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Сплощуючі властивості ліфтів аналітичних НР-перетворень келерових просторів.

У роботі вивчаються сплощуючі властивості інфінітезимальних перетворень дотичних розшарувань першого та другого порядків, породжені ліфтами аналітичних НР-перетворень келерових просторів.

Ключові слова: сплощення, порядок сплощення, р-геодезична крива, р-геодезичне відображення, р-геодезичне інфінітезимальне перетворення.

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Flattening properties of the lifts of analytic HP-transformations Kählerian manifolds.

In this paper we are study the flattening properties of the infinitesimal transformations of tangent bundles of orders 1 and 2, which generate the lifts of analytic HP-transformations of Kählerian manifold.

Key Words: flattening, the order of flattening, the p-geodesic curve, the p-geodesic map, the p-geodesic infinitesimal transformation.

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1 Introduction.

Generalisations of geodesic curves of a different aspect are known. In particular, A. Fialkow considers geodesics circles in Riemannian space ([1]). T. Otsuki, Y. Tashiro have introduced concept a holomorphically planar curve in Kählerian space ([2]). P. K. Rashevsky considers flattening curves of a arbitrary order in affine connected spaces, using concept of a flattening ([3]).

On the basis of these curves of generalisation of geodesic maps have been defined: concircular transformations K. Yano ([4]), holomorphically projective maps Y. Tashiro ([5]), p-geodesic maps S. G. Leiko ([6], [7]).

Their infinitesimal analogues were considered in works: for concircular transformations Riemannian spaces (S. Ishihara [8]), for holomorphic projective transformations Kählerian spaces (S. Tachibana, S. Ishihara [9]). P-geodesic infinitesimal transformations are defined S. G. Leiko in work [10].

Lifts of infinitesimal transformations were studied K. Yano and S. Ishihara ([11], [12]). By them it is established, that the complete lift X^{C} of the geodesic infinitesimal transformation X is infinitesimal geodesic transformation to a tangent bundle if and only if X is affine infinitesimal transformation. S. G. Leiko studied lifts of infinitesimal transformations from the point of view of the theory p-geodesic (flattening) maps. He has established, that for a tangent bundle of the first order, vertical lift X^{V} of the geodesic infinitesimal transformation X is canonical 2-geodesic infinitesimal transformation, and the complete lift X^{C} is not canonical 2-geodesic infinitesimal ([10]). The case of a tangent bundle of the second order also is considered S. G. Leiko in work [10]. Lifts of infinitesimal concircular transformation in a tangent bundle of the first order were studied S. G. Leiko ([13]). The case of a tangent bundle of the second order is considered in work [14].

The given work is devoted study of flattening properties of lifts analytical HP-transformations of Kählerian spaces.

2 Elements of the theory of flattening maps.

We will consider in affine connected space (M, ∇) curve $\mathscr{C}$ admiting parametre t; ξ - a field of tangent vectors along a curve $\mathscr{C}$. The vector q-th curvature ξ_q is defined by a rule $\xi_q = \nabla_t \xi_{q-1}$, $\xi_0 = \xi$.

Definition 2.1. ([10], [13]). Arbitrarily we take a point $p \in \mathscr{C}$ on a curve $\mathscr{C}$. If at a point p vectors ξ, $\xi_1, \ldots, \xi_{m-1}$ are linearly independent, and vectors

$\xi,\ \xi_1,\ \ldots,\ \xi_{m-1},\ \xi_m$ are linearly dependent, say, that the curve $\mathscr{C}$ at a point p has a flattening m-th order; the number m is called the order of flattening of point p the curve $\mathscr{C}$.

Considering properties of an external product, a condition

$$\xi \wedge \xi_1 \wedge ... \wedge \xi_{m-1} \wedge \xi_m = 0, \quad (1)$$
$$\xi \wedge \xi_1 \wedge ... \wedge \xi_{m-1} \neq 0, \quad (2)$$

are necessary and sufficient that the curve $\mathscr{C}$ have at a point p a flattening m-th order.

Definition 2.2. ([10], [13]). The curve $\mathscr{C}$ in affine connected space (M, ∇) is called m-geodesic if in each point it has m-th order.

That the curve $\mathscr{C}$ is m-geodesic necessary and sufficient that along it conditions (1) and (2) are satisfied.

On the other hand, if a curve $\mathscr{C}$ - m-geodesic along it holds equality

$$\xi_m = a_0\xi + a_1\xi_1 + ... + a_{m-1}\xi_{m-1}, \quad (3)$$

where $a_0,\ a_1,\ \ldots,\ a_{m-1}$ - some functions are defined along a curve $\mathscr{C}$.

Definition 2.3. ([10], [13]). The parametre t on m-geodesic curve $\mathscr{C}$ is called ι-canonical ($1 \leqslant \iota \leqslant m$), if $a_{m-\iota} = 0$ along a curve $\mathscr{C}$.

The parametre t on m-geodesic curve $\mathscr{C}$ is called $\iota_1,\ \iota_2,\ \ldots,\ \iota_k$ - canonical ($m \geqslant \iota_1 > \iota_2 > \ldots > \iota_k \geqslant 1$) if it is simultaneously ι_1-canonical, ι_2-canonical, $\ldots$, ι_k-canonical.

$1, 2..., m$-canonical the parametre t m-geodesic curve $\mathscr{C}$ is called as absolutely canonical.

From properties of an external product follows, that a necessary and sufficient condition of ι-canonical (resp. $\iota_1,\ \iota_2,\ \ldots,\ \iota_k$-canonical, absolute canonical) parametre t m-geodesic curve $\mathscr{C}$ is the equality $\xi \wedge \xi_1 \wedge ... \wedge \overset{\frown}{\xi}_{m-\iota} \wedge ... \wedge \xi_m = 0, \xi \wedge ... \wedge \overset{\frown}{\xi}_{m-\iota_1} \wedge ... \wedge \overset{\frown}{\xi}_{m-\iota_k} \wedge ... \wedge \xi_m = 0, \xi_m = 0$, which holds along a curve $\mathscr{C}$. Where the note $\overset{\frown}{\eta}$ shows, that η are not present a factor in an external product. We see that from this a condition of follows the condition (1).

Definition 2.4. ([10], [13]). Mapping $f\colon M \to \bar{M}$ is affine connected spaces (M, ∇) and $\left(\bar{M}, \bar{\nabla}\right)$ is called r-geodesic if this mapping translates all geodesic curves of the first space in curves of the second space at which points the greatest order of a flattening is equal r.

The number r is called as order of a flattening of mapping f.

r-geodesic diffeomorphism $\rho\colon M \to M$ it is affine connected space (M, ∇) on itself is called r-geodesic transformation affine connected space (M, ∇).

Geometrically r-geodesic mappings are characterised by that they geodesic curves translate in curves which on separate arcs are m-geodesic curves, and $m \leqslant r$, and r greatest of all numbers m.

S. G. Leiko the differential equations describing r-geodetic mappings are found. Let $\bar{u}^h = \bar{u}^h\left(u^1, u^2..., u^n\right)$ - representation of mapping $f\colon M \to \bar{M}$. Mapping f is r-geodesic necessary and sufficient that in general on a diffeomorphism local system coordinate are satisfied conditions

$$\begin{array}{l} \delta^{[h}_{(i}\,\mathrm{H}^{h_1}_{i_1 i_2} ... \mathrm{H}^{h_{r-1}}_{j_1 ... j_r}\,\mathrm{H}^{h_r]}_{j_1 ... j_r j_{r+1})} = 0, \\ \delta^{[h}_{(i}\,\mathrm{H}^{h_1}_{i_1 i_2} ... \mathrm{H}^{h_{r-1}]}_{j_1 ... j_r)} \neq 0, \end{array} \quad (4)$$

where $\mathrm{H}^h_{ij} = \breve{\nabla}_i \delta^h_j = \bar{\Gamma}^h_{ij} - \Gamma^h_{ij}$ - tensor of an affine deformation of mapping f, $\mathrm{H}^h_{j_1 ... j_m j_{m+1}} = \breve{\nabla}_{(j_{m+1}} \mathrm{H}^h_{j_1 ... j_m)}$, $\breve{\nabla}$ – the mixed covariant derivative in sense of the van der Waerden – Bortolotti concerning connections ∇ and $\bar{\nabla}$. Relations (4) are called as the basic equations r-geodesic mapping.

Let $\mu\colon M \to \bar{M}$ a diffeomorphism affine connected spaces (M, ∇) and $\left(\bar{M}, \bar{\nabla}\right)$.

In case of diffeomorphisms, investigation of orders of a flattening of points of a curve-image $\bar{\mathscr{C}} = \mu\left(\mathscr{C}\right)$ in manifold $\bar{M}$ with affine connection $\bar{\nabla}$ can be reduced to study of orders of a flattening of corresponding points of a geodesic curve $\mathscr{C}$ in manifold M with respect to special connection on manifold M - a pre-image of affine connection $\bar{\nabla}$ concerning a diffeomorphism μ. It allows us not to use a means of the mixed tensors and the mixed covariant derivative of the van der Waerden - Bortolotti (see [14], [15], [17]).

Definition 2.5. ([16]). Affine connection $\tilde{\nabla}$ on manifold M, defined by equality $\tilde{\nabla}_X Y = \left(\mu^{-1}\right)_* \left(\bar{\nabla}_{\mu_* X} \mu_* Y\right)$, for arbitrary smooth fields of vectors X and Y from $\mathfrak{X}\left(M\right)$, is called as a pre-image of affine connection $\bar{\nabla}$ with respect to (under) a diffeomorphism μ.

Let $\mu\colon M \to \bar{M}$ a diffeomorphism affine connected spaces (M, ∇) and $(\bar{M}, \bar{\nabla})$, $\tilde{\nabla}$ - a pre-image of affine connection $\bar{\nabla}$ with respect to a diffeomorphism μ.

Tensor $P(X,Y) = \tilde{\nabla}_X Y - \nabla_X Y$ we will call *a tensor of an affine deformation of a diffeomorphism μ.*

3 Flattening infinitesimal transformations.

Let X infinitesimal transformation, that is $X \in \mathfrak{X}(M)$ a field of vectors and $\tau_\varepsilon:\ \bar{u}^h = u^h + \varepsilon \cdot X^h$, $h = \overline{1,n}$, is infinitesimal the point-transformation is defined a field of vectors X, ε - infinitesimal parametre.

Infinitesimal transformation τ_ε translates a geodesic curve $\mathscr{C} \subset U$ in a curve $\bar{\mathscr{C}_\varepsilon}$. We now consider a field of tangent vectors $\bar{\xi}^{(\varepsilon)}$ and fields of vectors of curvature $\bar{\xi}^{(\varepsilon)}_m$, $m = 1, 2...$ along a curve $\bar{\mathscr{C}_\varepsilon}$.

Definition 3.1. ([10], [13]). We say, that infinitesimal transformation X adds a geodesic curve $\mathscr{C}$ a flattening r-th order in a point $p \in \mathscr{C}$, if

$$\lim_{\varepsilon\to 0} \frac{1}{\varepsilon^r} \bar{\xi}^{(\varepsilon)}_{\tau_\varepsilon(p)} \wedge \bar{\xi}^{(\varepsilon)}_1{}_{\tau_\varepsilon(p)} \wedge ... \wedge \bar{\xi}^{(\varepsilon)}_r{}_{\tau_\varepsilon(p)} = 0,$$

and number r least of the possible.

We take a pre-image $\tilde{\nabla}_{(\varepsilon)}$ affine connection ∇ concerning infinitesimal a point-transformation τ_ε. We build a field of tangent vectors ξ and fields of vectors of curvature $\tilde{\xi}^{(\varepsilon)}_m$, $m = 1, 2...$ along a geodesic curve $\mathscr{C}$ with respect to connection $\tilde{\nabla}_{(\varepsilon)}$. Definition 3.1 is equivalent to the following of

Definition 3.2. It is said that infinitesimal transformation X adds a geodesic curve $\mathscr{C}$ a flattening r-th order at a point $p \in \mathscr{C}$, if $\lim\limits_{\varepsilon\to 0} \frac{1}{\varepsilon^r} \xi_p \wedge \tilde{\xi}^{(\varepsilon)}_1{}_p \wedge ... \wedge \tilde{\xi}^{(\varepsilon)}_r{}_p = 0$, and number r least of the possible.

Definition 3.3. ([10], [13]). Infinitesimal transformation X is called r-geodesic infinitesimal transformation (short, r-g.i.t.) if on each geodesic curve $\mathscr{C}$ it adds each point $p \in \mathscr{C}$ a flattening m-th order, $m \le r$. The number m can depend as on a choice of a geodesic curve $\mathscr{C}$, and points on it, and number r greatest of all possible numbers m.

We denote r-g.i.t. by $\tau(r)$.

Theorem 1. *Let affine connection ∇ on manifold M in a map $c = (U; \varphi; \mathrm{R}^n)$ has components Γ^h_{ji}, the derivative of Lie $\mathcal{L}_X \nabla \in \mathfrak{T}^1_2(M)$ affine connection ∇ in a map c has components $\mathcal{L}_X \Gamma^h_{ij}$. Then a tensor $P^{(\varepsilon)}$ an affine deformation infinitesimal the point-transformation τ_ε in a map c has components $P^h_{ij}(p) = \varepsilon \cdot \mathcal{L}_X \Gamma^h_{ij}(p) + \underline{O}(\varepsilon^2)$.*

Proof. Let $\tilde{\nabla}_{(\varepsilon)}$ - a pre-image of affine connection concerning transformation τ_ε. Tensor $P^{(\varepsilon)}$ an affine deformation of infinitesimal transformation τ_ε has in a map c components P^h_{ij}. Considering expressions for representation $u^h \circ \tau_\varepsilon \circ \varphi^{-1}$ infinitesimal a point-transformation τ_ε, decomposing a difference $\Gamma^k_{ij}(\tau_\varepsilon(p)) - \Gamma^k_{ij}(p)$ by Taylor's formula, being limited to members not above ε^2, we will have the necessary. The theorem is proved.

N o t e . Since the received result holds in each map $(U; \varphi; \mathrm{R}^n)$ the equality can be noted in the invariant form $P^{(\varepsilon)} = \varepsilon \cdot \mathcal{L}_X \nabla + \underline{O}(\varepsilon^2)$.

Everywhere next, the symmetrization operator will denote by the letter S. Besides, for a field of tensors $T \in \mathfrak{T}^1_m(M)$, fields of vectors ξ, along a curve $\mathscr{C}$, a field of vectors $T(\underbrace{\xi ..., \xi}_{m})$, defined along a curve $\mathscr{C}$ will denote by $T(\xi^m)$.

Lemma 1. *Let in affine connected space (M, ∇) the geodesic curve $\mathscr{C}$, admiting canonical parametre t, is given and X - infinitesimal transformation M. Then a vectors of curvature of a geodesic curve $\mathscr{C}$, with respect to a pre-image $\tilde{\nabla}_{(\varepsilon)}$ affine connection ∇ under infinitesimal a point-transformation τ_ε, defined by a field of vectors X, have form:*

$$\tilde{\xi}^{(\varepsilon)}_m = \varepsilon \cdot L_{mX}\left(\xi^{m+1}\right) + \underline{O}\left(\varepsilon^2\right)$$

(resp. $\tilde{\xi}^{(\varepsilon)}_m = \varepsilon \cdot \mathcal{L}_{mX}\left(\xi^{m+1}\right) + \underline{O}\left(\varepsilon^2\right)$), where ξ - a field of tangent vectors along a curve $\mathscr{C}$, and a field of tensors $L_{mX} \in \mathfrak{T}^1_{m+1}(M)$ (resp. $\mathcal{L}_{mX} \in \mathfrak{T}^1_{m+1}(M)$) it is defined recurrently by a rule

$$L_{1X} = \mathcal{L}_X \nabla,\ L_{mX} = \nabla L_{m-1X}$$

(resp. $\mathcal{L}_{1X} = S(\mathcal{L}_X \nabla)$, $\mathcal{L}_{mX} = S(\nabla \mathcal{L}_{m-1X})$).

Proof. We will apply the theorem 2 (see [18]). Then $\tilde{\xi}^{(\varepsilon)}_m = P^{(\varepsilon)}_m\left(\xi^{m+1}\right)$, where the tensor field $P^{(\varepsilon)}_m \in \mathfrak{T}^1_m(U)$ is defined recurrently by a rule $P_1{}^{(\varepsilon)} = P^{(\varepsilon)}$, $P_m{}^{(\varepsilon)} = \nabla P_{m-1}{}^{(\varepsilon)} + P^{(\varepsilon)} \circ$

$\left(\delta \otimes P_{m-1}{}^{(\varepsilon)}\right)$. Using the previous lemma, we will receive $P_1^{(\varepsilon)} = P^{(\varepsilon)} = \varepsilon \cdot L_X\nabla + \underline{O}\left(\varepsilon^2\right) = \varepsilon \cdot L_{1X} + \underline{O}\left(\varepsilon^2\right)$. We suppose is shown, that $P_{m-1}^{(\varepsilon)} = \varepsilon \cdot L_{m-1X} + \underline{O}\left(\varepsilon^2\right) \in \mathfrak{T}_m^1(U)$. Then

$$P_m^{(\varepsilon)} = \nabla P_{m-1}^{(\varepsilon)} + P^{(\varepsilon)} \circ \left(\delta \otimes P_{m-1}^{(\varepsilon)}\right) = \\ = \varepsilon \cdot \nabla L_{m-1X} + \underline{O}\left(\varepsilon^2\right) = \varepsilon \cdot L_{mX} + \underline{O}\left(\varepsilon^2\right).$$

From here we will receive the necessary. Similarly for a field of tensors $\mathcal{L}_m$.

Take account the previous lemma, we will receive

$$\lim_{\varepsilon \to 0} \frac{1}{\varepsilon^r} \xi \wedge \tilde{\xi}_1^{(\varepsilon)} \wedge ... \wedge \tilde{\xi}_r^{(\varepsilon)} = \\ = \delta(\xi) \wedge L_{1X}\left(\xi^2\right) \wedge ... \wedge L_{rX}\left(\xi^{r+1}\right).$$

The given equality allows to receive a necessary and sufficient condition r-g.i.t. That infinitesimal transformation X was r-g.i.t. necessary and sufficient that conditions were satisfied

$$\delta(\xi) \wedge L_{1X}\left(\xi^2\right) \wedge ... \wedge L_{rX}\left(\xi^{r+1}\right) = 0$$
$$\delta(\xi) \wedge L_{1X}\left(\xi^2\right) \wedge ... \wedge L_{r-1X}\left(\xi^r\right) \neq 0$$

For arbitrary ξ. As ξ is arbitrary, we discover

$$S\left(\delta \wedge L_{1X} \wedge ... \wedge L_{r-1X} \wedge L_{rX}\right) = 0, \quad (5)$$
$$S\left(\delta \wedge L_{1X} \wedge ... \wedge L_{r-1X}\right) \neq 0. \quad (6)$$

Similarly, we will receive the conditions equivalent to conditions (5) and (6)

$$S\left(\delta \wedge \mathcal{L}_{1X} \wedge ... \wedge \mathcal{L}_{r-1X} \wedge \mathcal{L}_{rX}\right) = 0, \quad (7)$$
$$S\left(\delta \wedge \mathcal{L}_{1X} \wedge ... \wedge \mathcal{L}_{r-1X}\right) \neq 0. \quad (8)$$

Conditions (7) and (8) are discovered S. G. Leiko in the coordinate form and are the equations r-g.i.t. $\tau(r)$.

If for infinitesimal transformation X the condition is satisfied $S\left(L_{rX}\right) = 0$, and as a condition (6), then X is absolutely canonical r-geodesic infinitesimal transformation.

4 HP-transformations.

By a Kählerian space (see [2], [5], [9]) we mean manifold M dimensions $n = 2m > 2$, with the (pseudo)Riemannian metric given on it g and complex structure F, which satisfy to conditions:

1) holds equality $F^2 = -\delta$;

2) for arbitrary field of vectorses X and Y $g(F(X), Y) + g(X, F(Y)) = 0$,

3) holds equality $\nabla F = 0$, where ∇ there is a connection of the Levi-Civita of a metric tensor g.

It is said that the field of vectors X is infinitesimal holomorphically projective transformation, or is simple, HP-transformation (see [9]) if it satisfies to a condition

$$\mathcal{L}_X\nabla = \beta \otimes \delta + \delta \otimes \beta - \bar{\beta} \otimes F - F \otimes \bar{\beta},$$

where β - some field of covectors on M, $\bar{\beta} = \beta \circ F$ - dual with it a field of covectors .

If $\beta = 0$, then the HP-transformation reduces to the affine. This case will be trivial.

The field of vectors X is called *analytical* if the condition (see [9]) is satisfied $\mathcal{L}_X F = 0$.

As shown in [9], that *infinitesimal transformation preserved holomorphically planar curves necessary and sufficient that it was analytical HP-transformation.*

Fields of covectors β and $\bar{\beta}$ analytical HP-transformations possess properties which we will use next. It is shown in [9] (see the equality (3.8)), that in a coordinate neighbourhood $\left(U; u^h\right)$ hold equalities

$$\nabla_j \beta_i = -\frac{1}{n+2}\mathcal{L}_X R_{ji}, \quad (9)$$

where $R_{ji} = R^{\alpha}_{\alpha j,i}$ is a tensor of Ricci . As is known, the tensor of Ricci Riemannian spaces is symmetrical ([3]). Then the equality (9) shows, that *the covariant differential* $\nabla\beta$ *is symmetrical a tensor field.* Besides, the equality (see [9], equality (3.7)) holds

$$\nabla_j \bar{\beta}_i + \nabla_i \bar{\beta}_j = 0, \quad (10)$$

which shows, that

$$S\left(\nabla\bar{\beta}\right) = 0. \quad (11)$$

Theorem 2. *Nontrivial HP-transformation is 2-g.i.t.*

Proof. Let X is HP-transformation. We take an arbitrary geodesic curve $\mathscr{C}$ in M, admiting canonical parametre t. Let ξ is a field of tangent vectors along a curve $\mathscr{C}$. Then

$$L_{1X}\left(\xi^2\right) = 2\beta(\xi)\,\delta(\xi) - 2\bar{\beta}(\xi)\,F(\xi).$$

Besides, as $\nabla\delta = 0$ and $\nabla F = 0$,

$$L_{2X}\left(\xi^3\right) = 2\nabla\beta\left(\xi^2\right)\delta(\xi) - 2\nabla\bar{\beta}\left(\xi^2\right)F(\xi).$$

From here $\delta(\xi) \wedge L_{1X}\left(\xi^2\right) = -2\bar{\beta}(\xi)\,\delta(\xi) \wedge F(\xi)$ and $\delta(\xi) \wedge L_{1X}\left(\xi^2\right) \wedge L_{2X}\left(\xi^3\right) = 0$. The last shows, that when $\beta \neq 0$, infinitesimal transformation X are 2-g.i.t.. The theorem is proved

5 The flattening properties of vertical and complete lifts of analytical HP-transformations.

Proposition 1. *For an arbitrary field of vectors X with respect to connection of the complete lift ∇^{C}, holds equalities $L_{mX^{\mathrm{V}}} = (L_{mX})^{\mathrm{V}}$, $L_{mX^{\mathrm{C}}} = (L_{mX})^{\mathrm{C}}$,*

Proof is deduced by an induction on m. Considering a proposition 7.6 ([11]) we will obtain

$$(L_{1X})^{\mathrm{V}} = (L_X\nabla)^{\mathrm{V}} = L_{X^{\mathrm{V}}}\nabla^{\mathrm{C}} = L_{1X^{\mathrm{V}}},$$

$$(L_{1X})^{\mathrm{C}} = (L_X\nabla)^{\mathrm{C}} = L_{X^{\mathrm{C}}}\nabla^{\mathrm{C}} = L_{1X^{\mathrm{C}}}.$$

We suppose is shown, that $L_{m-1X^{\mathrm{V}}} = (L_{m-1X})^{\mathrm{V}}$ and $L_{m-1X^{\mathrm{C}}} = (L_{m-1X})^{\mathrm{C}}$. Then applying a proposition 6.5 ([11]) $L_{mX^{\mathrm{V}}} = \nabla^{\mathrm{C}} L_{m-1X^{\mathrm{V}}} = (L_{mX})^{\mathrm{V}}$, $L_{mX^{\mathrm{C}}} = \nabla^{\mathrm{C}} L_{m-1X^{\mathrm{C}}} = (L_{mX})^{\mathrm{C}}$.

Lemma 2. *Let in a point $p \in M$ holds equality*

$$S\left(\bar{\beta} \otimes \nabla\beta\right)\big|_p = 0, \qquad (12)$$

and for arbitrary fibre coordinates $y = \left(y^k\right) \in \mathbb{R}^n$ in a point $\tilde{p} = (p, y) \in TM$ holds equality

$$S\left(\bar{\beta} \otimes \nabla\beta\right)^{\mathrm{C}}\Big|_{\tilde{p}} = 0. \qquad (13)$$

Then is fulfilled the equality

$$\nabla\beta|_p = 0. \qquad (14)$$

Proof. C a s e 1. Let $\bar{\beta}\big|_p \neq 0$. The equality (12) can be noted in a form

$$S\left(\bar{\beta}\big|_p \otimes \nabla\beta|_p\right) = 0. \qquad (15)$$

Then applying the equality (15) to a lemma 1 (see [15]), considering symmetry a tensor field $\nabla\beta$, we will obtain equality (14).
C a s e 2. Let

$$\bar{\beta}\big|_p = 0. \qquad (16)$$

Then holds equality

$$\beta|_p = 0. \qquad (17)$$

From equalities (16) and (17) follow equalities

$$\bar{\beta}^{\mathrm{V}}\big|_{\tilde{p}} = \left(\bar{\beta}_i\big|_p, 0\right) = 0, \qquad (18)$$

$$\bar{\beta}^{\mathrm{C}}\big|_{\tilde{p}} = \left(\partial\bar{\beta}_i\big|_{\tilde{p}}, \bar{\beta}_i\big|_p\right) = \left(\partial\bar{\beta}_i\big|_{\tilde{p}}, 0\right),$$

$$\beta^{\mathrm{C}}\big|_{\tilde{p}} = \left(\partial\beta_i|_{\tilde{p}}, \beta_i|_p\right) = \left(\partial\beta_i|_{\tilde{p}}, 0\right). \qquad (19)$$

Applying a rule of a taking of the complete lift from a tensor product, from (13), considering equality (18), we will have

$$S\left(\bar{\beta}^{\mathrm{C}}\big|_{\tilde{p}} \otimes (\nabla\beta)^{\mathrm{V}}\Big|_{\tilde{p}}\right) = 0. \qquad (20)$$

C a s e 2.1. We suppose, that there is such collection of fibre coordinates $y = \left(y^k\right)$, that in a point $\tilde{p} = (p, y)$ условие The condition

$$\bar{\beta}^{\mathrm{C}}\big|_{\tilde{p}} \neq 0. \qquad (21)$$

is satisfied. Applying a lemma 1 (see [15]) to equality (20), taking into account a condition (21), we come to equality $(\nabla\beta)^{\mathrm{V}}\Big|_{\tilde{p}} = 0$, From which follows the equality (14).
C a s e 2.2. Let for arbitrary collections of fibre coordinates $y = \left(y^k\right)$ in a point $\tilde{p} = (p, y)$ holds equality $\bar{\beta}^{\mathrm{C}}\big|_{\tilde{p}} = 0$. Then holds also equality

$$\beta^{\mathrm{C}}\big|_{\tilde{p}} = 0. \qquad (22)$$

On the other hand, considering expressions for lifts (19) from equality (22) and definitions of the complete lift of function $\partial\beta_i|_{\tilde{p}} = y^s \cdot \partial_s\beta_i|_p$, we obtain equalities $y^s \cdot \partial_s\beta_i|_p = 0$ for any $i = \overline{1.n}$ and arbitrary collections $y = \left(y^k\right)$. As y^s it is arbitrary, from here to find $\partial_s\beta_i|_p = 0$ for any $i = \overline{1.n}$, $s = \overline{1.n}$. Then for arbitrary $i = \overline{1.n}$ and $j = \overline{1.n}$ it is had

$$\nabla_j\beta_i|_p = \partial_j\beta_i|_p - \Gamma^{\alpha}_{ji}\big|_p \cdot \beta_\alpha|_p = \partial_j\beta_i|_p = 0,$$

That gives equality $\nabla\beta|_p = 0$. The lemma is proved.

Theorem 3. *Let X is analytical HP-transformation of Kählerian spaces (M, g, F). Then:*

1. *lifts X^{V} and X^{C} are 1-g.i.t. if and only if $\beta = 0$ that is when X is an infinitesimal affinity;*
2. *lifts X^{V} and X^{C} are absolutely canonical 2-g.i.t. if and only if the field of covectors β is absolutely parallel, that is when $\nabla\beta = 0$.*
3. *generally lifts X^{V} and X^{C} are 3-g.i.t.*

Proof. We take in space M an arbitrary geodesic curve $\mathscr{C}$; Let ξ - a field of tangent vectors along a curve $\mathscr{C}$.

1) Condition $\tilde{\delta}(\xi) \wedge L_{1X^{\mathrm{V}}}(\xi^2) = 0$, to equivalently conditions $\beta^{\mathrm{V}}(\xi) = 0$, $\bar{\beta}^{\mathrm{V}}(\xi) = 0$. From ξ arbitrarily follows, that last conditions are equivalent to equalities

$$\beta = 0. \tag{23}$$

Similarly, condition $\tilde{\delta}(\xi) \wedge L_{1X^{\mathrm{C}}}(\xi^2) = 0$ to equivalently condition $\beta^{\mathrm{C}}(\xi) = 0$, $\bar{\beta}^{\mathrm{C}}(\xi) = 0$, $\bar{\beta}^{\mathrm{V}}(\xi) = 0$. From ξ arbitrarily follows, that last conditions are equivalent to equality (23). Thus, Lifts X^{V}, X^{C} are 1-g.i.t. if and only if $\beta = 0$ that is when X is an infinitesimal affinity.

2) Condition

$$\tilde{\delta}(\xi) \wedge L_{1X^{\mathrm{V}}}(\xi^2) \wedge L_{2X^{\mathrm{V}}}(\xi^3) = 0$$

to equivalently condition

$$M_{12} = \begin{vmatrix} \beta^{\mathrm{V}}(\xi) & \bar{\beta}^{\mathrm{V}}(\xi) \\ (\nabla\beta)^{\mathrm{V}}(\xi^2) & (\nabla\bar{\beta})^{\mathrm{V}}(\xi^2) \end{vmatrix} = 0,$$

that is to equality

$$\bar{\beta}^{\mathrm{V}}(\xi)\,(\nabla\beta)^{\mathrm{V}}(\xi^2) = 0. \tag{24}$$

Considering expression for components of lifts, we will obtain equality $\bar{\beta}_\iota \xi^\iota \nabla_j \beta_i \xi^i \xi^j = 0$, which whereas ξ is arbitrary, equivalent to equality

$$S(\bar{\beta} \otimes \nabla\beta) = 0. \tag{25}$$

Applying to equality (25) a lemma 2, we will obtain equality

$$\nabla\beta = 0. \tag{26}$$

On the other hand we see, that from equality (26) follows the equality $(\nabla\beta)^{\mathrm{V}} = 0$. So also equality $M_{12} = 0$.

Similarly, condition

$$\tilde{\delta}(\xi) \wedge L_{1X^{\mathrm{C}}}(\xi^2) \wedge L_{2X^{\mathrm{C}}}(\xi^3) = 0$$

to equivalently conditions $M_{12} = 0$, $M_{13} = 0$ и $M_{23} = 0$, where M_{12}, M_{13} and M_{23} are minors of the matrix

$$\begin{pmatrix} \beta^{\mathrm{C}}(\xi) & \bar{\beta}^{\mathrm{C}}(\xi) & \bar{\beta}^{\mathrm{V}}(\xi) \\ (\nabla\beta)^{\mathrm{C}}(\xi^2) & (\nabla\bar{\beta})^{\mathrm{C}}(\xi^2) & (\nabla\bar{\beta})^{\mathrm{V}}(\xi^2) \end{pmatrix}.$$

Equality $M_{13} = 0$ will take the form

$$\bar{\beta}^{\mathrm{V}}(\xi)\,(\nabla\beta)^{\mathrm{C}}(\xi^2) = 0; \tag{27}$$

from it we will obtain equality (26). Conversely, from equality (26) we will obtain equalities (27). Besides, from equality (26) follow equalities $L_{2X^{\mathrm{V}}}(\xi^3) = 0$ и $L_{2X^{\mathrm{C}}}(\xi^3) = 0$.

Thus, lifts X^{V} and X^{C} generate absolutely canonical 2-g.i.t. if and only if the covector field β is absolutely parallel.

3) Obviously

$$\tilde{\delta}(\xi) \wedge L_{1X^{\mathrm{V}}}(\xi^2) \wedge L_{2X^{\mathrm{V}}}(\xi^3) \wedge L_{3X^{\mathrm{V}}}(\xi^4) = 0$$

It is similarly shown, that

$$\tilde{\delta}(\xi) \wedge L_{1X^{\mathrm{C}}}(\xi^2) \wedge L_{2X^{\mathrm{C}}}(\xi^3) \wedge L_{3X^{\mathrm{C}}}(\xi^4) = \\ = 8M_{123}\tilde{\delta}(\xi) \wedge \delta^{\mathrm{V}}(\xi) \wedge F^{\mathrm{V}}(\xi) \wedge F^{\mathrm{C}}(\xi),$$

where M_{123} is a determinant

$$\begin{vmatrix} \beta^{\mathrm{C}}(\xi) & \bar{\beta}^{\mathrm{C}}(\xi) & \bar{\beta}^{\mathrm{V}}(\xi) \\ (\nabla\beta)^{\mathrm{C}}(\xi^2) & (\nabla\bar{\beta})^{\mathrm{C}}(\xi^2) & (\nabla\bar{\beta})^{\mathrm{V}}(\xi^2) \\ (\nabla^2\beta)^{\mathrm{C}}(\xi^3) & (\nabla^2\bar{\beta})^{\mathrm{C}}(\xi^3) & (\nabla^2\bar{\beta})^{\mathrm{V}}(\xi^3) \end{vmatrix}$$

Considering equality (10), we will obtain

$$\nabla_k \nabla_j \bar{\beta}_i + \nabla_k \nabla_i \bar{\beta}_j = \nabla_k(\nabla_j \bar{\beta}_i + \nabla_i \bar{\beta}_j) = 0. \tag{28}$$

From equality (28) we find

$$M_{123} = \begin{vmatrix} \beta^{\mathrm{C}}(\xi) & \bar{\beta}^{\mathrm{C}}(\xi) & \bar{\beta}^{\mathrm{V}}(\xi) \\ (\nabla\beta)^{\mathrm{C}}(\xi^2) & 0 & 0 \\ (\nabla^2\beta)^{\mathrm{C}}(\xi^3) & 0 & 0 \end{vmatrix} = 0.$$

In that case, a condition

$$\tilde{\delta}(\xi) \wedge L_{1X^{\mathrm{C}}}(\xi^2) \wedge L_{2X^{\mathrm{C}}}(\xi^3) \wedge L_{3X^{\mathrm{C}}}(\xi^4) = 0,$$

It is satisfied identically.

Thus, generally lifts X^{V} and X^{C} generate 3-g.i.t. The theorem is proved.

6 The flattening properties of 0th, I th and II th lifts of analytical HP-transformations.

Proposition 2. *For an arbitrary field of vectors X with respect to connection of the II-lift ∇^{II}, holds equalities $L_{mX^0} = (L_{mX})^0$, $L_{mX^{\mathrm{I}}} = (L_{mX})^{\mathrm{I}}$, $L_{mX^{\mathrm{II}}} = (L_{mX})^{\mathrm{II}}$.*

Proof to similarly proof of the proposition 1.

Lemma 3. *Let in a point $p \in M$ holds equality $S(\bar{\beta} \otimes \nabla\beta)\big|_p = 0$, and for arbitrary fibre coordinates $y = (y^k) \in \mathbb{R}^n$, $z = (z^k) \in \mathbb{R}^n$ in a point $\tilde{p} = (p, y, z) \in T^2M$ holds equality $S(\bar{\beta} \otimes \nabla\beta)^{\mathrm{I}}\big|_{\tilde{p}} = 0$. Then we have $\nabla\beta|_p = 0$.*

Proof To similarly proof of a lemma 2.

Lemma 4. *Let holds equality*

$$S\left(\bar{\beta}\otimes\nabla\beta\otimes\nabla\nabla\beta\right)=0. \tag{29}$$

Then we have equality

$$S\left(\nabla\nabla\beta\right)=0. \tag{30}$$

Proof. We will establish equality

$$\nabla\beta\otimes S\left(\nabla\nabla\beta\right)=0. \tag{31}$$

For this it verify in each point $p\in M$. The equality (29), according to a lemma 3 (see [18]), will take the form

$$S\left(S\left(\bar{\beta}\otimes\nabla\beta\right)\otimes S\left(\nabla\nabla\beta\right)\right)=0. \tag{32}$$

Case 1. Let in a point $p\in M$ the condition $S\left(\bar{\beta}\otimes\nabla\beta\right)\big|_p\neq 0$ is satisfied. Then taking account a lemma 1 (see [15]), applying the given condition, from equality (32) we will obtain equality of

$$S\left(\nabla\nabla\beta\right)|_p=0. \tag{33}$$

From here we will obtain equality of

$$\nabla\beta|_p\otimes S\left(\nabla\nabla\beta\right)|_p=0. \tag{34}$$

Case 2. Let holds equality of

$$S\left(\bar{\beta}\otimes\nabla\beta\right)\big|_p=0. \tag{35}$$

We take the I-lift from a tensor product, and from equality (32), taking account equalities (35) and 0-lift definitions, we will have equality of

$$S\left(S\left(\bar{\beta}\otimes\nabla\beta\right)^{\mathrm{I}}\Big|_{\tilde{p}}\otimes S\left(\nabla\nabla\beta\right)^{0}\Big|_{\tilde{p}}\right)=0. \tag{36}$$

Case 2.1. Let there will be such collections of fibre coordinates $y=\left(y^k\right)$, $z=\left(z^k\right)$, that in a point $\tilde{p}=(p,y,z)$ the condition of $S\left(\bar{\beta}\otimes\nabla\beta\right)^{\mathrm{I}}\Big|_{\tilde{p}}\neq 0.$ is satisfied. Then to equality (36) we can apply a lemma 1 (see [15]). From this we will obtain equality of $S\left(\nabla\nabla\beta\right)^{0}\Big|_{\tilde{p}}=0,$, from which the equality (33), and from here and equality (34) follows.

Case 2.2. Let for arbitrary collections of fibre coordinates $y=\left(y^k\right)$, $z=\left(z^k\right)$ in a point $\tilde{p}=(p,y,z)$ holds equality of

$$S\left(\bar{\beta}\otimes\nabla\beta\right)^{\mathrm{I}}\Big|_{\tilde{p}}=0. \tag{37}$$

Equalities (35) and (37) allow to apply a lemma 3 from which we will obtain equality of

$$\nabla\beta|_p=0. \tag{38}$$

From here we will obtain equality (34).

Now we will prove equalities (30). For this purpose we will show, that in each point $p\in M$ holds equality (33). In the given point the equality (31) takes the form (34).

Case 1. Let $\nabla\beta|_p\neq 0$. Then from equality (34) we will obtain (33).

Case 2. Let the equality (38) holds. We take the I-lift from equality (31) and we assume, that collections of fibre coordinates $y=\left(y^k\right)$, $z=\left(z^k\right)$ are arbitrary in a point $\tilde{p}=(p,y,z)$. Then we will obtain

$$(\nabla\beta)|^{\mathrm{I}}_{\tilde{p}}\otimes S\left(\nabla\nabla\beta\right)|^{0}_{\tilde{p}}=0. \tag{39}$$

Case 2.1. Let for some collection of fibre coordinates $y=\left(y^k\right)$, $z=\left(z^k\right)$ in a point $\tilde{p}=(p,y,z)$ the condition of $(\nabla\beta)|^{\mathrm{I}}_{\tilde{p}}\neq 0.$ is satisfied. Then from equality (39) we will obtain $S\left(\nabla\nabla\beta\right)|^{0}_{\tilde{p}}=0,$ that implies equality (33).

Case 2.2. Let now for any collections of fibre coordinates $y=\left(y^k\right)$, $z=\left(z^k\right)$ in a point $\tilde{p}=(p,y,z)$ the condition of

$$(\nabla\beta)|^{\mathrm{I}}_{\tilde{p}}=0. \tag{40}$$

is satisfied. Taking account I-lift definition, equality (40), which is true for any collections $y=\left(y^k\right)\in\mathbb{R}^n$, we will obtain $\partial_s\left(\nabla_j\beta_i\right)|_p=0$, for arbitrary $s,i,j=\overline{1,n}$. From this, we find $\nabla\nabla\beta|_p=0$, It reduces to equality (33). The lemma is proved.

Theorem 4. *Let X is analytical HP-transformation Kählerian spaces (M,g,F). Then:*

1. *lifts X^0, X^{I}, X^{II} are 1-g.i.t. if and only if $\beta=0$ that is when X is an infinitesimal affinity;*
2. *lifts X^0, X^{I}, X^{II} are absolutely canonical 2-g.i.t. if and only if the covector field β is absolutely parallel, that is when $\nabla\beta=0$.*
3. *в общем случае лифт X^0 является 3-г.и.п.; lifts X^{I}, X^{II} are absolutely canonical 3-g.i.t. if and only if the covector field β is not absolutely parallel and satisfies to equality of $S\left(\nabla^2\beta\right)=0$;*
4. *generally lifts X^{I}, X^{II} are 4-g.i.t.*

Proof. We take in space M an arbitrary geodesic curve $\mathscr{C}$; Let ξ - a field of tangent vectors along a curve $\mathscr{C}$. Taking account properties of lifts, we will obtain.

1) Obviously conditions of $\tilde{\delta}(\xi) \wedge L_{1X^0}(\xi^2) = 0$, $\tilde{\delta}(\xi) \wedge L_{1X^{\mathrm{I}}}(\xi^2) = 0$, $\tilde{\delta}(\xi) \wedge L_{1X^{\mathrm{II}}}(\xi^2) = 0$, are equivalent respectively to conditions of $\beta^0(\xi) = 0$, $\bar{\beta}^0(\xi) = 0$, $\beta^{\mathrm{I}}(\xi) = 0$, $\beta^0(\xi) = 0$, $\bar{\beta}^0(\xi) = 0$, $\bar{\beta}^{\mathrm{I}}(\xi) = 0$, $\beta^{\mathrm{II}}(\xi) = 0$, $\beta^{\mathrm{I}}(\xi) = 0$, $\bar{\beta}^{\mathrm{II}}(\xi) = 0$, $\bar{\beta}^{\mathrm{I}}(\xi) = 0$, $\bar{\beta}^0(\xi) = 0$. It is equivalent to a condition $\beta = 0$ as ξ is arbitrary.

Thus, lifts X^0, X^{I}, X^{II} are 1-g.i.t. if and only if $\beta = 0$ that is when X is an infinitesimal affinity.

2) Obviously

$$\begin{aligned}\tilde{\delta}(\xi) \wedge L_{1X^0}(\xi^2) \wedge L_{2X^0}(\xi^3) &= \\ &= -4M_{12}^0 \tilde{\delta}(\xi) \wedge \delta^0(\xi) \wedge F^0(\xi),\end{aligned} \tag{41}$$

where $M_{12}^0 = \begin{vmatrix} \beta^0(\xi) & \bar{\beta}^0(\xi) \\ (\nabla\beta)^0(\xi^2) & (\nabla\bar{\beta})^0(\xi^2) \end{vmatrix}$. From equality (11) we will obtain equalities of

$$S\left(\nabla\bar{\beta}\right)^0 = 0,\ S\left(\nabla\bar{\beta}\right)^{\mathrm{I}} = 0,\ S\left(\nabla\bar{\beta}\right)^{\mathrm{II}} = 0. \tag{42}$$

Condition of $\tilde{\delta}(\xi) \wedge L_{1X^0}(\xi^2) \wedge L_{2X^0}(\xi^3) = 0$, to equivalently equality of

$$\begin{vmatrix} \beta^0(\xi) & \bar{\beta}^0(\xi) \\ (\nabla\beta)^0(\xi^2) & 0 \end{vmatrix} = 0, \tag{43}$$

which is equivalent to equality of

$$\bar{\beta}^0(\xi)(\nabla\beta)^0(\xi^2) = 0.$$

Taking account expressions for lifts, and ξ is arbitrary, we come to equality (25). Applying to equality (25) a lemma 3, we will have (26). On the other hand, from equality (26) the equality (43) follows.

Condition $\tilde{\delta}(\xi) \wedge L_{1X^{\mathrm{I}}}(\xi^2) \wedge L_{2X^{\mathrm{I}}}(\xi^3) = 0$ to equivalently equalities of

$$\begin{aligned}&M_{12}^{\mathrm{I}} = 0,\ M_{13}^{\mathrm{I}} = 0,\ M_{14}^{\mathrm{I}} = 0,\\ &M_{23}^{\mathrm{I}} = 0,\ M_{24}^{\mathrm{I}} = 0,\end{aligned} \tag{44}$$

where M_{12}^{I}, M_{13}^{I}, M_{14}^{I}, M_{23}^{I}, M_{24}^{I}, M_{34}^{I} minors of a matrix of

$$\begin{pmatrix} \beta^{\mathrm{I}}(\xi) & \beta^0(\xi) & \bar{\beta}^{\mathrm{I}}(\xi) & \bar{\beta}^0(\xi) \\ (\nabla\beta)^{\mathrm{I}}(\xi^2) & (\nabla\beta)^0(\xi^2) & (\nabla\bar{\beta})^{\mathrm{I}}(\xi^2) & (\nabla\bar{\beta})^0(\xi^2) \end{pmatrix}$$

The equality $M_{24}^{\mathrm{I}} = 0$, taking into account equalities (42), is (43) from which the equality (26) follows; On the other hand, equalities (26) imply equalities (44).

Condition $\tilde{\delta}(\xi) \wedge L_{1X^{\mathrm{II}}}(\xi^2) \wedge L_{2X^{\mathrm{II}}}(\xi^3) = 0$ to equivalently equalities of

$$\begin{aligned}&M_{12}^{\mathrm{II}} = 0,\ M_{13}^{\mathrm{II}} = 0,\ M_{14}^{\mathrm{II}} = 0,\ M_{15}^{\mathrm{II}} = 0,\\ &M_{23}^{\mathrm{II}} = 0,\ M_{24}^{\mathrm{II}} = 0,\ M_{25}^{\mathrm{II}} = 0,\end{aligned} \tag{45}$$

where M_{12}^{II}, M_{13}^{II}, M_{14}^{II}, M_{15}^{II}, M_{23}^{II}, M_{24}^{II}, M_{25}^{II}, M_{34}^{II}, M_{35}^{II}, M_{45}^{II} minors of a matrix of

$$\begin{pmatrix} \beta^{\mathrm{II}}(\xi) & \beta^{\mathrm{I}}(\xi) & \bar{\beta}^{\mathrm{II}}(\xi) & \bar{\beta}^{\mathrm{I}}(\xi) & \bar{\beta}^0(\xi) \\ (\nabla\beta)^{\mathrm{II}}(\xi^2) & (\nabla\beta)^{\mathrm{I}}(\xi^2) & (\nabla\bar{\beta})^{\mathrm{II}}(\xi^2) & (\nabla\bar{\beta})^{\mathrm{I}}(\xi^2) & (\nabla\bar{\beta})^0(\xi^2) \end{pmatrix}$$

From equality $M_{25}^{\mathrm{II}} = 0$, taking account equalities (42), we will obtain equality (26). On the other hand, the equality (26) reduces to equalities (45) and to equalities $L_{2X^0}(\xi^3) = 0$, $L_{2X^{\mathrm{I}}}(\xi^3) = 0$ and $L_{2X^{\mathrm{II}}}(\xi^3) = 0$.

Thus, lifts X^0, X^{I}, X^{II} are absolute canonical 2-g.i.t. if and only if the covector field β is absolute parallel.

3) Obviously

$$\tilde{\delta}(\xi) \wedge L_{1X^0}(\xi^2) \wedge L_{2X^0}(\xi^3) \wedge L_{3X^0}(\xi^4) = 0$$

So generally, the lift X^0 generates 3-g.i.t. It is similarly shown, that a condition of

$$\tilde{\delta}(\xi) \wedge L_{1X^{\mathrm{I}}}(\xi^2) \wedge L_{2X^{\mathrm{I}}}(\xi^3) \wedge L_{3X^{\mathrm{I}}}(\xi^4) = 0$$

to equivalently conditions of

$$M_{123}^{\mathrm{I}} = 0,\ M_{124}^{\mathrm{I}} = 0,\ M_{134}^{\mathrm{I}} = 0,\ M_{234}^{\mathrm{I}} = 0, \tag{46}$$

where M_{123}^{I}, M_{124}^{I}, M_{134}^{I}, M_{234}^{I} minors of a matrix of

$$\begin{pmatrix} \beta^{\mathrm{I}}(\xi) & \beta^0(\xi) & \bar{\beta}^{\mathrm{I}}(\xi) & \bar{\beta}^0(\xi) \\ (\nabla\beta)^{\mathrm{I}}(\xi^2) & (\nabla\beta)^0(\xi^2) & (\nabla\bar{\beta})^{\mathrm{I}}(\xi^2) & (\nabla\bar{\beta})^0(\xi^2) \\ (\nabla^2\beta)^{\mathrm{I}}(\xi^3) & (\nabla^2\beta)^0(\xi^3) & (\nabla^2\bar{\beta})^{\mathrm{I}}(\xi^3) & (\nabla^2\bar{\beta})^0(\xi^3) \end{pmatrix} \tag{47}$$

Taking account expressions for lifts, from equality (28) it is had

$$\begin{aligned}&\left(\nabla^2\bar{\beta}\right)^0(\xi^3) = 0,\ \left(\nabla^2\bar{\beta}\right)^{\mathrm{I}}(\xi^3) = 0,\\ &\left(\nabla^2\bar{\beta}\right)^{\mathrm{II}}(\xi^3) = 0\end{aligned} \tag{48}$$

Taking account equalities (42) and (48) in (46), we will obtain

$$\begin{aligned}\bar{\beta}^0(\xi)\Big((\nabla\beta)^{\mathrm{I}}(\xi^2)\left(\nabla^2\beta\right)^0(\xi^3) &- \\ - (\nabla\beta)^0(\xi^2)\left(\nabla^2\beta\right)^{\mathrm{I}}(\xi^3)\Big) &= 0.\end{aligned}$$

From here we will obtain

$$S\left(\bar{\beta}\otimes\nabla\beta\otimes\nabla\nabla\beta\right)=0. \qquad (49)$$

From a lemma 4 the equality of

$$S\left(\nabla\nabla\beta\right)=0. \qquad (50)$$

follows. Conversely, let the equality (50) is true. Then it is obvious $S\left(\nabla\nabla\beta\right)^0=0$, $S\left(\nabla\nabla\beta\right)^{\mathrm{I}}=0$. In that case the matrix (47) will take the form of

$$\begin{pmatrix} \beta^{\mathrm{I}}(\xi) & \beta^0(\xi) & \bar{\beta}^{\mathrm{I}}(\xi) & \bar{\beta}^0(\xi) \\ (\nabla\beta)^{\mathrm{I}}(\xi^2) & (\nabla\beta)^0(\xi^2) & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

From here conditions (46) follow. Besides, the equality (50) implies equality $L_{3X^{\mathrm{I}}}\left(\xi^4\right)=0$.

Thus, the lift X^{I} is absolutely canonical 3-g.i.t. if and only if the covector field β is not absolute parallel and satisfies to equality (50).

It is easy to show, that a condition of

$$\tilde{\delta}\left(\xi\right)\wedge L_{1X^{\mathrm{II}}}\left(\xi^2\right)\wedge L_{2X^{\mathrm{II}}}\left(\xi^3\right)\wedge L_{3X^{\mathrm{II}}}\left(\xi^4\right)=0$$

to equivalently condition of

$$\begin{aligned} &M^{\mathrm{II}}_{123}=0, M^{\mathrm{II}}_{124}=0, M^{\mathrm{II}}_{125}=0, M^{\mathrm{II}}_{134}=0,\\ &M^{\mathrm{II}}_{135}=0, M^{\mathrm{II}}_{145}=0, M^{\mathrm{II}}_{234}=0, M^{\mathrm{II}}_{235}=0,\\ &M^{\mathrm{II}}_{245}=0, M^{\mathrm{II}}_{345}=0, \end{aligned} \qquad (51)$$

where M^{II}_{123}, M^{II}_{124}, M^{II}_{125}, M^{II}_{134}, M^{II}_{135}, M^{II}_{145}, M^{II}_{234}, M^{II}_{235}, M^{II}_{245}, M^{II}_{345} are minors of a matrix of

$$\begin{pmatrix} \beta^{\mathrm{II}}(\xi) & \beta^{\mathrm{I}}(\xi) & \bar{\beta}^{\mathrm{II}}(\xi) & \bar{\beta}^{\mathrm{I}}(\xi) & \bar{\beta}^0(\xi) \\ (\nabla\beta)^{\mathrm{II}}(\xi^2) & (\nabla\beta)^{\mathrm{I}}(\xi^2) & 0 & 0 & 0 \\ (\nabla^2\beta)^{\mathrm{II}}(\xi^3) & (\nabla^2\beta)^{\mathrm{I}}(\xi^3) & 0 & 0 & 0 \end{pmatrix} \qquad (52)$$

The equality $M^{\mathrm{II}}_{125}=0$ will take the form of

$$\bar{\beta}^0\left(\xi\right)\Big(\left(\nabla\beta\right)^{\mathrm{II}}\left(\xi^2\right)\left(\nabla^2\beta\right)^{\mathrm{I}}\left(\xi^3\right)- \\ -\left(\nabla\beta\right)^{\mathrm{I}}\left(\xi^2\right)\left(\nabla^2\beta\right)^{\mathrm{II}}\left(\xi^3\right)\Big)=0.$$

From here we will obtain

$$\bar{\beta}_\alpha\xi^\alpha\nabla_\beta\beta_\iota\xi^\iota\xi^\beta\nabla_k\nabla_j\beta_i\xi^i\xi^j\xi^k=0.$$

As last equality is satisfied for arbitrary ξ we will obtain equality (49) which taking account a lemma 4, implies equality (50).

Conversely, if the equality (50) is valid the matrix (52) will take the form of

$$\begin{pmatrix} \beta^{\mathrm{II}}(\xi) & \beta^{\mathrm{I}}(\xi) & \bar{\beta}^{\mathrm{II}}(\xi) & \bar{\beta}^{\mathrm{I}}(\xi) & \bar{\beta}^0(\xi) \\ (\nabla\beta)^{\mathrm{II}}(\xi^2) & (\nabla\beta)^{\mathrm{I}}(\xi^2) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

that implies conditions (51) are satisfied. Besides, the equality (50) reduces to equality $L_{3X^{\mathrm{II}}}\left(\xi^4\right)$.

Thus, the lift X^{II} is 3-g.i.t. if and only if the covector field β is not absolute parallel and satisfies to equality (50).

4) We take from equality (28) a covariant differential; we will obtain

$$\left(\nabla^3\bar{\beta}\right)^0\left(\xi^4\right)=0,\ \left(\nabla^3\bar{\beta}\right)^{\mathrm{I}}\left(\xi^4\right)=0,\\ \left(\nabla^3\bar{\beta}\right)^{\mathrm{II}}\left(\xi^4\right)=0.$$

It is easy to show, that a condition of

$$\tilde{\delta}\left(\xi\right)\wedge L_{1X^{\mathrm{I}}}\left(\xi^2\right)\wedge L_{2X^{\mathrm{I}}}\left(\xi^3\right)\wedge \\ \wedge L_{3X^{\mathrm{I}}}\left(\xi^4\right)\wedge L_{4X^{\mathrm{I}}}\left(\xi^5\right)=0$$

to equivalently condition $M^{\mathrm{I}}_{1234}=0$, where

$$M^{\mathrm{I}}_{1234}=\begin{vmatrix} \beta^{\mathrm{I}}(\xi) & \beta^0(\xi) & \bar{\beta}^{\mathrm{I}}(\xi) & \bar{\beta}^0(\xi) \\ (\nabla\beta)^{\mathrm{I}}(\xi^2) & (\nabla\beta)^0(\xi^2) & 0 & 0 \\ (\nabla^2\beta)^{\mathrm{I}}(\xi^3) & (\nabla^2\beta)^0(\xi^3) & 0 & 0 \\ (\nabla^3\beta)^{\mathrm{I}}(\xi^4) & (\nabla^3\beta)^0(\xi^4) & 0 & 0 \end{vmatrix}=0$$

That it is easy to obtain application of the theorem of Laplace about determinant expansion on columns 3, 4; All minors of the second order arranged in columns 3 and 4 vanish. Means, the given condition is satisfied identically. Thus, generally the lift X^{I} is 4-g.i.t. It is easy to show, that a condition of

$$\tilde{\delta}\left(\xi\right)\wedge L_{1X^{\mathrm{II}}}\left(\xi^2\right)\wedge L_{2X^{\mathrm{II}}}\left(\xi^3\right)\wedge \\ \wedge L_{3X^{\mathrm{II}}}\left(\xi^4\right)\wedge L_{4X^{\mathrm{II}}}\left(\xi^5\right)=0$$

to equivalently condition of

$$\begin{aligned} M^{\mathrm{II}}_{1234}=0,\ M^{\mathrm{II}}_{1235}=0,\\ M^{\mathrm{II}}_{1345}=0,\ M^{\mathrm{II}}_{2345}=0, \end{aligned} \qquad (53)$$

where M^{II}_{1234}, M^{II}_{1235}, M^{II}_{1345}, M^{II}_{2345} minors of a matrix of

$$\begin{pmatrix} \beta^{\mathrm{II}}(\xi) & \beta^{\mathrm{I}}(\xi) & \bar{\beta}^{\mathrm{II}}(\xi) & \bar{\beta}^{\mathrm{I}}(\xi) & \bar{\beta}^0(\xi) \\ (\nabla\beta)^{\mathrm{II}}(\xi^2) & (\nabla\beta)^{\mathrm{I}}(\xi^2) & 0 & 0 & 0 \\ (\nabla^2\beta)^{\mathrm{II}}(\xi^3) & (\nabla^2\beta)^{\mathrm{I}}(\xi^3) & 0 & 0 & 0 \\ (\nabla^3\beta)^{\mathrm{II}}(\xi^4) & (\nabla^3\beta)^{\mathrm{I}}(\xi^4) & 0 & 0 & 0 \end{pmatrix}$$

Each of minors M^{II}_{1234}, M^{II}_{1235}, M^{II}_{1345}, M^{II}_{2345} has two columns arranged in which all minors of the second order vanish; Under the theorem of Laplace from here follows, that conditions (53) are satisfied identically.

Thus, generally the lift X^{II} is 4-g.i.t. The theorem is proved.

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