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TIME-LIKE SINGULARITIES IN GENERAL RELATIVITY

We present some results obtained from study the properties of naked time-like singularities in General Relativity and quantum effects in their vicinity.

Представлені деякі результати дослідження властивостей голих часоподібних сінгулярностей в загальній теорії відносності та квантових ефектів у їх околі.

This is a brief review of the properties of the naked time-like singularities in the General Relativity, partially based on our results obtained from late 1970. First of all let us note that singularity in this paper means the part of space-time where some curvature invariants diverge not in the δ-function way. So, we drop out all pathological and directional singularities, as well as conical singularities. Some directional singularities will appear later, but only as a by-product. If an infinitely close to the singularity hypersurface is space-like, we deal with the space-like singularity, e.g. cosmological one or Schwarzschild singularity. If the hypersurface is time-like, the singularity is also time-like one. There are two possibilities. There is an even number of horizons around a time-like singularity, so time-like singularity is under these horizons. This object looks like a black hole. An example of such object is the Reisner-Nordström black hole with two horizons. Observer cannot see a singularity from the outside.

But if there is no horizon at all, this time-like singularity are called a naked singularity. There are a lot of exact solutions of the Einstein equations with such singularities. But there is the problem with this type of singularity. Distant observer can see it. It could inject radiation, matter and information. So one cannot set a Cauchy problem for our space-time (if there exists at least one naked singularity) without knowledge about the boundary condition on it. To discard this problem, Penrose proposed the so-called the Cosmic Censorship Principle [10]. It states that all singularities produced by a collapse must be inside the horizons. Theoretically, it tolerates naked singularities produced by the Big Bang. But during the inflation they must fly away from the visible part of the Universe. So we get the practical conclusion: there are no naked singularities in our Universe inside a cosmological horizon. Nevertheless the Cosmic Censorship Principle is just a hypothesis.

By now we have no real possibility to prove or disprove it by studying a process of the collapse. So we proposed to study the properties of the naked singularities in the General Relativity in order to get some conclusions in this connection. Some results are described below.

A note about the cosmological constant. Nowadays we believe in the existence of the cosmological constant or dark energy, which acts similarly to it. Its influence on a metric vanishes in the vicinity of time-like singularities. So, one can neglect cosmological constant when studying naked singularities.

We begin from types, examples and hierarchy of naked singularities. The simplest example with the metric depending only on one spatial coordinate x is the spatial Kasner solution [3]

$$ds^2 = -dx^2 + x^{2p_1}dt^2 - x^{2p_2}dy^2 - x^{2p_3}dz^2 \tag{1}$$

with one negative and two positive Kasner indices p_i satisfying conditions

$$p_1 + p_2 + p_3 = 1, \qquad p_1^2 + p_2^2 + p_3^2 = 1. \tag{2}$$

In the paper [4] it was identified as the metric around the infinitely long straight thread with the constant linear mass density. Let us start from the Weil metric describing static axial-symmetric space-times:

$$\begin{aligned} &ds^2 = e^{\nu}dt^2 - \rho^2 e^{-\nu}d\phi^2 - e^{\gamma-\nu}\left(d\rho^2 + dz^2\right), \quad \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\nu}{\partial\rho}\right) + \frac{\partial^2\nu}{\partial z^2} = 0, \\ &\frac{\partial\gamma}{\partial z} = \rho\frac{\partial\nu}{\partial\rho}\frac{\partial\nu}{\partial z}, \qquad \frac{\partial\gamma}{\partial z} = \frac{\rho}{2}\left[\left(\frac{\partial\nu}{\partial\rho}\right)^2 - \left(\frac{\partial\nu}{\partial z}\right)^2\right]. \end{aligned} \tag{3}$$

Here a function $\nu(\rho, z)$ is a harmonic axial-symmetric function. In the conditional flat space with the cylindrical coordinate set ρ, ϕ, z it describes the Newtonian potential of some axial-symmetric mass distribution. If this source is the thread ρ=0 with the constant linear mass density μ, then one can get after the transformation $x = \rho^{\mu^2-\mu+1}$ the Kasner metric (1) with

$$p_1 = \frac{\mu}{\mu^2 - \mu + 1}, \qquad p_2 = \frac{1-\mu}{\mu^2 - \mu + 1}, \qquad p_3 = \frac{\mu^2 - \mu}{\mu^2 - \mu + 1}. \tag{4}$$

But if μ>1, we get some problems. To solve them let's consider the case of the source in the form of a finite thread ρ=0 with the constant linear mass density μ and the length L. Using the oblate spheroid coordinate set *v*, *u*, φ, one can simply obtain the function *v*=2μ ln(tanh(*v*/2)) and, after substitution into the Weil metric, get the Zipoy-Voorhees metric [11,12].

$$ds^2 = \text{th}^{2\mu}\frac{v}{2}dt^2 - \frac{L^2}{4}\text{th}^{-2\mu}\frac{v}{2}\text{sh}^2 v\left[\left(1+\frac{\cos^2 u}{\text{sh}^2 v}\right)^{1-\mu^2}\left(dv^2+du^2\right)+\cos^2 u d\phi^2\right]. \qquad (5)$$

Naked singularity corresponds to v=0. But what is the type of this singularity? Is it line, or point-like or something else? This problem was investigated in [13] by using diagrams, describing the simplest properties of the space-time. It was shown that case $\mu<0$ corresponds to a point-like singularity with negative mass, case $0<\mu<1$ corresponds to a line-like singularity with positive mass. In the most complex case $\mu>1$ we deal with the new type of singularity, which was named paradox-like in [14]. It has positive mass. If $\mu\geq 2$ there are two additional directional singularities on the "ends" v=0, $u=\pm\pi/2$. In this case space-time (6) has three different spatial infinities.

The metric (6) was generalized for the case $\mu\neq$const, but all generalized solutions are approximate ones near a singularity. If μ depends on z, we get Weil singularities [13], on z and t - "simple line sources" [2], on z, t, φ - generalized spatial Kasner metric [5]

$$ds^2 = -dx^2 + (x^{2p_1} l_i l_k - x^{2p_2} m_i m_k - x^{2p_3} n_i n_k) dx^i dx^k. \qquad (6)$$

All these solutions are approximate at $x\to 0$ or $r\to 0$. Their properties were analyzed in [13]. It was shown that all types of naked singularities described by these solutions are point-like singularity with negative mass or line-like with positive mass or paradox-like. But point-like singularities repulse collapsing matter and cannot be formed by collapse. Paradox-like singularities must have a linear density exceeding the critical value and also cannot be formed by collapse. Only line-like singularities could be considered as a candidate to break the Cosmic Censorship Principle.

But all these solutions are not enough general. The general solution of Einstein equations near time-like singularity was found and analyzed in the paper [9]. It is an oscillating solution (naturally, approximate one) very similar to the well-known Belinsky-Khalatnikov-Lifshitz (BKL) solution near space-like singularities [1]. In order to get a general solution near the arbitrary singularity it has to be matched with BKL solution. This was done in the paper [5].

An influence of non-gravitational fields was analyzed in the papers [14, 9] and some other ones. Only scalar fields can "kill" a general oscillatory metric. In this case the generalized spatial Kasner metric (6) with all positive indices is the most general solution near a naked singularity. In the case of real collapse we have to take into account quantum effects. If a classical collapse leads to the formation of the naked singularity, it could cause a strong radiation due to the quantum pair's production and changing of a vacuum polarization. Its backreaction could slow the collapse in such a way, that it forms a black hole instead of naked singularity. So we need to calculate the mass loss due to quantum radiation during a formation of the naked singularity. A simple model with massive shell, shrinking up to the Plank length was used. Conclusions: mass loss is very small at the formation of the linear singularity [7], but very large at the formation of the Reisner-Nordström singularity with $Q>M$ [8].

Also the already formed naked singularities could be "dressed up" due to quantum radiation and its backreaction. E.g. naked Kerr singularity with $a>M$ acts in the same way [15]. We have the Kerr metric with mass M and angular momentum $J=Ma$. If $a>M$ it describes the naked singularity. Quantum particles production near it leads to loss of the angular momentum J and its mass M. The ratio a decreases faster than M, so the naked Kerr singularity can turn into a rotating black hole. Estimation of a the time of "dressed up" in the Plank units is

$$T_{dress} \approx 50M\alpha^9 \exp\left(2\alpha^2\right)\ln\left(\frac{M^2}{\alpha}\right), \qquad \alpha = \frac{a}{M} > 1. \qquad (7)$$

During the time since the Big Bang, the singularities with solar mass can "dress up" if $1+\varepsilon < \alpha < 4$, where $\varepsilon \approx 10^{-20}$.

Coming back to the Cosmic Censorship Principle we get some brief conclusions. Some types of naked singularities cannot be formed by a collapse. Point-like singularities repulse collapsing matter. Paradox-like singularities must have a linear density exceeding the critical value. General oscillatory singularities must have a strong influence of the quantum effects. We cannot estimate it, but it is possible that it formation is also forbidden due to these effects. But line-like singularities have no such problems. So, in order to prove or disprove The Cosmic Censorship Principle one has to study a collapse with the formation of a line-like naked singularity.

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