УДК 519.9

Rukhaia Kh.[1], prof., Tibua L., Chankvetadze G., Mikhanadze G.

An Algorithm of the Classification of Unranked Propositional Logic Formulas

Formulas of propositional logic are proved as algorithmic processes. It is established that all these processes can be completed and their final values are one of the following halting constants: T (a valid formula), F (an inconsistent formula) and S (an indefinite formula).

Key words: Unranked logic, Classification of formulas, Directive operator of the process.

[1] VIAM, Tbilisi State University and Sokhumi State University, Georgia,
e-mail: khimuri.rukhaia@viam.sci.tsu.ge

Х.М. Рухая[1], проф., Л.М. Тибуа, Г.О. Чанкветадзе, Г.М. Миканадзе

Об одном алгоритме классификации формул безранговой логики высказываний

В работе рассматривается безранговая логика высказываний, каждая формула которой рассматривается, как определенный алгоритмический процесс стратегиями T, F и $T+F$. Установлено, что все алгоритмические процессы завершаются и ответом является либо тождественно истинная формула, либо тождественно ложная формула, либо строго выполнимая формула.

Ключевые слова: безранговая логика, классификация формул, оператор управляющий процессом.

[1] Тбилисский Государственный Университет, Сухумский Университет

Статтю представив д.ф.-м.н., проф. Буй Д.Б. (за результатами конференції TAAPSD'2012)

The following symbols of unranked propositional logic are considered in the present paper

I. **Basic symbols::**

1) Propositional constants: t, f, s
2) Propositional variables: $P_0, P_1, \ldots$
3) Logical connectives: $\wedge, \vee, \neg$
4) **[,]**

II. **Signs introduced by the definition of type I from [2].**

III. Auxiliary symbols T (halting constant «a valid formula»), F (halting constant «an inconsistent formula»), S (halting constant «an indefinite formula»);

IV. Meta-variables $A, B, A_1, \ldots$ to denote formulae and Meta-variables $\Phi, \Phi_1, \ldots$ to denote sequence formulae.

We define formulas as follows:

1) The simplest formulas are propositional constants and propositional variables.

2) $\neg A$ is a formula.

3) $[A \wedge B]$, $[A \vee B]$, $\wedge \Phi A$ and $\vee \Phi A$ are formulas.

4) If σ is an $n-$ place logical-relative operator introduced by the definition of type I, then $[\sigma A_1 \ldots A_n]$ are formulas.

5) We do not have other formulas than those defined by items 1-4.

As an example of $n+1$-ary logical operator we define the operators $\bigvee^{n+1}$ and $\bigwedge^{n+1}$ as:

1. $\vee^1 A$ — A. $\vee^{n+1} A_1 \ldots A_n A$ — $\vee \vee^n A_1 \ldots A_n A$ to be read as: «disjunction of formulas $A_1, \ldots, A_n, A$», $n = 1, 2, \ldots$

2. $\wedge^1 A$ — A. $\wedge^{n+1} A_1 \ldots A_n A$ — $\wedge \wedge^n A_1 \ldots A_n A$ to be read as: «Conjunction of formulas $A_1, \ldots, A_n, A$», $n = 1, 2, \ldots$

Note that if we remove the place measure from the operators $\bigvee^{n+1}$ and $\bigwedge^{n+1}$, then we get usual unranked connectives $\bigvee$ and $\bigwedge$.

Rank of these operators can be any natural number, which can be read from the context.

The given occurrence of an operator σ in a formula A is called **main operator,** if it is not in the scope of any other operator occurring in A.

The given occurrence of an operator σ in a part B of a formula A is called **main operator,** if it is not in the scope of any other operator occurring in B.

Assume that $A_1, \ldots, A_n$, $(n \geq 2)$ are the main operators of the formulas, which differ in the sign $\vee$, then any formula constructed by means of these formulas and disjunctive sign $\vee$ is **called a formula of disjunctive type** and $A_1, \ldots, A_n$, are called the disjunctive parts of the constructed formula.

Assume that $A_1, \ldots, A_n$, $(n \geq 2)$ are the main operators of the formulas, which differ in the sign $\wedge$, then any formula constructed by means of these formulas and conjunctive sign $\vee$ is called a formula of conjunctive type and $A_1,\dots,A_n$, are called the conjunctive parts of the constructed formula.

We define **the main part of a formula** as follows:

1) If a formula is of type $\neg A$, then its main part is A.
2) If a formula is of disjunctive type, then its main parts are disjunctive ones.
3) If a formula is of conjunctive type, then its main parts are conjunctive ones.
4) If a formula is of type $\sigma A_1,\dots,A_n$, then its main parts are $A_1,\dots,A_n$.
5) If a formula is of type $\wedge\Phi A$ and $\vee\Phi A$, then its main parts are A and Φ.

Every formula is considered as the definite algorithmic process. We describe this process as follows.

An algorithmic process begins from the main operator of the formula, which is called **the directive operator** of the process.

An immediate **consequence of the directive operator of the process** is formulated in the manner as:

1) An immediate consequence of the formula(operator) *t is* T;
2) An immediate consequence of the formula(operator) *f* is F;
3) An immediate consequence of the formula(operator) *s* is *S*;
4) An immediate consequence of the propositional variable(operator) *p* and propositional *Φ* is *s*;
5) If a formula is of type $\neg A$, then we have the following immediate consequences of the operator $\neg$:
   a) *t*. if $A \cong f$;
   b) *f*, if $A \cong t$ A≅t;
   c) *B*, if $A \cong \neg B$;
   d) $\wedge\neg A_1 \dots \neg A_n$ is a formula of conjunctive type if A is a formula of disjunctive type and a complete enumeration of its disjunctive parts is $A_1,\dots,A_n$.
   e) $\vee\neg A_1 \dots \neg A_n$ is a formula of disjunctive type if A is a formula of conjunctive type and a complete enumeration of its conjunctive t parts is $A_1,\dots,A_n$.
   f) s, if A≅s;
   g) expectation in all other cases.
6) If a formula is of disjunctive type, then an immediate consequence of the operator $\vee$ is:
   a) *t*, if some disjunctive part is *t*.
   b) *f*, if all its disjunctive parts are *f*.
   c) *t*, if some disjunctive part is the negation of one or several disjunctive parts;
   d) a formula which is obtained from a formula of disjunctive type by elimination of disjunctive parts which are equal to *f*.
   e) a formula which is obtained from a formula of disjunctive type by elimination of repeated disjunctive parts.
   f) a formula of conjunctive type $\wedge[\vee A_1 \dots A_{i-1} A_{i+1} \dots A_n B_k] \dots [\vee A_1 \dots A_{i-1} A_{i+1} \dots A_n B_1]$ where $A_1,\dots,A_n$ is a complete enumeration of disjunctive parts of a given formula, while A_i *(i∈{1,...,n})* is the first disjunctive part which is a formula of conjunctive type and $B_1,\dots,B_k$ is a complete enumeration of its conjunctive parts;
   g) expectation in all other cases.
7) If a formula is of conjunctive type, then an immediate consequence of the operator $\vee$ is:
   a) *f*, if some conjunctive part is *f*.
   b) *t*, if all its conjunctive parts are *t*.
   c) *f*, if some conjunctive part is the negation of one or several conjunctive parts;
   d) a formula which is obtained from a formula of conjunctive type by elimination of conjunctive parts which are equal to *t*.
   e) a formula which is obtained from a formula of conjunctive type by elimination of repeated conjunctive parts.
   f) a formula of disjunctive type $\vee[\wedge A_1 \dots A_{i-1} A_{i+1} \dots A_n B_1] \cdots [\wedge A_1 \dots A_{i-1} A_{i+1} \dots A_n B_k]$ where $A_1,\dots,A_n$ is a complete enumeration of conjunctive parts of a given formula, while A_i *(i∈{1,...,n})* is the first conjunctive part which is a formula of disjunctive type and $B_1,\dots,B_k$ is a complete enumeration of its disjunctive parts;
   g) expectation in all other cases.
8) If a formula is of the form $\sigma A_1,\dots,A_n$, where σ is the operator introduced by the definition, then an immediate consequence of the operator σ is the formula on the right-hand side of its definition.

When we get an immediate consequence of the directive operator or passes in the expectation state, then control is given to some other operator. After a defined operation, it gets the control. In particular:

If during the performance of the process defined by the formula $\neg A$, the negation operator passed in the state of expectation, then control is given over to the main operator of the main part A. After obtaining an immediate consequence of this operator, the negation operator recovers control.

If during the performance of the process defined by a formula of disjunctive(conjunctive) type the directive operator passes in the expectation state, then control is given to the first from the right main operator of the main part of this formula; after ending its performance, this operator recovers control. If after recovering control, the disjunctive(conjunctive) operator again passes in the expectation state, then control is given to the main operator of the main part having the same property. This process goes on until the main operator of all those main parts, the length of which does not exceed 2, receives control.

If after this, the directive operator again passes in the expectation state, then control is given to the main operator of the disjunctive(conjunctive) part of maximal length until this part becomes a formula of conjunctive (disjunctive) type or a propositional variable or the negation of a variable.

Since at every stage of the process performance there may occur a change in the main operator of the obtained formula, the process has one or several control operators.

In particular, we consider the algorithmic process by the T-strategy, F-strategy and $T+F$-strategy. A difference between these strategies arises only at the time of the process performance when the conditions f) for formulas of disjunctive and conjunctive type are satisfied. For the T-strategy the condition f) of the disjunctive type formula is fulfilled, but the condition f) of the conjunctive type formula is left out. For the F-strategy the condition f) of the conjunctive type formula is fulfilled, but the condition f) for the disjunctive type formula is left out. In the case of the $T+F$-strategy a parallel process begins to take place from a certain stage. In particular, the latter process begins when the formula obtained during the first realization of the condition f) is fulfilled simultaneously with the preceding formula according to the following rule. If the realized condition f) is a formula of disjunctive(conjunctive) type, then the fulfillment of the obtained formula of conjunctive (disjunctive) types continued by T-strategy (F-strategy), while the processing of the given formula of disjunctive(conjunctive) type is continued by the -F-strategy (T-strategy). The $T+F$ process is halted only if some parallel process produces the halting constant T or F or two halting constants S.

The following theorems are immediate consequence of the algorithm process and [4;5]

Theorem 1. A formula A considered as an algorithmic process is fulfilled by the $T+F$-strategy and its final value is equal to: a) the halting constant T, if A is a valid formula; b) the halting constant F, if A is an inconsistent formula, and c) the halting constant S, if A is a satisfiable formula.

Theorem 2. A valid formula considered as an algorithmic process is fulfilled by the T-strategy and its final value is equal to the halting constant T.

Theorem 3. An inconsistent formula considered as an algorithmic process is fulfilled by the F-strategy and its final value is equal to the halting constant F.

Theorem 4. A satisfiable formula differing from a valid formula and considered as an algorithmic process T or F or $T+F$-strategy is fulfilled and its final value is the halting constant S.

Theorem 5. A formula A considered as an algorithmic process is fulfilled.

Acknowledgements

This research has been Supported by the Georgian Rustaveli National Science Foundation under the project № D/16/4 - 120/11.

Authors would like to thank the editorial board of this bulletin for their sincere help with the paper.

References

1) Rukhaia K.H; Tibua L.M.; Chankvetadze G., Mikhanadze G.; Unranked Formal Mathematical Theory. // Reports of The Russian Conference of the Sobolov Institute of Mathematics.- 2012. (in Russian)

2) Pkhakadze Sh.; Some Problems of the Notation Theory// TSU-1977.-p.195.

3) Kutsia T., Theorem Proving with Sequence Variables and Flexible Arity Symbols.// In: Proceedings of LPAR 2002.-p. 278-291.

4) N. Nikitchenko.; Applicative compositions of Partial Predicates. Kibernetika i systemny analiz, No.2 (2000) 3-19. (In Russian)

5) C. Chen, R. Li; Mathematical Logic and Automated Theorem Proving. a//- M.- 1983.

6) Burbaki N. ; Set Theory. M.- 1965;.- p.453.

7) Rukhaia Kh.M.,Tibua L., Chankvetadze G., Dundua B.; One Method of constructing a formal system // AMIM -2006. – T. 11. – № 2. –p. 3–15.

Надійшла до редколегії 03.12.2012