix, is perspective way to increase the speed of measurements. Furthermore, analysis carried out by us earlier shows that the measurement of incomplete matrix is characterized by higher precision than that of complete matrix.

Thus, relying on model symmetry which will be determined in scope of intended research; it will be possible to determine the incomplete Mueller matrices which are sufficient for full description of media with orthogonal eigenpolarizations in general case and for solving of corresponding classes of the inverse problems. In its turn, this allows to determine the scheme of polarimeter which is optimally fit for the measurement of these structures of incomplete Mueller matrices. This would permit to increase both the speed and accuracy of polarimetric measurements. This has ultimate importance for imaging Mueller polarimetry as well.

Methods, Assumptions and Procedures. To describe the linear interaction of polarized radiation with the medium, the Jones and Mueller matrix methods, which is uniquely related in case of a homogeneous anisotropic media [2], are used.

When Jones matrix method is used than

$$\mathbf{E}^{\text{out}} = \mathbf{T} \cdot \mathbf{E}^{\text{in}}, \qquad (1)$$

where $\mathbf{E}^{\text{in(out)}}$ is the Jones vector of input (output) radiation, and $\mathbf{T}$ denotes the Jones matrix.

Jones matrix **T** (2x2 matrix with complex elements t_{mn}) describes anisotropic properties of homogeneous medium:

$$\mathbf{T} = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix}; \quad \text{where } t_{mn} = |t_{mn}| \exp(-i\varphi_{mn}). \qquad (2)$$

Eigenpolarizations χ of such matrix can be obtained as:

$$\chi_{1,2} = \frac{1}{2} \frac{t_{22} - t_{11} \pm \sqrt{(t_{22} - t_{11})^2 + 4t_{12} \cdot t_{21}}}{t_{21}}; \qquad (3)$$

where $\chi = E_x / E_y$ – complex variable [2]; $E_{x,y}$ – components of the Jones vector **E**.

For orthogonal eigenpolarizations the following relation:

$$\chi_1 \chi_2^* = -1, \qquad (4)$$

is satisfied.

In experimental studies the Mueller method is used for description of interaction between electromagnetic radiation and medium because it operates with intensities of radiation that can be directly measured.

Then, in scope of the Muller matrix method (1) can be rewritten as:

$$\mathbf{S}^{\text{out}} = \mathbf{M}\mathbf{S}^{\text{in}}, \qquad (5)$$

where $\mathbf{S}^{\text{in(out)}}$ – denotes input (output) Stokes vectors.

The definition of Stokes vector is follows:

$$\mathbf{S} = \left(I \;\; Ip\cos(2\theta)\cos(2\varepsilon) \;\; Ip\sin(2\theta)\cos(2\varepsilon) \;\; Ip\sin(2\varepsilon) \right)^T, \qquad (6)$$

where I – overall intensity of radiation; p – polarization degree; θ – azimuth and ε – ellipticity angle of polarization ellipse; T – transposing.

In accordance with (5) the Mueller matrix **M** is a 4x4 matrix with real elements

$$\mathbf{M} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{bmatrix}. \qquad (7)$$

This matrix, as well as Jones matrix, describes completely anisotropic properties of homogeneous medium for a given input and output (scattering) directions and wavelength of input radiation.

Direct solving of the spectral problem in scope of Mueller formalism, i.e., finding the conditions on Mueller matrix elements for eigenpolarizations to be orthogonal, is quite complicated task because of Mueller matrix dimension. However, this problem, as it was demonstrated in [8], can be solved in scope of the Jones formalism both in terms of matrix. Taking into account the fact that Mueller **M** and Jones **T** matrices for homogeneous medium are interconnected by relation

$$\mathbf{M} = \mathbf{A}(\mathbf{T} \otimes \mathbf{T}^*)\mathbf{A}^{-1}, \qquad (8)$$

where $*$ – conjugation; $\otimes$ – Kronecker product; and

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & i & -i & 0 \end{bmatrix},$$

all results, which are obtained for the Jones formalism, can be translated to the Mueller formalism with 8). Note that the main condition for that is the medium under consideration does not depolarize input radiation.

It is important to note that any 2x2 matrix could be called "Jones matrix", i.e., arbitrary 2x2 matrix with complex elements describes always a physical realizable transformation of polarization trough the (1). The same can not to be said about any 4x4 matrix with real elements. To be named as "Mueller matrix" this matrix has to meet an ample of requirements [4].

Due to above conditions the solving of spectral problem even for Mueller-Jones matrix (which is determined by (8)) case is generally difficult. We can write it in the following form:

$$\mathbf{M}\left(\nu_1^{-1}\mathbf{S}_1+\nu_2^{-1}\mathbf{S}_2\right)=\begin{pmatrix}1 & 0 & 0 & 0\end{pmatrix}^T . \quad (9)$$

Conditions on matrices elements. In [8] was established that for orthogonality of eigenpolarizations the following relations between amplitudes and phases of Jones matrix elements need to be satisfied:

$$|t_{12}|=|t_{21}| , \quad (10)$$

$$2\phi_{22-11}=\phi_{12}+\phi_{21}\pm n\pi , \quad (11)$$

$$2\phi_{21+12}=\phi_{12}+\phi_{21}\pm n\pi , \quad (12)$$

$$2\phi_{i(21-12)}=\phi_{12}+\phi_{21}\pm n\pi . \quad (13)$$

where $\varphi_{mn\pm kl}=\arg\left(t_{mn}\pm t_{kl}\right)$, $\varphi_{i(mn\pm kl)}=\arg\left(i(t_{mn}\pm t_{kl})\right)$.

Using interrelation (8) between Jones and Mueller methods, (10)-(12) and description of spectral problem as (9) we can also study symmetry of the Mueller matrix for medium with orthogonal polarizations.

In particular, from (8) it follows:

$$|t_{21}|=\sqrt{\frac{m_{11}-m_{21}+m_{12}-m_{22}}{2}} ,$$

$$|t_{12}|=\sqrt{\frac{m_{11}+m_{21}-m_{12}-m_{22}}{2}} ,$$

$$|t_{11}|=\sqrt{\frac{m_{11}+m_{21}+m_{12}+m_{22}}{2}} ,$$

$$|t_{22}|=\sqrt{\frac{m_{11}-m_{21}-m_{12}+m_{22}}{2}} , \quad (14)$$

$$\cos(\varphi_{12})=\frac{m_{13}+m_{23}}{2|t_{11}||t_{12}|};\quad \sin(\varphi_{12})=\frac{m_{14}+m_{24}}{2|t_{11}||t_{12}|};$$

$$\cos(\varphi_{21})=\frac{m_{31}+m_{32}}{2|t_{11}||t_{21}|};\quad \sin(\varphi_{21})=\frac{m_{41}+m_{42}}{2|t_{11}||t_{21}|};$$

$$\cos(\varphi_{22})=\frac{m_{33}+m_{44}}{2|t_{11}||t_{22}|};\quad \sin(\varphi_{22})=\frac{m_{43}-m_{34}}{2|t_{11}||t_{12}|}. \quad (15)$$

Here we assumed that all phases of Jones matrix elements (2) are normalized on phase of the first matrix element (i.e. $\varphi_{mn}\to\varphi_{mn}-\varphi_{11}$). Thus, in this case the phase of element t_{11} is $\varphi_{11}\to 0$.

Condition (4) for orthogonality of eigenpolarizations of medium in terms of Jones matrix elements can be transformed using (14) into relation:

$$m_{11}-m_{21}+m_{12}-m_{22}=m_{11}+m_{21}-m_{12}-m_{22} ,$$

or

$$m_{12}=m_{21} . \quad (16)$$

Considering (10)–(12) as equivalent $2\phi_{11+22}=2\phi_{(21+12)}=$ $=2\phi_{i(21+12)}=\phi_{12}+\phi_{21}\pm n\pi$ we can write:

$$\frac{\operatorname{Im}(t_{22}-t_{11})}{\operatorname{Re}(t_{22}-t_{11})}=\frac{\operatorname{Im}\left(i(t_{21}-t_{12})\right)}{\operatorname{Re}\left(i(t_{21}-t_{12})\right)}=\frac{\operatorname{Im}\left(t_{21}+t_{12}\right)}{\operatorname{Re}\left(t_{21}+t_{12}\right)} , \quad (17)$$

then, invoking (14) and (15) we'll get:

$$\frac{m_{13}+m_{23}-m_{31}-m_{32}}{m_{41}+m_{42}+m_{14}+m_{24}}=\frac{m_{41}+m_{42}-m_{14}-m_{24}}{m_{13}+m_{23}+m_{31}+m_{32}}= =\frac{m_{43}-m_{34}}{m_{11}+m_{21}+m_{12}+m_{22}-m_{33}-m_{44}} , \quad (18)$$

and

$$\left(m_{13}+m_{23}\right)^2-\left(m_{31}+m_{32}\right)^2=\left(m_{41}+m_{42}\right)^2-\left(m_{14}+m_{24}\right)^2 \quad (19)$$

After exploiting of relation (8) we tried to expand (9) in assumption that polarization degree of eigenpolarizations is p=1, i.e. $s_1^2=s_2^2+s_3^2+s_4^2$.

From (9) for normalized Stokes vectors we can write:

$$\mathbf{S}_1=\begin{pmatrix}1 & s_2 & s_3 & s_4\end{pmatrix}^T,\ \mathbf{S}_2=\begin{pmatrix}1 & -s_2 & -s_3 & -s_4\end{pmatrix}^T \quad (20)$$

As the Stokes vectors (20) are eigenvectors of Mueller matrix $\mathbf{M}$ it is right that:

$$\mathbf{M}\cdot\mathbf{S}_{1,(2)}=\begin{pmatrix} m_{11}\pm m_{12}s_2\pm m_{13}s_3\pm m_{14}s_4\\ m_{21}\pm m_{22}s_2\pm m_{23}s_3\pm m_{24}s_4\\ m_{31}\pm m_{32}s_2\pm m_{33}s_3\pm m_{34}s_4\\ m_{41}\pm m_{42}s_2\pm m_{43}s_3\pm m_{44}s_4\end{pmatrix}= =\underbrace{\left(m_{11}\pm m_{12}s_2\pm m_{13}s_3\pm m_{14}s_4\right)}_{\nu_{1,(2)}}\mathbf{S}_{1,(2)} , \quad (21)$$

From (21) it follows:

$$\begin{cases} m_{21}+m_{22}s_2+m_{23}s_3+m_{24}s_4-s_2\,\nu_1=0\\ m_{31}+m_{32}s_2+m_{33}s_3+m_{34}s_4-s_3\,\nu_1=0\\ m_{41}+m_{42}s_2+m_{43}s_3+m_{44}s_4-s_4\,\nu_1=0\\ m_{21}-m_{22}s_2-m_{23}s_3-m_{24}s_4-s_2\,\nu_2=0\\ m_{31}-m_{32}s_2-m_{33}s_3-m_{34}s_4-s_3\,\nu_2=0\\ m_{41}-m_{42}s_2-m_{43}s_3-m_{44}s_4-s_4\,\nu_3=0\end{cases} \quad (22)$$

Combining (22) we obtain:

$$\begin{aligned} m_{21}&=s_2(m_{12}s_2+m_{13}s_3+m_{14}s_4),\\ m_{31}&=s_3(m_{12}s_2+m_{13}s_3+m_{14}s_4),\\ m_{41}&=s_4(m_{12}s_2+m_{13}s_3+m_{14}s_4).\end{aligned} \quad (23)$$

Multiplying (23) by s_2, s_3, s_4, respectively, and, taking into account that for completely polarized light $\left(s_2^2+s_3^2+s_4^2\right)^{1/2}=1$, we have:

$$s_2\left(m_{21}-m_{12}\right)+s_3\left(m_{31}-m_{13}\right)+s_4\left(m_{41}-m_{14}\right)=0 \quad (24)$$

$$m_{21}\left(m_{21}-m_{12}\right)+m_{31}\left(m_{31}-m_{13}\right)+m_{41}\left(m_{41}-m_{14}\right)=0 , \quad (25)$$

$$\frac{m_{12}^2+m_{13}^2+m_{14}^2}{m_{12}m_{21}+m_{13}m_{31}+m_{14}m_{41}}=1 . \quad (26)$$

As for homogeneous media it is true that [1,4]

$$m_{12}^2+m_{13}^2+m_{14}^2=m_{21}^2+m_{31}^2+m_{41}^2 . \quad (27)$$

From (26) and (27) we can gat that:

$$\left(m_{12}-m_{21}\right)^2+\left(m_{13}-m_{31}\right)^2+(m_{14}-m_{41})^2=0 . \quad (28)$$

From (28) it results that:

$$m_{12}=m_{21},\quad m_{13}=m_{31},\quad m_{14}=m_{41}. \quad (29)$$

From (30) it follows that for complete description of anisotropy of medium with orthogonal eigenpolarizations the knowledge of second, third and fourth columns(rows) of Mueller matrix is sufficient. This result can be used during optimization of polarimeter for studying of given polarization class of media.

Conditions on anisotropy. To derive orthogonality condition in terms of anisotropy parameters we use the matrix model of arbitrary homogeneous anisotropy that has recently been presented in [6] and combines both mathematical generality and physical interpretability:

$$\mathbf{T}(\varphi,\Delta,\alpha,R,P,\gamma)=\mathbf{T}^{CP}(\varphi)\mathbf{T}^{LP}(\Delta,\alpha)\mathbf{T}^{CA}(R)\mathbf{T}^{LA}(P,\gamma) , \quad (30)$$

where: value of linear dichroism is in the range $P \in [0;1]$ and the azimuth of maximum transition is in the range $\gamma \in \left[-\frac{\pi}{2};\frac{\pi}{2}\right]$; value of circular dichroism $R \in [-1;1]$; value of linear birefringence $\Delta \in [0;2\pi]$ with it's fast axis orientation $\alpha \in \left[-\frac{\pi}{2};\frac{\pi}{2}\right]$; value of optical activity is $\varphi \in [-\pi;\pi]$.

Let write next useful relations:

$$t_{22} - t_{11} = (a_1 + ib_1)\exp\left(-i\frac{\Delta}{2}\right), \quad (31)$$

$$t_{21} - t_{12} = (a_2 + ib_2)\exp\left(-i\frac{\Delta}{2}\right), \quad (32)$$

$$t_{21} + t_{12} = (a_3 + ib_3)\exp\left(-i\frac{\Delta}{2}\right). \quad (33)$$

In compliance with (31) parameters a_i and b_i have the following form:

$$a_1 = (1+P)R\sin\frac{\Delta}{2}\sin(2\alpha-\varphi) - (1-P)\cos\frac{\Delta}{2}\cos(2\gamma-\varphi),$$

$$b_1 = (1-P)R\cos\frac{\Delta}{2}\sin(2\gamma-\varphi) - (1+P)\sin\frac{\Delta}{2}\cos(2\alpha-\varphi),$$

$$a_2 = (1-P)R\sin\frac{\Delta}{2}\cos(2\alpha-2\gamma-\varphi) - (1+P)\cos\frac{\Delta}{2}\sin(\varphi),$$

$$b_2 = (1+P)R\cos\frac{\Delta}{2}\cos(\varphi) - (1-P)\sin\frac{\Delta}{2}\sin(2\alpha-2\gamma-\varphi),$$

$$a_3 = (1+P)R\sin\frac{\Delta}{2}\cos(2\alpha-\varphi) + (1-P)\cos\frac{\Delta}{2}\sin(2\gamma-\varphi),$$

$$b_3 = (1-P)R\cos\frac{\Delta}{2}\cos(2\gamma-\varphi) + (1+P)\sin\frac{\Delta}{2}\sin(2\alpha-\varphi), \quad (34)$$

From (34) and (10) can be written:

$$a_2a_3 + b_2b_3 = 0; \quad (35)$$

(11)–(13) are transformed to:

$$a_1b_3 - a_3b_1 = 0, \quad (36)$$

$$a_1b_2 + a_2b_1 = 0. \quad (37)$$

(35)–(37) can be used for determination of anisotropy parameters. In particular we had got that for media have orthogonal eigenpolarizations its dichroism can be arbitrary but birefringence need to satisfy:

$$\mathrm{tg}\frac{\Delta}{2} = -\frac{1}{2R}\frac{P-1}{P+1}\left[\mathrm{A} - \sqrt{\mathrm{A}^2 + 4R^2}\right],$$

$$\phi = \alpha - \gamma + \frac{1}{2}\arccos\left[\cos 2(\alpha-\gamma)\frac{R^2-1}{R^2+1}\right] - \frac{\pi}{2}, \quad (38)$$

where $\mathrm{A} = (1-R^2)\sin 2(\alpha-\gamma)$.

It turns convenient to determine the parameters Δ and ϕ, while other four parameters are fixed in a range of definition. For example:

$$\mathbf{T}(9.4°, 51.8°, 84°, 0.7, 0.2, 25°) = \begin{bmatrix} 0.803+0.61i & 0.457+0.098i \\ 0.188+0.428i & 0.261+0.168i \end{bmatrix} \quad (39)$$

It is easy to see that eigenpolarizations $\chi_{1,2}$ of the Jones matrix (39) are orthogonal

$$\chi_1 = 0.446+0.228i; \quad \chi_2 = -1.777-0.908i; \quad \chi_1\chi_2^* = -1. \quad (40)$$

Taking to account (8) and (30), in terms of Mueller matrix we can obtain:

$$\mathbf{M}(\varphi,\Delta,\alpha,R,P,\gamma) = \begin{bmatrix} 1.000 & 0.593 & 0.707 & 0.361 \\ 0.593 & 0.436 & 0.394 & 0.155 \\ 0.707 & 0.356 & 0.568 & 0.260 \\ 0.361 & 0.229 & 0.197 & 0.238 \end{bmatrix} \quad (41)$$

Eigenpolarizations of the Mueller matrix $\mathbf{M}$:

$$\mathbf{S}_1 = (1 \;\; 0.599 \;\; 0.713 \;\; 0.365)^T;$$

$$\mathbf{S}_2 = (1 \;\; -0.599 \;\; -0.713 \;\; -0.365)^T;$$

$$\mathbf{S}_1 + \mathbf{S}_2 = (1 \;\; 0 \;\; 0 \;\; 0)^T. \quad (42)$$

Summary. Summarizing obtained results we can formulate next conclusions. Anisotropy of homogeneous media with orthogonal eigenpolarizations has limitations. In particular when we use for modeling such media an equivalence theorem in form (30) eigenpolarizations will be orthogonal always if linear Δ and circular birefringence φ will have values as (38), whereas other four anisotropy parameters (R, P,γ,α) can take an arbitrary values. In other words a media with orthogonal eigenpolarizations can demonstrate all six possible anisotropy behaviors as separately as simultaneously. One exclusion need to be taking into account: when the medium has simultaneously only two dichroismes (circular and linear) it can't have orthogonal eigenpolarizations under no conditions.

Limited anisotropy of media with orthogonal eigenpolarizations leads to symmetry in Mueller matrix (see (29)) and to other more difficult relations between elements on those matrices. These relations can be useful for simplify an analysis of properties of objects and utilized for optimization of measurement procedure. In last context we have used an equality of first column and row of Mueller matrix of studied objects to reduction measurement time with saving accuracy on polarimeter with four LC-transducers [5]. If we could to remove limitation on shift of LC transducers than measurement error limit could be reduced to δm_{13}=1.9%, in comparison δm_{16}=2.1%.

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СТРУКТУРА МАТРИЦІ МЮЛЛЕРА СЕРЕДОВИЩ З ОРТОГОНАЛЬНИМИ ПОЛЯРИЗАЦІЯМИ ВЛАСНИХ ХВИЛЬ

В роботі досліджено зв'язок між елементами матриці Мюллера однорідних анізотропних середовищ з ортогональними поляризаціями власних хвиль. Отримані співвідношення перевірено для випадку середовищ з простими та змішаними типами анізотропії. Встановлено, що ортогональність поляризацій власних хвиль зменшує кількість незалежних елементів матриці Мюллера до 12. Досліджено зв'язок між параметрами анізотропії такого типу середовищ та встановлено, що їх анізотропія може утворюватись поєднанням всіх чотирьох елементарних типів дихроїзму та двопроменезаломлення, проте, з обмеженим співвідношенням. Зокрема, показано, що за довільних значень дихроїзму, міри лінійного та циркулярного двопроменезаломлення можуть приймати тільки конкретні значення. З огляду на встановлену симетрію матриці Мюллера, вимірювання тільки необхідної кількості її елементів дозволяє скоротити час вимірювання зберігши або підвищивши точність.

Ключові слова: *Матриця Мюллера, ортогональність власних поляризацій, параметри анізотропії.*

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СТРУКТУРА МАТРИЦЫ МЮЛЛЕРА СРЕД С ОРТОГОНАЛЬНЫМИ ПОЛЯРИЗАЦИЯМИ СОБСТВЕННЫХ ВОЛН

В работе исследована связь между элементами матрицы Мюллера однородных анизотропных сред с ортогональными поляризациями собственных волн. Полученные соотношения проверены для случая сред с простыми и смешанными типами анизотропии. Установлено, что ортогональность поляризаций собственных волн уменьшает число независимых элементов матрицы Мюллера до 12. Исследована связь между параметрами анизотропии такого типа сред и установлено, что их анизотропия может образовываться сочетанием всех четырех элементарных типов дихроизма и двулучепреломления, однако, с ограниченным соотношением. В частности, показано, что при произвольных значениях дихроизма, значения линейного и циркулярного двулучепреломления могут принимать только конкретные значения. Учитывая установленную симметрию матрицы Мюллера, измерения только необходимого количества ее элементов позволяет сократить время измерений, сохранив или повысив точность.

Ключевые слова: *Матрица Мюллера, ортогональность собственных поляризаций, параметры анизотропии.*

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DIGITAL PROCESSING OF MEDICAL IMAGE WITH MAPLE

Typical medical images are most often in gray scale and their matrices of pixel intensities are convenient for digital processing. Recent versions of popular computer mathematics system Maple have special program packages for this aim. We are going to illustrate the possibilities of these packages by example of usual medical image. Our case study presents the discrete dyadic wavelet analysis (DWT), block thresholding of detail coefficients and evaluations of quality for reconstructed image. We investigated in addition the dependences of quality parameters on the digital filters used within DWT procedure.

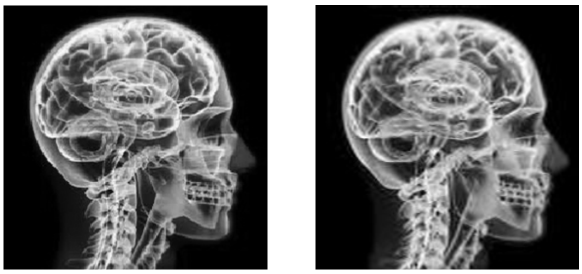
Keywords: *medical image, discrete wavelet analysis, dyadic decomposition, block thresholding, factors of image quality, image compression*

Introduction. Digital processing is important for many applications that involve huge data storage, transmission and retrieval such as for multimedia, documents, videoconferencing, and medical imaging. Uncompressed images require considerable storage capacity. The objective of image compression technique is to reduce redundancy of the image data in order to be able to store or transmit data in an efficient form. This results in the reduction of file size and allows more images to be stored in a given amount of disk or memory space [1].

Medical image compression may be based on wavelet decomposition. It can produce notably better medical image results compared to the compression results that are generated by Fourier transform based methods such as the discrete cosine transform (DCT) used by JPEG [6].

The aim of this paper is to illustrate how discrete dyadic wavelet analysis (DWT) and block thresholding may be applied in medical image compression to reduce the volume of data.

Experimental. If we consider the quadratic image with sizes $N \times N = 2^m$, we can get at least $m = 2\log_2 N$ consecutive Wavelet-Transformations (so called the scale levels). In practice, usually from two to six scale levels are enough for useful analysis [3]. We were starting with image size $N = 224$ and $m = 2$ (see Fig. 1).



a b

Fig. 1. A medical image before (A) and after processing (B)

Two-dimensional wavelet transform can be represented as a composition of one-dimensional wavelet decompositions in rows and columns of the matrix image.

Image after single-level wavelets transformation in rows and columns is divided into four frequency blocks with different interpretation (Fig. 2) [5]:

1. LL-blok is top left block of approximation coefficients, which were filtered out in the analysis (decomposition) with two low-pass filters. The block contains a copy of the primary image with half resolution.