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Ганна В.Верьовкіна[1], *к.ф.-м.н., доцент*

Інтерполяційні зображення деяких класів випадкових процесів із групуванням вузлів інтерполяції

Досліджуються інтерполяційні зображення певного класу випадкових процесів за нерівновіддаленими вузлами інтерполяції.

Ключові слова: інтерполяційне зображення, випадковий процес, вузли інтерполяції .

[1]Київський національний університет імені Тараса Шевченка, механіко-математичний факультет, 01601, Київ, вул. Володимирська, 64

Ganna V.Verovkina[1], *PhD, Associate Professor*

The interpolation representation of some classes of random processes with a set of interpolation knots

Some interpolation representations of random processes with non equidistance interpolation knots are investigated.

Key Words: interpolation representation, random process, interpolation knots.

[1]Taras Shevchenko National University of Kyiv, Department of Mechanics and Mathematics, 01601, Kyiv, Volodymyrska str., 64

E-mail: ganna.verov@jmail.com
The article presented Dr. Sci. Science, Full Professor Yu.V.Kozachenko

1 Introduction

Interpolation representations of a class of random process with non equidistance interpolation knots are investigated. Obtained interpolation formula that uses the value of the process and its derivatives at the knots of interpolation. The convergence with probability 1 of the corresponding interpolation series to a random process is proved.

2 The interpolation representations of some kinds of random processes

Consider the interpolation representation of random processes [1] on non equidistance interpolation knots of the type $t_{n0} = n\frac{7\pi}{\alpha}, t_{n1} = n\frac{7\pi}{\alpha}+\frac{\pi}{\alpha}, t_{n2} = n\frac{7\pi}{\alpha}+\frac{2\pi}{\alpha}$, $n \in$ Z, based on observations of the process and its derivatives of the first and second orders at knots t_{n0}, t_{n1}, $n \in$ Z and observations of the process at knots t_{n2}, $n \in$ Z .

Let us formulate the necessary results from the theory of entire functions of complex variable.

Lemma 1. *Let $f(z)$ be an entire bounded on the real axis function of exponential type with indicator σ. Then for any $\alpha, \alpha > \sigma$, the representation holds true*

$$f(z) = \sum_{n=-\infty}^{\infty} \Bigg(\Bigg[\frac{f(t_{n0})}{\left(\frac{\alpha}{7}\right)^3 (z-t_{n0})^3} + \frac{f'(t_{n0})}{\left(\frac{\alpha}{7}\right)^3 (z-t_{n0})^2} + \frac{f''(t_{n0})}{\left(\frac{\alpha}{7}\right)^3 (z-t_{n0})}\frac{1}{2} \Bigg] \times \frac{1}{\sin^3\frac{\pi}{7}\sin^2\frac{2\pi}{7}} - $$

$$- \Bigg[\frac{f(t_{n0}+\frac{\pi}{\alpha})}{\left(\frac{\alpha}{7}\right)^3 (z-t_{n0}-\frac{\pi}{\alpha})} + \frac{f'(t_{n0}+\frac{\pi}{\alpha})}{\left(\frac{\alpha}{7}\right)^3 (z-t_{n0}-\frac{\pi}{\alpha})^2} + \frac{f''(t_{n0}+\frac{\pi}{\alpha})}{\left(\frac{\alpha}{7}\right)^3 (z-t_{n0}-\frac{\pi}{\alpha})} \Bigg] \times \frac{1}{\sin^4\frac{\pi}{7}} + $$

$$+ \frac{f(t_{n0}+\frac{2\pi}{\alpha})}{\frac{\alpha}{7}(z-t_{n0}-\frac{2\pi}{\alpha})} \times \frac{1}{2\sin^3\frac{\pi}{7}\sin^3\frac{2\pi}{7}} \Bigg) \times$$

$$\times \sin^3\frac{\alpha}{7}(z-t_{n0})\sin^3\frac{\alpha}{7}(z-t_{n0}-\frac{\pi}{\alpha})\times$$

$$\times \sin\frac{\alpha}{7}(z-t_{n0}-\frac{2\pi}{\alpha}), \qquad (1)$$

where $t_{n0} = n\frac{7\pi}{\alpha}, n \in Z$, provided that the interpolation series (1) converges uniformly in any bounded region of the complex plane.

Proving Lemma 1, we obtain estimate of the residual of the interpolation series (1), which has the following form

$$|R_n(z)| \leqslant LG(z)C_f \frac{\alpha}{\alpha-\sigma}\frac{1}{n}, \tag{2}$$

where L is a constant, $C_f = \sup\limits_{t\in R}|f(t)|$, $G(z) = \left|\sin^3 \frac{\alpha}{7}z \times \sin^3 \frac{\alpha}{7}(z-\frac{\pi}{\alpha}) \times \sin\frac{\alpha}{7}(z-\frac{2\pi}{\alpha})\right|$ is a function bounded on any bounded region of the complex plane.

Consider a random process $\xi(t)$, $t \in R$ with $M\xi(t) = 0$ and covariance function which the representation is

$$B(t,s) = \int_{\Lambda\times\Lambda} f(t,\lambda)\overline{f(s,\mu)}F(d\lambda, d\mu), \tag{3}$$

where Λ is a set of parameters, $F(.,.)$ is a positive definite additive complex function on $\Lambda\times\Lambda$ such that

$$\int_{\Lambda\times\Lambda} |F(d\lambda, d\mu)| < +\infty. \tag{4}$$

The function $f(t,\lambda)$ with respect to t is an entire function of exponential type with indicator $\sigma(\lambda)$ such that

$$\sup_{\lambda\in\Lambda}\sup_{-\infty<t<+\infty}|f(t,\lambda)| = C_f < +\infty, \tag{5}$$

$$\sup_{\lambda\in\Lambda}\sigma(\lambda) = \sigma < +\infty. \tag{6}$$

The following theorem holds true.

Theorem 2.1. *Let $\xi(t)$ be a separable random process that satisfies conditions* (3)–(6). *Then for any $\alpha, \alpha > \sigma$ with probability 1 the following representation holds true*

$$\xi(t) = \sum_{n=-\infty}^{\infty}\Bigg(\Bigg[\frac{\xi(t_{n0})}{\left(\frac{\alpha}{7}\right)^3(t-t_{n0})^3} + \frac{\xi'(t_{n0})}{\left(\frac{\alpha}{7}\right)^3(t-t_{n0})^2} + \frac{\xi''(t_{n0})}{\left(\frac{\alpha}{7}\right)^3(t-t_{n0})}\frac{1}{2}\Bigg]\times\frac{1}{\sin^3\frac{\pi}{7}\sin^2\frac{2\pi}{7}} -$$
$$-\Bigg[\frac{\xi(t_{n0}+\frac{\pi}{\alpha})}{\left(\frac{\alpha}{7}\right)^3(t-t_{n0}-\frac{\pi}{\alpha})^3} + \frac{\xi'(t_{n0}+\frac{\pi}{\alpha})}{\left(\frac{\alpha}{7}\right)^3(t-t_{n0}-\frac{\pi}{\alpha})^2} + \frac{\xi''(t_{n0}+\frac{\pi}{\alpha})}{\left(\frac{\alpha}{7}\right)^3(t-t_{n0}-\frac{\pi}{\alpha})}\Bigg]\times\frac{1}{\sin^4\frac{\pi}{7}} +$$
$$+\frac{\xi(t_{n0}+\frac{2\pi}{\alpha})}{\frac{\alpha}{7}(t-t_{n0}-\frac{2\pi}{\alpha})}\times\frac{1}{2\sin^3\frac{\pi}{7}\sin^3\frac{2\pi}{7}}\Bigg)\times$$
$$\times\sin^3\frac{\alpha}{7}(t-t_{n0})\sin^3\frac{\alpha}{7}(t-t_{n0}-\frac{\pi}{\alpha})\times$$
$$\times\sin\frac{\alpha}{7}(t-t_{n0}-\frac{2\pi}{\alpha}) \tag{7}$$

Proof. The random processes $\xi(t)$ has the spectral representation of the form [1]

$$\xi(t) = \int_{\Lambda} f(t,\lambda)\,Z(d\lambda), \tag{8}$$

where $Z(d\lambda)$ is a random measure on Λ such that $MZ(A_1)\cdot\overline{Z(A_2)} = F(A_1, A_2)$. For any natural n consider the process $\xi_n(t)$ which is defined as a partial sum of series (7):

$$\xi_n(t) = \sum_{k=-n}^{n}\Bigg(\Bigg[\frac{\xi(t_{k0})}{\left(\frac{\alpha}{7}\right)^3(t-t_{k0})^3} + \frac{\xi'(t_{k0})}{\left(\frac{\alpha}{7}\right)^3(t-t_{k0})^2} + \frac{\xi''(t_{k0})}{\left(\frac{\alpha}{7}\right)^3(t-t_{k0})}\frac{1}{2}\Bigg]\times\frac{1}{\sin^3\frac{\pi}{7}\sin^2\frac{2\pi}{7}} -$$
$$-\Bigg[\frac{\xi(t_{k0}+\frac{\pi}{\alpha})}{\left(\frac{\alpha}{7}\right)^3(t-t_{k0}-\frac{\pi}{\alpha})^3} + \frac{\xi'(t_{k0}+\frac{\pi}{\alpha})}{\left(\frac{\alpha}{7}\right)^3(t-t_{k0}-\frac{\pi}{\alpha})^2} + \frac{\xi''(t_{k0}+\frac{\pi}{\alpha})}{\left(\frac{\alpha}{7}\right)^3(t-t_{k0}-\frac{\pi}{\alpha})}\Bigg]\times\frac{1}{\sin^4\frac{\pi}{7}} +$$
$$+\frac{\xi(t_{k0}+\frac{2\pi}{\alpha})}{\frac{\alpha}{7}(t-t_{k0}-\frac{2\pi}{\alpha})}\times\frac{1}{2\sin^3\frac{\pi}{7}\sin^3\frac{2\pi}{7}}\Bigg)\times$$

$$\times \sin^3 \frac{\alpha}{7}(t - t_{k0}) \sin^3 \frac{\alpha}{7}(t - t_{k0} - \frac{\pi}{\alpha})\times$$

$$\times \sin \frac{\alpha}{7}(t - t_{k0} - \frac{2\pi}{\alpha}).$$

Using representation (8) and the assertion of Lemma 1, we can present $\xi_n(t)$ in the form

$$\xi_n(t) = \sum_{k=-n}^{n} \int_\Lambda \left(\left[\frac{f(t_{k0})}{\left(\frac{\alpha}{7}\right)^3 (t - t_{k0})^3} + \right.\right.$$

$$+ \frac{f'(t_{k0})}{\left(\frac{\alpha}{7}\right)^3 (t - t_{k0})^2} +$$

$$\left. + \frac{f''(t_{k0})}{\left(\frac{\alpha}{7}\right)^3 (t - t_{k0})} \frac{1}{2} \right] \times \frac{1}{\sin^3 \frac{\pi}{7} \sin^2 \frac{2\pi}{7}} -$$

$$- \left[\frac{f(t_{k0} + \frac{\pi}{\alpha})}{\left(\frac{\alpha}{7}\right)^3 (t - t_{k0} - \frac{\pi}{\alpha})^3} + \right.$$

$$+ \frac{f'(t_{k0} + \frac{\pi}{\alpha})}{\left(\frac{\alpha}{7}\right)^3 (t - t_{k0} - \frac{\pi}{\alpha})^2} +$$

$$\left. + \frac{f''(t_{k0} + \frac{\pi}{\alpha})}{\left(\frac{\alpha}{7}\right)^3 (t - t_{k0} - \frac{\pi}{\alpha})} \right] \times \frac{1}{\sin^4 \frac{\pi}{7}} +$$

$$\left. + \frac{f(t_{k0} + \frac{2\pi}{\alpha})}{\frac{\alpha}{7}(t - t_{k0} - \frac{2\pi}{\alpha})} \times \frac{1}{2 \sin^3 \frac{\pi}{7} \sin^3 \frac{2\pi}{7}} \right) \times$$

$$\times \sin^3 \frac{\alpha}{7}(t - t_{k0}) \sin^3 \frac{\alpha}{7}(t - t_{k0} - \frac{\pi}{\alpha})\times$$

$$\times \sin \frac{\alpha}{7}(t - t_{k0} - \frac{2\pi}{\alpha}) \times Z(d\alpha) \qquad (9)$$

Then based on the representations (1), (8), (9) and estimate (2), we obtain

$$\mathrm{M}|\xi(t)) - \xi_n(t)|^2 \leqslant$$

$$\leqslant R_n^2(t) \int_{\Lambda\times\Lambda} |F\,(d\lambda, , d\mu)| =$$

$$= L^2 G^2(t) C_f^2 \left(\frac{\alpha}{\alpha - \sigma}\right)^2 \frac{1}{n^2} \times$$

$$\times \int_{\Lambda\times\Lambda} |F\,(d\lambda, d\mu)|. \qquad (10)$$

From inequality (10), taking into account condition (4), we find that the interpolation series (7) converges to a random process $\xi(t)$ in the mean-square sense. Based on separability of the process $\xi(t)$ and convergence of the series

$$\sum_{n=-\infty}^{\infty} |\xi(t) - \xi_n(t)|^2,$$

we obtain that the interpolation series (7) converges with probability 1 to a random process $\xi(t)$ in any bounded domain of changes of parameter t.

3 Conclusions

The work is devoted to investigation of interpolation representations of a class of random process. The main result is a theorem on the convergence of interpolating series to a random process with probability 1.

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