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Вимушені коливання системи «резервуар – капілярна рідина з вільною поверхнею»

Досліджується задача виходу механічної системи «циліндричний резервуар – рідина з вільною поверхнею» на усталений режим. Поведінка системи розглядається в рамках нелінійної моделі на тривалому проміжку часу при гармонічному збудженні руху системи періодичною силою, прикладеною до резервуара. Рух системи відбувається в полі слабкої гравітації з урахуванням сил поверхневого натягу на вільній поверхні рідини. Показано, що тільки в деяких окремих випадках можливий вихід системи на режим, близький до усталеного.

Ключові слова: нелінійна динаміка, слабка гравітація, вільна поверхня рідини, вихід на усталений режим.

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Forced oscillations of the system «reservoir – capillary liquid with a free surface»

The problem of attainment of the mechanical system «cylindrical reservoir – liquid with a free surface» on stationary mode is under consideration. Behaviour of the system is considered within the framework of nonlinear model on durable time interval under periodic force, applied to the reservoir. The system performs motion in a weak gravitational field with considering capillary forces on a free surface of liquid. It was shown that only rarely system attains mode of oscillations closed to stationary one.

Keywords: nonlinear dynamics of liquid, weak gravity, free surface of liquid, attainment of stationary mode.

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Introduction

We consider the problem of dynamics of liquid with a free surface in the cylindrical reservoir. Mechanical system performs motion in weak gravity field with considering capillary forces. We stated the problem of investigation of attainment of the system on steady mode within the framework of multimode nonlinear model.

In theoretical publications only low-dimensional models were used for investigation of steady oscillations. Such investigations were done under the assumption that oscillations of the system are performed only with frequencies multiple to the frequency of forced oscillations. However, experimenttal investigations [2, 3] showed that on disturbance of oscillations with respect to main mode excitation of oscillations of higher normal modes with their normal frequencies, which can be not multiple with frequency of system disturbance, are by all means take place. In publication [3] for the reservoir of rectangular shape it was shown that under real conditions attainment of the system to steady mode of oscillations does not manifested in pure form in resonant and above resonant frequency range.

For investigation of the stated problem we make use the model [1], which was examined by the examples of transient processes for problems of dynamics of cylindrical reservoirs with free surfaced liquid. We state the problem of studying behavior of the system on durable time interval under disturbance of the system motion system by harmonic force applied to reservoir. System behavior was considered for frequencies in below and above resonance range as well as in a vicinity of resonance frequency.

Discrete model of the mechanical system

Let us consider the problem about oscillations of the mechanical system “cylindrical reservoir and free surfaced liquid”. We consider the reservoir as absolutely rigid body, which can perform translational motion under action of active forces. Liquid is supposed to be ideal, incompressible, homogeneous and its initial motion is irrotational. The system is considered under the conditions of weak capillarity, when free falling acceleration is smaller than the ground-based one 1000 times, so, it is necessary to take into account action of capillary forces on a free surface of liquid.

We construct the discrete model of the mechanical system “reservoir – liquid with a free surface” on the basis of the Hamilton-Ostyrogradskiy variation al principle, method of modal decomposition and the method [1], which enables analytical elimination of kinematic boundary condition on free surface of liquid. This model includes the following independent parameters, a_i are coefficients of decomposition of elevation of a free surface of liquid ξ with respect to normal modes of oscillation of a free surface of liquid ψ_i, ε_i are components of the vector of displacement of the reservoir (center of undisturbed free surface of liquid)

$$\sum_{n=1}^{N} p_{rn}\ddot{a}_n + \sum_{n=N+1}^{N+3} p_{rn}\ddot{\varepsilon}_{n-N} = q_r, \quad r = \overline{1, N+3},$$

here p_{rn} is quadratic matrix and q_r is vector, which depend on a_n, $\dot{a}_n$, ε_n, $\dot{\varepsilon}_n$ and the vector of active external forces $\vec{F}$. The obtained discrete model describes dynamics of combined motion of the system “reservoir – free surfaced liquid” for different types of kinematic perturbations and dynamic disturbances of liquid free surface and reservoir.

Results of numerical modeling

We consider the circular cylindrical reservoir with vertical longitudinal axis *Oz*, which performs translational motion in the coordinate plane *Oy*. The reservoir of radius R = 0.3 m and the mass M_T is partially filled by liquid with the mass M_F tile the depth $H=R$. In numerical experiments we accepted M_T = $0,1M_F$, physical constants (surface tension σ and density ρ) were accepted for water, values of contact angular θ_1 varies within the range $70^o \div 90^o$, we accept the value g = $0,001g_0$ for free falling acceleration, where g_0 = 9,81 m/s^2 pis free falling acceleration on the Earth surface. Motion of the system occurs due to action of horizontal force $F = F_y \cos pt$, applied to the reservoir walls, initial elevation of a free surface of liquid is supposed to be absent. For all below considered cases values of the amplitude of external horizontal force, applied to the reservoir, were selected such that oscillations of a liquid free surface occur in nonlinear range of variation of wave amplitudes, namely, amplitudes of liquid free surface near reservoirs walls were within the range $(0,2 \div 0,25)R$.

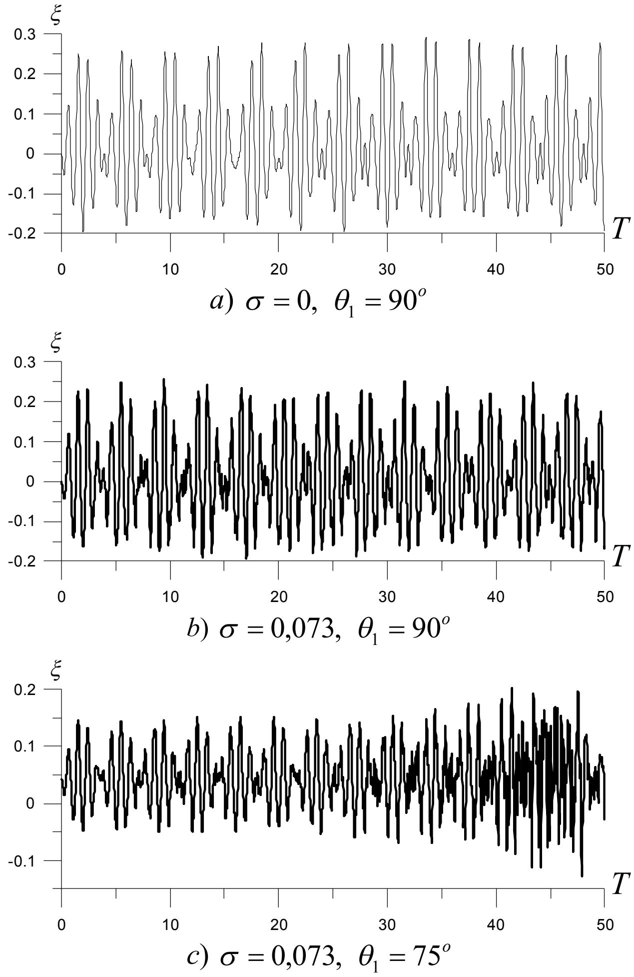


Fig. 1. Elevation on liquid free surface near reservoir walls for below resonance frequency of motion disturbance

Let us consider nonlinear oscillations of liquid free surface when frequency of external force is in below resonance range, namely, $p = \omega_1$, where ω_1 is partial frequency of the first antisymmetric normal mode. Normal (resonance) frequency of the mechanical system for the specified ratio of masses M_T = $0,1M_F$ is equal to $\omega_e = 1,2834\omega_1$.

Amplitudes of elevation of liquid free surface is shown in Fig. 1 for three cases of considering of capillary forces, namely, the absence of capillary forces (Fig. 1 *a*), the presence of capillary forces on a free surface of liquid (Fig. 1 *b*), the presence of capillary forces o both liquid free surface and contour of contact “liquid-gas-reservoir walls” (Fig. 1 *c*).

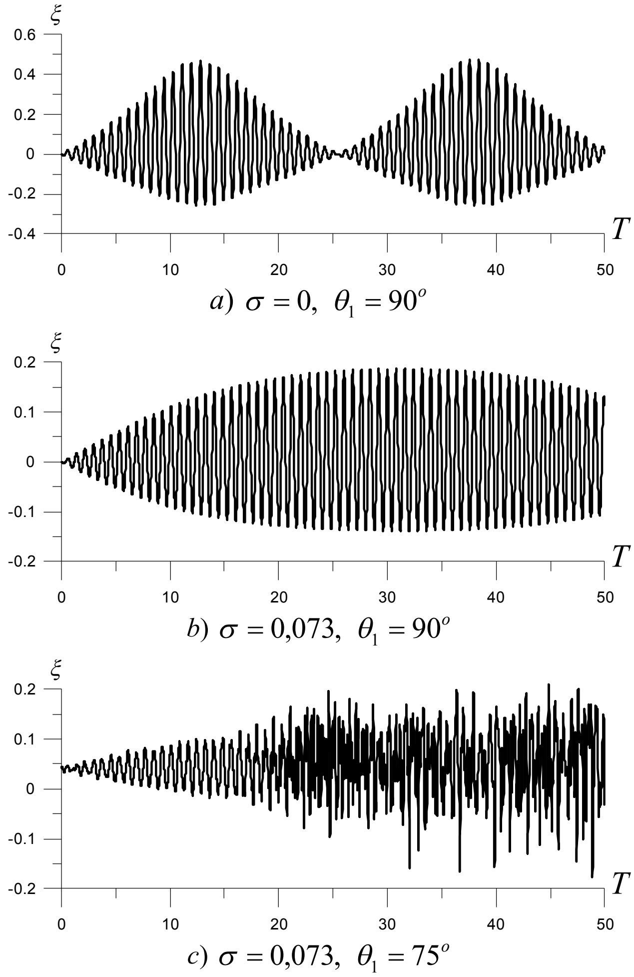
As it is seen from Fig. 1 account of capillary forces (Fig. 1 *b,c*) results in manifestation of decays and kinks on graphs, moreover, on decrease of contact angle oscillations tends to irregular character, which is the result of deepening of internal nonlinear constraints between normal modes and strengthening of contribution of highest normal modes.

We can also see from the graph that on the absence of surface tension (Fig. 1 *a*) and under the presence of surface tension on a free surface (Fig. 1 *b*) oscillations occur with noticeable modulation with practically immutable mean value of amplitudes. However, under the presence of surface tension on the contour of three phase contact (Fig. 1 *c*) contribution of high modes so considerable that two dominating frequencies disappear and the phenomenon of amplitude modulation is not manifested at all.

In general in the below resonance range of external disturbance of reservoir motion the system attains the mode close to steady one only under the absence of surface tension (Fig. 1 *a*). For this mode of the system motion noticeable amplitude modulation with practically immutable mean value of amplitudes is typical. Introduction of capillary forces into the system results in considerable increase of contribution of higher normal modes of oscillations and transition onto steady mode is not observed at all (Fig. 1 *b, c*).

Let us consider nonlinear oscillations of a free surface of liquid, when frequency of the external force is in a vicinity of resonance, namely $p = 1{,}25\omega_1$ (Fig. 2). As it is seen from the graph, on excitation of motion in a vicinity of resonance and under the absence of capillary forces (Fig. 2 *a*) тand under the presence of capillary forces on a free surface (Fig. 2 *b*) oscillations are performed with noticeable modulation of amplitude, but mean value of amplitude considerably varies in time. If we reduce contact angle (Fig. 2 *c*) amplitudes of highest normal modes increase and tends to the values of amplitudes of oscillations of two dominations harmonics with external and normal frequency, and character of oscillations become irregular.

Moreover, if we disturb system motion in a vicinity of resonance in the case of absence of capillary forces (Fig. 2 *a*) and under the presence of capillary forces on a free surface of liquid (Fig. 2 *b*) phenomenon of antiresonance is character, when for intervals of time comparable with several periods of oscillations with normal frequencies of the first antisymmetrical normal mode amplitudes of elevateon of a free surface of liquid reduces to the level $\xi \approx (0{,}01 \div 0{,}015)R$, so for certain time amplitudes reduced considerably (10–15 times). At the same time phenomenon of antiresonance is not manifested in the case of motion disturbance for below and above range of variation of frequencies. This phenolmenon differs from well-known beatings, because in the considered case we observe behavior of the multifrequency (but not double-frequency) system.



a) $\sigma = 0,\ \theta_1 = 90^o$

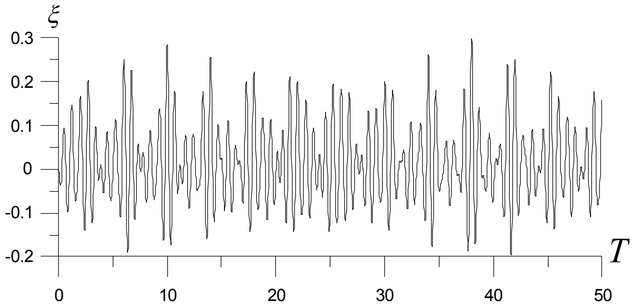
b) $\sigma = 0{,}073,\ \theta_1 = 90^o$

c) $\sigma = 0{,}073,\ \theta_1 = 75^o$

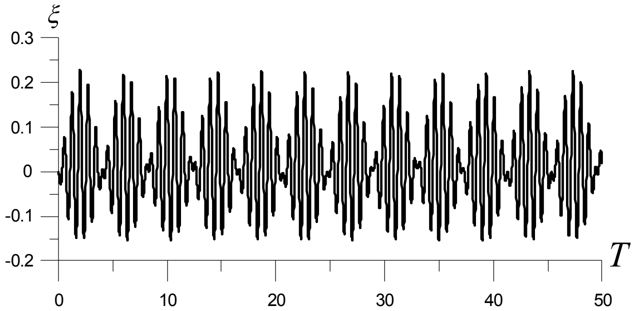
Fig. 2. Elevation on liquid free surface near reservoir walls for near resonance frequency of motion disturbance

On disturbance of system motion in a vicinity resonant frequency transition to the mode, close to steady one, occurs in the case, when capillary forces are absent (Fig. 2 *a*) of in the case of presence of capillary forces on a free surface of liquid (Fig. 2 *b*). Reduction of contact angle results in the effect that steady modes in the system are not manifested at all, and law of oscillations is considerably distorted by higher harmonic of spectrum (superharmonica and double-peaks are significantly manifested).

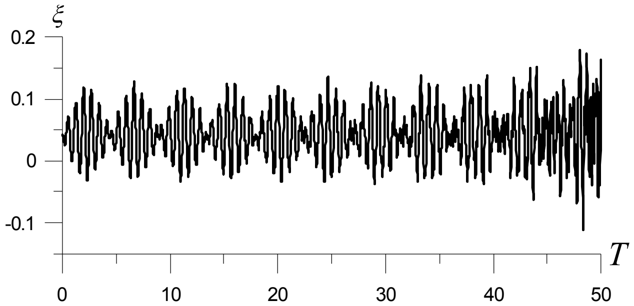
Let us consider nonlinear oscillations of a free surface of liquid, when frequency of the external force is in the above range, for example, $p = 1{,}5\omega_1$ (Fig. 3). As it is seen from Fig. 3 *a*, for oscillations in the above resonance range of motion disturbance under the absence of capillary forces oscillations of a free surface occur with noticeable amplitude modulation and time variable mean value, moreover, shape of the amplitude envelope is essentially distorted by higher harmonics of spectrum.



a) $\sigma = 0,\ \ \theta_1 = 90^o$



b) $\sigma = 0{,}073,\ \ \theta_1 = 90^o$



c) $\sigma = 0{,}073,\ \ \theta_1 = 75^o$

Fig. 3. Elevation on liquid free surface near reservoir walls for above resonance frequency of motion disturbance

In contrast to below resonance range of disturbances, where taking into account capillary forces on a free surface results in increase of contribution of high modes of oscillations (Fig. 1 *b*), in resonant and above resonance modes of disturbance the presence of capillary forces on a free surface of liquid on the contrary leads to the effect, when manifestation of of high modes of spectrum is practically absent, and harmonics with two frequencies (normal and forced) dominate (Fig. 2 *b*, 3 *c*). This means that capillary forces on a free surface of liquid act similar to selective filter, which restricts harmonics on combined and high frequencies and conserves only harmonics with normal and forced frequencies.

Taking into account capillary forces on the contour of three-phase contact (Fig. 3 *c*), for both below resonance and resonance range of excitation results in irregular oscillations, when on oscillations practically all harmonics manifest.

Conclusions

For the problem of dynamics of liquid with a free surface in the cylindrical reservoir with considering capillary forces we investigate potential of transition of the system to steady mode of oscillations. Behavior of the system is considered for frequency range below resonance, in a small vicinity of resonance and above resonance. In all cases transition to the mode of steady oscillations in classical sense does not occur.

The considered examples showed that the presence of capillary forces leads to increase of contribution of high modes of oscillations and, therefore, does not promote transition of the system to steady mode of oscillations.

Analysis showed that the main reason of differrence of the obtained results from the obtained before ones is usage of multimodal model of the system and refuse from the hypothesis about potential of neglect of oscillations with normal frequencies of modes, which is valid only for systems with one degree of freedoms.

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