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Б.В. Бондарев[1], *д.ф.-м.н., професор*
О.Є.Сосницький[2], *аспірант*

Оцінки для ймовірности нерозорення страхової компанії у дискретний час

У даній роботі будуються оцінки знизу для ймовірності нерозорення страхової компанії, яка еволюціонує у дискретному часі. Розглянуто випадкові впливи: незалежні, пов'язані мартінгальною залежністю, а також впливи, які перемішуються "по Ібрагімову" або перемішуються "по Розенблатту".

Ключові слова: ймовірність нерозорення, нерівність Розенталя, перемішування "по Ібрагімову".

[1,2]Донецький національний університет, 83001, Донецьк, вул. Університетська, 24

B.V. Bondarev[1], *Professor*
O.E.Sosnytskyy[2], *post-graduate student*

Estimates for non-ruin probability of the insurance company in the discrete time model

In this paper lower estimations for the non-ruin probability of insurance company in the discrete time model are built. The following random effects have been considered: independent, connected by martingale dependence, and also effects, mixed "according to Ibragimov" or mixed "according to Rosenblatt".

Key Words: non-ruin probability, Rosenthal inequality, mixing "according to Ibragimov".

[1,2]*Donetsk National University, 83001, Donetsk, 24 Universitetskaya st.*

E-mails: [1]bondarev.mart@gmail.com, [2]yahooomg@gmail.com

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1 Introduction

In the monograph by A.Melnikov [1, p.64] the following model of evolution of insurance company's capital is presented: at initial (zero) time $n = 0$ the capital of insurance company is equal to some value $x > 0$, at the same time the company invests this capital into the bank account with simple interest rate $r > 0$, during unit time, at the moment $t = 1$ there will be already the sum $x(1+r)$ on the bank account. It is considered that during the time from $t = 0$ to $t = 1$ the premium of value c and the claim, described by Z_1, which must be payed at the time $n = 1$, are received.

So at the time $n = 1$ the capital of the insurance company will be equal to $R_1 = x\,(1+r) + c - Z_1$, if the way of insurance company's functioning doesn't change, then at the time $t = n$ we will have $R_n = R_{n-1}\,(1+r) + c - Z_n$, where the sequence of independent identically distributed positive random variables Z_n has distribution law $F\,(x) = P\,(Z_1 < x)$. In the study [1] recurrence formula for the probability of ruin of the insurance company for the finite time were deduced, the formula for the finding of solution was proposed. In the research by B.Bondarev, T. Zhmyhova [2], [3] infinite time interval of insurance company's functioning is considered and values of premiums C_n can be also random, then, it is obvious that $R_n = R_{n-1}\,(1+r) + C_n - Z_n$. In these studies it is proposed to look for the solution in the form of functional series of a definite kind, with coefficients which can be estimated by recurrence relation. A very interesting application of similar scheme of capital functioning to research of the probability of bank's ruin was given by N.Gonchar [4]. It should be noted that in all mentioned above cases Markov scheme was observed - random effects were expected to be independent. In the given article the following problem is observed: suppose that insurance company has m branches in different regions of the country, where deposit interest rate, premium flow and claims can be various, besides this flows are not necessarily led by sequence of independent random variables - this sequence can be martingale, or can even satisfy any of the conditions of weak dependence. Using inequalities of Kolmogorov, Kolmogorov-Hajek-Renyi, inequalities of S.Utev for the sum of weakly dependant random variables, lower estimates are deduced for probability of non-ruin of insurance company. From our point of view this approach was for the first time used in the study [5].

2 Problem statement and main results

Lets have $m \geqslant 1$ regions, where there are insurance company's branches. In the region with a number $k, 1 \leqslant k \leqslant m$ a branch has initial capital $x^{(k)}, k = 1, ..., m$, at the time interval from $l-1$ to l deposit interest rate is $r_{l-1}^k \geqslant r_k > 0, 1 \leqslant k \leqslant m, l = 1, ..., n$, to the k-th branch of the company at the time interval from $l-1$ to l random premiums $C_l^k, l = 1, ..., n, k = 1, ..., m$ with distribution function $G_l^k(x) = P\left(C_l^k < x\right), M\left|C_l^k\right| < +\infty$, $l = 1, ..., n, k = 1, ..., m$ and claims $Z_l^k, l = 1, ..., n, k = 1, ..., m$ with the distribution function $F_l^k(y) = P\left(Z_l^k < y\right), M\left|Z_l^k\right| < +\infty, l = 1, ..., n, k = 1, ..., m$ are received. The capital evolution at the time interval $[l-1, l), l = 1, ..., n$ will be described by recurrence relation $R_l^k = R_{l-1}^k\left(1 + r_{l-1}^k\right) + C_l^k - Z_l^k, l = 1, ..., n, k = 1, ..., m$, then

$$R_n^k = x^{(k)} \prod_{l=0}^{n-1}\left(1 + r_l^k\right) + \sum_{l=1}^{n}\left(C_l^k - Z_l^k\right)\prod_{\nu=l}^{n-1}\left(1 + r_\nu^k\right) = x^{(k)} \prod_{l=0}^{n-1}\left(1 + r_l^k\right) + \prod_{l=0}^{n-1}\left(1 + r_l^k\right) \times \sum_{l=1}^{n}\left(C_l^k - Z_l^k\right)\prod_{\nu=0}^{l-1}\left(1 + r_\nu^k\right)^{-1} \quad (1)$$

Let $\bar{C}_l^k = MC_l^k, \tilde{C}_l^k = C_l^k - \bar{C}_l^k, \bar{Z}_l^k = MZ_l^k, \tilde{Z}_l^k = Z_l^k - \bar{Z}_l^k, l = 1, ..., n, k = 1, ..., m$ then from (1) we obtain

$$P\left(R_l^k \geqslant 0, l = 1, ..., n, k = 1, ..., m\right) = P\left(x^{(k)}\prod_{j=0}^{l-1}\left(1 + r_j^k\right) + \prod_{j=0}^{l-1}\left(1 + r_j^k\right) \times \sum_{j=1}^{l}\left(C_j^k - Z_j^k\right)\prod_{\nu=0}^{j-1}\left(1 + r_\nu^k\right)^{-1} \geqslant 0, l = 1, ..., n, k = 1, ..., m\right) =$$

$$= P\left(x^{(k)} + \sum_{j=1}^{l}\left(C_j^k - Z_j^k\right)\prod_{\nu=0}^{j-1}\left(1 + r_\nu^k\right)^{-1} \geqslant 0, l = 1, ..., n, k = 1, ..., m\right) =$$

$$= P\left(\sum_{j=1}^{l}\left(\tilde{Z}_j^k - \tilde{C}_j^k\right)\prod_{\nu=0}^{j-1}\left(1 + r_\nu^k\right)^{-1} \leqslant x^{(k)} + \sum_{j=1}^{l}\left(\bar{C}_j^k - \bar{Z}_j^k\right)\prod_{\nu=0}^{j-1}\left(1 + r_\nu^k\right)^{-1}, l = 1, ..., n, k = 1, ..., m\right) \geqslant$$

$$\geqslant 1 - \sum_{k=1}^{m} P\left(\left[\sum_{j=1}^{l}\left(\tilde{Z}_j^k - \tilde{C}_j^k\right)\prod_{\nu=0}^{j-1}\left(1 + r_\nu^k\right)^{-1}\right] > x^{(k)} + \sum_{j=1}^{l}\left(\bar{C}_j^k - \bar{Z}_j^k\right)\prod_{\nu=0}^{j-1}\left(1 + r_\nu^k\right)^{-1}, \exists l \in \{1, ..., n\}\right) \geqslant 1 -$$

$$-\sum_{k=1}^{m}\left(1 - P\left(\left[\sum_{j=1}^{l}\left(\tilde{Z}_j^k - \tilde{C}_j^k\right)\prod_{\nu=0}^{j-1}\left(1 + r_\nu^k\right)^{-1}\right] \leqslant x^{(k)} + \sum_{j=1}^{l}\left(\bar{C}_j^k - \bar{Z}_j^k\right)\prod_{\nu=0}^{j-1}\left(1 + r_\nu^k\right)^{-1}, l = 1, ..., n\right)\right) \quad (2)$$

Theorem 2.1. *Suppose that at the time interval from $l-1$ to l deposit interest rate is $r_{l-1}^k \geqslant r_k > 0, 1 \leqslant k \leqslant m, l = 1, \ldots$. Let Z_j^k, C_j^k be such that for each fixed $k, k = 1, ..., m$ random variables $Z_j^k - C_j^k, j = 1, 2, ...$ are independent, $M\left(\tilde{Z}_j^k - \tilde{C}_j^k\right)^2 \leqslant \sigma_k^2 < \propto, \bar{C}_j^k \geqslant \bar{Z}_j^k, j = 1, ...; k = 1, ..., m$, $\left|M\left(\tilde{Z}_j^k - \tilde{C}_j^k\right)^\tau\right| \leqslant \frac{\tau!}{2}M\left(\tilde{Z}_j^k - \tilde{C}_j^k\right)^2 H_k^{\tau-2}, \tau \geqslant 2$ is integer, $H_k > 0$ is constant, $k = 1, ..., m$. If $B_\propto^k = \sum_{j=1}^{\propto} M\left(\tilde{Z}_j^k - \tilde{C}_j^k\right)^2 \prod_{\nu=0}^{j-1}\left(1 + r_\nu^k\right)^{-2}$, then for $x^{(k)} > \sqrt{2B_\propto^k}$ the following estimates are correct:*

$$P\left(R_l^k \geqslant 0, l = 1, \ldots k = 1, ..., m\right) \geqslant \geqslant 1 - 2\sum_{k=1}^{m} \exp\left\{-\frac{\left[x^{(k)} - \sqrt{2B_\propto^k}\right]^2}{4B_\propto^k}\right\} \quad (3)$$

where

$$0 \leqslant x^{(k)} - \sqrt{2B_\propto^k} \leqslant \frac{B_\propto^k}{H_k}, k = 1, ..., m, \quad (4)$$

$$P\left(R_l^k \geqslant 0, l = 1, ... \; k = 1, ..., m\right) \geqslant$$
$$\geqslant 1 - 2\sum_{k=1}^{m} \exp\left\{-\frac{x^{(k)} - \sqrt{2B_\propto^k}}{4H_k}\right\} \quad (5)$$

where

$$0 \leqslant x^{(k)} - \sqrt{2B_\propto^k} \geqslant \frac{B_\propto^k}{H_k}, k = 1, ..., m. \quad (6)$$

Proof. Suppose that

$$B_n^k = \sum_{j=1}^{n} M\left(\tilde{Z}_j^k - \tilde{C}_j^k\right)^2 \prod_{\nu=0}^{j-1}\left(1 + r_\nu^k\right)^{-2},$$

then we have the following estimate

$$P\left(R_l^k \geqslant 0, l = 1, ..., n, k = 1, ..., m\right) \geqslant$$
$$\geqslant 1 - \sum_{k=1}^{m} P\left(\sup_{1\leqslant l\leqslant n}\left[\sum_{j=1}^{l}\left(\tilde{Z}_j^k - \tilde{C}_j^k\right) \times \right.\right.$$
$$\left.\left.\times \prod_{\nu=0}^{j-1}\left(1 + r_\nu^k\right)^{-1}\right] > x^{(k)}\right) \geqslant$$
$$\geqslant 1 - 2\sum_{k=1}^{m} P\left(\sum_{j=1}^{n}\left(\tilde{Z}_j^k - \tilde{C}_j^k\right) \times \right.$$
$$\left.\times \prod_{\nu=0}^{j-1}\left(1 + r_\nu^k\right)^{-1} > x^{(k)} - \sqrt{2B_n^k}\right) \quad (7)$$

which is a consequence of estimates from theorem 12 from [6, p. 68]. Then using estimates of theorem 17 from [6, p. 73] we obtain

$$P\left(R_l^k \geqslant 0, l = 1, ..., n, k = 1, ..., m\right) \geqslant$$
$$\geqslant 1 - 2\sum_{k=1}^{m} \exp\left\{-\frac{\left[x^{(k)} - \sqrt{2B_n^k}\right]^2}{4B_n^k}\right\} \quad (8)$$

where

$$0 \leqslant x^{(k)} - \sqrt{2B_n^k} \leqslant \frac{B_n^k}{H_k}, k = 1, ..., m, \quad (9)$$

$$P\left(R_l^k \geqslant 0, l = 1, ..., n, k = 1, ..., m\right) \geqslant$$
$$\geqslant 1 - 2\sum_{k=1}^{m} \exp\left\{-\frac{x^{(k)} - \sqrt{2B_n^k}}{4H_k}\right\} \quad (10)$$

where

$$0 \leqslant x^{(k)} - \sqrt{2B_n^k} \geqslant \frac{B_n^k}{H_k}, k = 1, ..., m. \quad (11)$$

Since $B_n^k < B_{n+1}^k$ is increasing bounded sequence, it has the limit B_∞^k. Tending n to infinity in (8), (9), (10), (11) and taking into account that $\lim\limits_{n\to+\infty} P\left(R_l^k \geqslant 0, l = 1, ..., n, k = 1, ..., m\right) = P\left(R_l^k \geqslant 0, l = 1, ...; k = 1, ..., m\right)$ we get the assertion of the theorem 2.1.□

In the theorem 2.1 the existence of exponential moment of random variables $\left(\tilde{Z}_j^k - \tilde{C}_j^k\right), j = 1, 2, ...$ was supposed. This requirement can be weaken, for instance, in assumption of existence of the $p - th$ moment, $p > 2$.

Theorem 2.2. *Suppose that at the time interval from $l-1$ to l deposit interest rate is $r_{l-1}^k \geqslant r_k > 0, 1 \leqslant k \leqslant m, l = 1, ...$. Let Z_j^k, C_j^k be such that for each fixed $k, k = 1, ..., m$ random variables $Z_j^k - C_j^k, j = 1, 2, ...$ are independent, $\bar{C}_j^k \geqslant \bar{Z}_j^k, j = 1, ...; k = 1, ..., m,$. Assume, that for $p > 1$ the following estimates are correct:*

$$M\left|\tilde{Z}_j^k - \tilde{C}_j^k\right|^{2p} \leqslant \sigma_k^{2p} < +\infty,$$
$$j = 1, ...; k = 1, ..., m, \quad (12)$$

Then the following estimate takes place

$$P\left(R_l^k \geqslant 0, l = 1, ...; k = 1, ..., m\right) \geqslant$$
$$\geqslant 1 - 2M(\theta - 1)^{2p}\sum_{k=1}^{m}\left[x^{(k)}\right]^{-2p} \times$$
$$\times \max\left(\sum_{j=1}^{\propto} M\left(\tilde{Z}_j^k - \tilde{C}_j^k\right)^{2p}\prod_{\nu=0}^{j-1}\left(1 + r_\nu^k\right)^{-2p},\right.$$
$$\left.\left[\sum_{j=1}^{\propto} M\left(\tilde{Z}_j^k - \tilde{C}_j^k\right)^{2}\prod_{\nu=0}^{j-1}\left(1 + r_\nu^k\right)^{-2}\right]^p\right) \quad (13)$$

where θ is Poisson random variable with the parameter 1.

Proof. From (7), using Kolmogorov's inequality we obtain the estimate

$$P\left(R_l^k \geqslant 0, l = 1, ..., n, k = 1, ..., m\right) \geqslant$$
$$\geqslant 1 - \sum_{k=1}^{m} P\left(\sup_{1\leqslant l\leqslant n}\left[\sum_{j=1}^{l}\left(\tilde{Z}_j^k - \tilde{C}_j^k\right) \times \right.\right.$$
$$\left.\left.\times \prod_{\nu=0}^{j-1}\left(1 + r_\nu^k\right)^{-1}\right] > x^{(k)}\right) \geqslant 1 - 2\sum_{k=1}^{m}\left[x^{(k)}\right]^{-2p} \times$$
$$\times M\left[\sum_{j=1}^{n}\left(\tilde{Z}_j^k - \tilde{C}_j^k\right)\prod_{\nu=0}^{j-1}\left(1 + r_\nu^k\right)^{-1}\right]^p \quad (14)$$

Then, using Rosenthal's inequality with exact constant from the work [7], we obtain the following estimate

$$P\left(R_l^k \geqslant 0, l = 1, ..., n, k = 1, ..., m\right) \geqslant$$
$$\geqslant 1 - 2M(\theta - 1)^{2p} \sum_{k=1}^{m} \left[x^{(k)}\right]^{-2p} \times$$
$$\times \max\left(\sum_{j=1}^{n} M\left(\tilde{Z}_j^k - \tilde{C}_j^k\right)^{2p} \prod_{\nu=0}^{j-1}\left(1 + r_\nu^k\right)^{-2p},\right.$$
$$\left.\left[\sum_{j=1}^{n} M\left(\tilde{Z}_j^k - \tilde{C}_j^k\right)^{2} \prod_{\nu=0}^{j-1}\left(1 + r_\nu^k\right)^{-2}\right]^p\right) \quad (15)$$

Tending n to infinity in (16) we will obtain (14). Theorem 2.2 is proved.□ It is easy to see that the following assertion is correct

Corollary 2.1. *Suppose that at the time interval from $l-1$ to l deposit interest rate is $r_{l-1} = r_k > 0, 1 \leqslant k \leqslant m, l = 1, ...$ Let Z_j^k, C_j^k be such that for each fixed $k, k = 1, ..., m$ random variables $Z_j^k - C_j^k, j = 1, 2, ...$ are independent, $\bar{C}_j^k \geqslant \bar{Z}_j^k, j = 1, ...; k = 1, ..., m,$.*

Assume, that for $p > 1$ the following is valid

$$M\left|\tilde{Z}_j^k - \tilde{C}_j^k\right|^{2p} \leqslant \sigma_k^{2p} < +\infty,$$
$$M\left|\tilde{Z}_j^k - \tilde{C}_j^k\right|^{2} \leqslant \sigma_k^{2} < +\infty,$$
$$j = 1, ...; k = 1, ..., m.$$

Then the following estimate takes place

$$P\left(R_l^k \geqslant 0, l = 1, ...; k = 1, ..., m\right) \geqslant$$
$$\geqslant 1 - 2M(\theta - 1)^{2p} \sum_{k=1}^{m} \left[x^{(k)}\right]^{-2p} \times \quad (16)$$
$$\times \max\left(\frac{\sigma_k^{2p}}{1 - [1 + r_k]^{-2p}}, \left[\frac{\sigma_k^2}{1 - [1 + r_k)]^{-2}}\right]^p\right)$$

where θ - Poisson random variable with the parameter 1.

Since sequence $S_n^k = \sum_{j=1}^{n}\left(\tilde{Z}_j^k - \tilde{C}_j^k\right)\prod_{\nu=0}^{j-1}\left(1 + r_\nu^k\right)^{-1}$ is a martingale not only for independent variables $\left(\tilde{Z}_j^k - \tilde{C}_j^k\right), j = 1, 2, ...$ but also for variables which are connected only by martingale dependence, it makes sense to raise the question of getting estimates of the form (14) also for this case, in other words, to try to weaken the requirement about the independence of $(\tilde{Z}_j^k - \tilde{C}_j^k), j = 1, 2, ...$ considerably, replacing it by martingale's requirement for $(\tilde{Z}_j^k - \tilde{C}_j^k), j = 1, ..., n$.

Theorem 2.3. *Suppose that at the time interval from $l-1$ to l deposit interest rate is $r_{l-1}^k \geqslant r_k > 0, 1 \leqslant k \leqslant m, l = 1, ...,$ if the sequence $(\tilde{Z}_j^k - \tilde{C}_j^k), j = 1, 2, ...$ is a martingale for each fixed $1 \leqslant k \leqslant m$, and for $p > 1$ the following is valid*

$$M\left|\tilde{Z}_j^k - \tilde{C}_j^k\right|^{2p} \leqslant \sigma_k^{2p} < +\infty,$$
$$j = 1, ...;, k = 1, ..., m$$

Then the following estimate takes place

$$P\left(R_l^k \geqslant 0, l = 1, ...; k = 1, ..., m\right) \geqslant$$
$$\geqslant 1 - 2\sum_{k=1}^{m}\left[x^{(k)}\right]^{-2p} \times$$
$$\times \sum_{j=1}^{\infty} M\left(\tilde{Z}_j^k - \tilde{C}_j^k\right)^{2p} \prod_{\nu=0}^{j-1}\left(1 + r_\nu^k\right)^{-p} \times$$
$$\times\left[\sum_{j=1}^{\infty}\prod_{\nu=0}^{j-1}\left(1 + r_\nu^k\right)^{-\frac{p}{2p-1}}\right]^{2p-1} \quad (17)$$

Proof. Since the sequence $\left\{S_l^k\right\}, l = 1, 2, ...; 1 \leqslant k \leqslant m$ is in this case a martingale, then using Kolmogorov's inequality we obtain

$$P\left(R_l^k \geqslant 0, l = 1, ..., n, k = 1, ..., m\right) \geqslant$$
$$\geqslant 1 - 2\sum_{k=1}^{m}\left[x^{(k)}\right]^{-2p} \times$$
$$\times M\left[\sum_{j=1}^{n}\left(\tilde{Z}_j^k - \tilde{C}_j^k\right)\prod_{\nu=0}^{j-1}\left(1 + r_\nu^k\right)^{-1}\right]^{2p} \quad (18)$$

Next, since in the conditions of the theorem the following estimate takes place

$$M\left[\sum_{j=1}^{n}\left(\tilde{Z}_j^k - \tilde{C}_j^k\right)\prod_{\nu=0}^{j-1}\left(1 + r_\nu^k\right)^{-1}\right]^{2p} \leqslant$$
$$\leqslant \sum_{j=1}^{n} M\left(\tilde{Z}_j^k - \tilde{C}_j^k\right)^{2p}\prod_{\nu=0}^{j-1}\left(1 + r_\nu^k\right)^{-p} \times$$
$$\times\left[\sum_{j=1}^{n}\prod_{\nu=0}^{j-1}\left(1 + r_\nu^k\right)^{-\frac{p}{2p-1}}\right]^{2p-1}, \quad (19)$$

tending n to infinity in (18) and (19), we will obtain (17). Theorem 2.3 is proved.□

Corollary 2.2. *Suppose that at the time interval from $l-1$ to l deposit interest rate is $r_{l-1} \geqslant r_k > 0, 1 \leqslant k \leqslant m, l = 1, ...,$ let Z_j^k, C_j^k be such that for each fixed $k, k = 1, ..., m$ random variables $\tilde{Z}_j^k - \tilde{C}_j^k, j = 1, 2, ...$ are connected by martingale dependence,$\bar{C}_j^k \geqslant \bar{Z}_j^k, j = 1, ...; k = 1, ..., m,$ and for $p > 1$ the following is valid*

$$M\left|\tilde{Z}_j^k - \tilde{C}_j^k\right|^{2p} \leqslant \sigma_k^{2p} < +\infty,$$
$$j = 1, ...;, k = 1, ..., m$$

Then the following estimate takes place

$$P\left(R_l^k \geqslant 0, l = 1, ...; k = 1, ..., m\right) \geqslant$$
$$\geqslant 1 - 2\sum_{k=1}^{m}\left[x^{(k)}\right]^{-2p} \times$$
$$\times \frac{\sigma_k^{2p}}{\left(1 - [1 + r(k)]^{-p}\right)\left(1 - [1 + r(k)]^{-p/(2p-1)}\right)^{(2p-1)}}$$

There is another interesting statement. Since

$$P\left(R_l^k \geqslant 0, l = 1, ..., n, k = 1, ..., m\right) \geqslant$$
$$\geqslant 1 - \sum_{k=1}^{m}\left(1 - P\left(\left|\sum_{j=1}^{l}\left(\tilde{Z}_l^k - \tilde{C}_l^k\right) \times \right.\right.\right.$$
$$\left. \times \prod_{\nu=0}^{j-1}\left(1 + r_\nu^k\right)^{-1}\right| \leqslant x^{(k)} +$$
$$\left.\left. + \sum_{j=1}^{l}\left(\bar{C}_j^k - \bar{Z}_j^k\right)\prod_{\nu=0}^{j-1}\left(1 + r_\nu^k\right)^{-1}, l = 1, ..., n\right)\right) \tag{20}$$

the following theorem takes place.

Theorem 2.4. *Suppose that at the time interval from $l-1$ to l deposit interest rate is $r_{l-1}^k \geqslant r_k > 0, 1 \leqslant k \leqslant m, l = 1, ...$ and suppose that the sequence $\left\{\tilde{Z}_l^k - \tilde{C}_l^k\right\}, l = 1, ..., n$ forms square-integrable martingale for each fixed $k, k = 1, ..., m,$ and the following is valid*

$$M\left|\tilde{Z}_j^k - \tilde{C}_j^k\right|^2 \leqslant \sigma_k^2 < +\infty,$$
$$j = 1, ...;, k = 1, ..., m$$

Then the following estimate takes place

$$P\left(R_l^k \geqslant 0, l = 1, ... k = 1, ..., m\right) \geqslant$$
$$\geqslant 1 - \sum_{k=1}^{m}\sum_{l=1}^{\infty}\left[\left(x^{(k)} + \right.\right.$$
$$\left.\left. + \sum_{j=1}^{l}\left(\bar{C}_j^k - \bar{Z}_j^k\right)\prod_{\nu=0}^{j-1}\left(1 + r_\nu^k\right)^{-1}\right)\right]^{-2} \times$$
$$\times \left[M\left(\tilde{Z}_l^k - \tilde{C}_l^k\right)^2 \prod_{\nu=0}^{l-1}\left(1 + r_\nu^k\right)^{-2}\right] \tag{21}$$

Proof. Using Kolmogorov-Hajek-Renyi's inequality [8, p.218], for the function $y = x^2$ from (20) we obtain

$$P\left(R_l^k \geqslant 0, l = 1, ... k = 1, ..., m\right) \geqslant$$
$$\geqslant 1 - \sum_{k=1}^{m}\sum_{l=1}^{n}\left[\left(x^{(k)} + \right.\right.$$
$$\left.\left. + \sum_{j=1}^{l}\left(\bar{C}_j^k - \bar{Z}_j^k\right)\prod_{\nu=0}^{j-1}\left(1 + r_\nu^k\right)^{-1}\right)\right]^{-2} \times$$
$$\times \left[M\left(\tilde{Z}_l^k - \tilde{C}_l^k\right)^2 \prod_{\nu=0}^{l-1}\left(1 + r_\nu^k\right)^{-2}\right] \tag{22}$$

Tending n to infinity in (22), we obtain (21). Theorem 2.4 is proved.□

Earlier we observed the case of independent random effects and effects connected by martingale dependence. It also makes sense to consider random effects which are connected by any properties of weak dependence. We shall restrict ourselves to the case of u.s.m. - uniform strong mixing - mixing "according to Ibragimov" or φ-mixing and the case of s.m. - strong mixing - mixing "according to Rosenblatt" or α-mixing. Let's recall the definition of $\alpha(\tau), \varphi(\tau)$. Centered sequence $\tilde{Z}_j^k - \tilde{C}_j^k$ for each fixed $1 \leqslant k \leqslant m$ satisfies the condition of uniform strong mixing (u.s.m.) if

$$\sup_{0 \leqslant t < +\infty}\ \sup_{A \in \Im_0^t, B \in \Im_{t+\tau}^+} |P(B/A) - P(B)| =$$
$$= \varphi(\tau) \to 0, \tau \to +\infty$$

here $\Im_0^t$ is σ-algebra, generated by the sequence

$$\tilde{Z}_j^k - \tilde{C}_j^k, j = 1, ..., t$$

$\Im_{t+\tau}^{+\infty}$ is σ-algebra, generated by the sequence

$$\tilde{Z}_j^k - \tilde{C}_j^k, j = t + \tau, ...$$

$\varphi(\tau)$ is called the coefficient of mixing.

Centered sequence $\tilde{Z}_j^k - \tilde{C}_j^k$ for each fixed $1 \leqslant k \leqslant m$ satisfies the condition of strong mixing (s.m.) if

$$\sup_{0\leqslant t<+\infty} \sup_{A\in\Im_0^t, B\in\Im_{t+\tau}^+} |P(B\cap A) - P(A)P(B)| = \\ = \alpha(\tau) \to 0, \tau \to +\infty$$

here $\Im_0^t$ is σ-algebra, generated by the sequence

$$\tilde{Z}_j^k - \tilde{C}_j^k, j = 1, ..., t$$

$\Im_{t+\tau}^{+\infty}$ is σ-algebra, generated by the sequence

$$\tilde{Z}_j^k - \tilde{C}_j^k, j = t+\tau, ...$$

$\alpha(\tau)$ is called the coefficient of mixing.

Denote $\bar{S}_n = \max_{1\leqslant t\leqslant n} |S_t|, S_t = \sum_{i=1}^{t} \eta_i$, where $\{\eta_k\}, k = 1, 2, ...$ satisfies the condition of (s.m.). In this case the following estimates take place (see [11], p. 67). Suppose that

$$j(t) = 2\min\{k \in N : 2k \geqslant t\},$$
$$b(\alpha, t, \delta) = \sum_{k=0}^{+\infty} \alpha^{\frac{\delta}{\delta+j(t)}}(k)(k+1)^{j(t)-2},$$
$$\varepsilon(\alpha, t, \delta) = \sum_{k=0}^{+\infty} \alpha^{\frac{\delta}{(t\delta+t)}}(k), \tag{23}$$
$$L_t(n, \delta) = \sum_{k=1}^{n} \left(M|\eta_k|^{t+\delta}\right)^{\frac{t}{t+\delta}},$$
$$(D_n(\delta))^2 = L_2(n, \delta)$$

The following inequalities take place (see [11],p. 67):

$$P\left\{\bar{S}_n^k \geqslant x\right\} \leqslant x^{-t} c_2 \varepsilon(\alpha, t, \delta)(b(\alpha, t, \delta) + \\ + \varepsilon^t(\alpha, t, \delta))\left(L_t(n, \delta) + (D_n(\delta))^t\right), t \geqslant 2 \tag{24}$$

And if $\alpha(k) \leqslant Aq^k, k = 0, 1, 2, ... 0 \leqslant q < 1, t \geqslant 2$, then

$$P\left\{\bar{S}_n^k \geqslant x\right\} \leqslant x^{-t} c_3 L_t(n, \delta) + \\ + c_4 \exp\left\{-c_5\left(\frac{x}{D_n(\delta)}\right)^{1/3}\right\}, t \geqslant 2 \tag{25}$$

Denote $\eta_j^k = (Z_j^k - C_j^k)\prod_{\nu=0}^{j-1}\left(1 + r_\nu^k\right)^{-1}, S_t^k = \sum_{j=1}^{t} \eta_j^k$, $\bar{S}_n^k = \max_{1\leqslant t\leqslant n}\left|S_t^k\right|$

Theorem 2.5. *Suppose that at the time interval from $l-1$ to l deposit interest rate is $r_{l-1}^k \geqslant r_k > 0, 1 \leqslant k \leqslant m, l = 1, ...$ and suppose that for each fixed $1 \leqslant k \leqslant m$ the sequence $\tilde{Z}_j^k - \tilde{C}_j^k, j = 1, 2, ...$ satisfies the condition of (s.m.) with the coefficient of mixing $\alpha(\tau)$ such that all functions are defined in (23) for $t = 2p, p > 1$, and the following is valid*

$$M\left|\tilde{Z}_j^k - \tilde{C}_j^k\right|^{2p+\delta} \leqslant \sigma_k^{2p+\delta} < +\infty, \\ j = 1, ...; k = 1, ..., m$$

Then the following estimate takes place

$$P\left(R_l^k \geqslant 0, l = 1, ... \ k = 1, ..., m\right) \geqslant 1- \\ - c_2\varepsilon(\alpha, 2p, \delta)\left(b(\alpha, 2p, \delta) + \varepsilon^{2p}(\alpha, 2p, \delta)\right) \times \\ \times \left(L_{2p}(\propto, \delta, k) + (D_\propto(\delta, k))^{2p}\right)\sum_{k=1}^{m}\left[x^{(k)}\right]^{-2p} \tag{26}$$

Here

$$L_{2p}(\propto, \delta, k) = \sum_{j=1}^{\propto}\left(M\left|\eta_j^k\right|^{2p+\delta}\right)^{\frac{2p}{2p+\delta}} = \\ = \sum_{j=1}^{\propto}\left(M\left|\tilde{Z}_j^k - \tilde{C}_j^k\right|^{2p+\delta}\right)^{\frac{2p}{2p+\delta}}\prod_{\nu=0}^{j-1}\left(1 + r_\nu^k\right)^{-2p},$$

$$(D_\propto(\delta, k))^2 = L_2(\propto, \delta, k) \tag{27}$$

Proof. Using the estimate (24) we obtain

$$P\left(R_l^k \geqslant 0, l = 1, ..., n, \ k = 1, ..., m\right) \geqslant 1- \\ - c_2\varepsilon(\alpha, 2p, \delta)\left(b(\alpha, 2p, \delta) + \varepsilon^{2p}(\alpha, 2p, \delta)\right) \times \\ \times \left(L_{2p}(n, \delta, k) + (D_n(\delta, k))^{2p}\right)\sum_{k=1}^{m}\left[x^{(k)}\right]^{-2p} \tag{28}$$

Here

$$L_{2p}(n, \delta, k) = \sum_{j=1}^{n}\left(M\left|\eta_j^k\right|^{2p+\delta}\right)^{\frac{2p}{2p+\delta}} = \\ = \sum_{j=1}^{n}\left(M\left|\tilde{Z}_j^k - \tilde{C}_j^k\right|^{2p+\delta}\right)^{\frac{2p}{2p+\delta}}\prod_{\nu=0}^{j-1}\left(1 + r_\nu^k\right)^{-2p},$$

$$(D_n(\delta, k))^2 = L_2(n, \delta, k) \tag{29}$$

Tending n to infinity in (28) and (29), we will obtain (26). Theorem 2.5 is proved.□

Theorem 2.6. *Suppose that at the time interval from $l-1$ to l deposit interest rate is $r_{l-1}^k \geqslant r_k > 0, 1 \leqslant k \leqslant m, l = 1, ...$ and suppose that for each fixed $1 \leqslant k \leqslant m$ the sequence $\tilde{Z}_j^k - \tilde{C}_j^k$ satisfies the condition of (s.m.) with the coefficient of mixing $\alpha(\tau) \leqslant Aq^\tau, 0 \leqslant q < 1$ such that all functions are defined in (23) for $t = 2p, p > 1$. Then the following is valid*

$$P\left(R_l^k \geqslant 0, l = 1, ... \ k = 1, ..., m\right) \geqslant \geqslant 1 - \sum_{k=1}^{m} \left(\left[x^{(k)}\right]^{-2p} c_3 L_{2p}(\propto, k, \delta) + + c_4 \exp\left\{ -c_5 \left(\frac{x^{(k)}}{D_\propto(\delta, k)} \right)^{1/3} \right\} \right) \quad (30)$$

Proof. Using the estimation (24) we obtain

$$P\left(R_l^k \geqslant 0, l = 1, .., n \ k = 1, ..., m\right) \geqslant \geqslant 1 - \sum_{k=1}^{m} \left(\left[x^{(k)}\right]^{-2p} c_3 L_{2p}(n, k, \delta) + + c_4 \exp\left\{ -c_5 \left(\frac{x^{(k)}}{D_n(\delta, k)} \right)^{1/3} \right\} \right) \quad (31)$$

Tending n to infinity in (31), we will obtain (30). Theorem 2.6 is proved.□

Suppose that sequence $\{\eta_j\}, j = 1, 2, ..., n$ satisfies the condition of (u.s.m.) with the coefficient of mixing $\varphi(\tau)$, let $\bar{S}_n = \max\limits_{1 \leqslant k \leqslant n} \left| \sum\limits_{i=1}^{k} \eta_i \right|$ and the following conditions are satisfied

$$L_t(n) = \sum_{j=1}^{n} M |\eta_j|^t < +\infty, \ (D_n)^2 = L_2(n),$$

$$a(\varphi, t) = \sum_{k=0}^{+\infty} \varphi^{1/j(t)}(k) (k+1)^{j(t)-2} < +\infty,$$

$$\varphi(0) = 1, t \geqslant 2, \quad (32)$$

then the following inequalities are valid (see [11, p.67]):

$$P\left\{\bar{S}_n \geqslant x\right\} \leqslant c_2 a(\varphi, t) a(\varphi, 2) \times \times \left(L_t(n) + (D_n)^t\right) x^{-t}, t \geqslant 2 \quad (33)$$

where c_2 depends only on t ,

$$P\left\{\bar{S}_n \geqslant x\right\} \leqslant c_3 x^{-t} L_t(n) + + c_4 \exp\left(-c_5 \left({}^x/_{D_n} \right)^{1/3} \right), t \geqslant 2 \quad (34)$$

where $\varphi(k) \leqslant Aq^k, 0 \leqslant q < 1, c_3, c_4, c_5$ depends on A, t, q

Theorem 2.7. *Suppose that at the time interval from $l-1$ to l deposit interest rate is $r_{l-1}^k \geqslant r_k > 0, 1 \leqslant k \leqslant m, l = 1, ...$ and suppose that for each fixed $1 \leqslant k \leqslant m$ the sequence $\tilde{Z}_j^k - \tilde{C}_j^k$ satisfies the condition of (u.s.m.) with the coefficient $\varphi(\tau)$ such that following functions are defined*

$$L_{2p}(\propto, k) = = \sum_{j=1}^{\propto} M \left| \tilde{Z}_j^k - \tilde{C}_j^k \right|^{2p} \prod_{\nu=0}^{j-1} \left(1 + r_\nu^k\right)^{-2p} < +\infty,$$

$$a(\varphi, t) = \sum_{k=0}^{+\infty} \varphi^{1/j(t)}(k) (k+1)^{j(t)-2} < +\infty,$$

$$\varphi(0) = 1, p \geqslant 1, \left(D_\propto^k\right)^2 = L_2(\propto, k) \quad (35)$$

Then the following estimate takes place

$$P\left(R_l^k \geqslant 0, l = 1, ... \ k = 1, ..., m\right) \geqslant \geqslant 1 - c_2 a(\varphi, 2p) a(\varphi, 2) \times \times \sum_{k=1}^{m} \left[x^{(k)}\right]^{-2p} \left(L_{2p}(\propto, k) + \left(D_\propto^k\right)^{2p} \right) \quad (36)$$

Proof. Under the conditions of theorem the following estimate takes place

$$P\left(R_l^k \geqslant 0, l = 1, ..., n, \ k = 1, ..., m\right) \geqslant \geqslant 1 - c_2 a(\varphi, 2p) a(\varphi, 2) \times \times \sum_{k=1}^{m} \left[x^{(k)}\right]^{-2p} \left(L_{2p}(n, k) + \left(D_n^k\right)^{2p} \right) \quad (37)$$

where

$$L_{2p}(n, k) = = \sum_{j=1}^{n} M \left| \tilde{Z}_j^k - \tilde{C}_j^k \right|^{2p} \prod_{\nu=0}^{j-1} \left(1 + r_\nu^k\right)^{-2p} < +\infty,$$

$$a(\varphi, t) = \sum_{k=0}^{+\infty} \varphi^{1/j(t)}(k) (k+1)^{j(t)-2} < +\infty,$$

$$\varphi(0) = 1, p \geqslant 1, \left(D_n^k\right)^2 = L_2(n, k), \quad (38)$$

Tending n to infinity in (37) and (38) we will obtain (36). Theorem 2.7 is proved.□

Theorem 2.8. *Suppose that at the time interval from $l-1$ to l deposit interest rate is $r_{l-1}^k \geqslant r_k > 0, 1 \leqslant k \leqslant m, l = 1, \ldots$ and suppose that for each fixed $1 \leqslant k \leqslant m$ the sequence $\tilde{Z}_j^k - \tilde{C}_j^k$ satisfies the condition of (u.s.m.) with the coefficient of mixing $\varphi(k) \leqslant Aq^k, 0 \leqslant q < 1$ such that all functions are defined in (35) for $t = 2p, p > 1$, then the following estimate takes place*

$$P\left(R_l^k \geqslant 0, l = 1, \ldots k = 1, \ldots, m\right) \geqslant \geqslant 1 - \sum_{k=1}^{m}\left(\left[x^{(k)}\right]^{-2p} c_3 L_{2p}(\propto, k) + +c_4 \exp\left(-c_5\left(x^{(k)}/D_{\propto}^k\right)^{1/3}\right)\right) \quad (39)$$

Proof. Under the conditions of theorem the following estimate takes place

$$P\left(R_l^k \geqslant 0, l = 1, \ldots, n\ k = 1, \ldots, m\right) \geqslant \geqslant 1 - \sum_{k=1}^{m}\left(\left[x^{(k)}\right]^{-2p} c_3 L_{2p}(n, k) + +c_4 \exp\left(-c_5\left(x^{(k)}/D_{n}^k\right)^{1/3}\right)\right) \quad (40)$$

Tending n to infinity in (40) we will obtain (39). Theorem 2.8 is proved.□

3 Conclusion

Established lower estimates for the non-ruin probability of insurance company (meaning the probability that non of its branches has ruined during the time from $t = 0$ to $\propto$) as a function of the initial capital, allow us to say about acceptable reliability of conclusions in classical scheme - in the case of independent effects, in the case of random effects connected by martingale dependence, and also in the cases of effects with weak dependence - (u.s.m.) and (s.m.).

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