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Linear and nonlinear analysis of time series: correlation dimension, Lyapunov exponents, and prediction

This paper focuses on reconstruction of dynamic models of geomagnetic indixes using satellite data. The possibility to calculate Lyapunov exponents for prediction the predictability.

Key Words: modelling, Lyapunov exponents, correlation dimension, prediction, chaos.

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Лінійний і нелінійний аналіз часового ряду: кореляційна розмірність, показники Ляпунова та прогнозування

В даній роботі на основі методу реконструкції динамічної моделі побудована модель геомагнітного індексу на основі супутникових даних. Показана можливість застосування показників Ляпунова для обчислення горизонту прогнозу.

Ключові слова: моделювання, показники Ляпунова, кореляційна розмірність, прогнозування, хаос.

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1 Introduction

Reconstruction of dynamic models of geomagnetic indexes focuses on time domain techniques to identify a dynamical model for the evolution of the *Dst*-index under the influence of the solar wind. Frequency domain analysis of this model is used to study spectral properties of the *Dst*-index dynamics. These properties indicate that the dynamics of the *Dst*-index is similar to that of a linear oscillator under the action of an external force.

2 Methods

2.1. Localized Lyapunov exponents and the prediction of predictability

The prediction of chaotic systems is a significant real-world but challenging open problem [1-16]. By definition, chaotic systems display sensitive dependence on initial conditions: two initially close trajectories can diverge exponentially in the phase space with a rate given by the largest Lyapunov exponent λ_1. If the initial perturbation is of size δ, and the accepted error tolerance, Δ, is still small, then the largest Lyapunov exponent λ_1 gives a rough estimate of the predictability time: $T_p = \frac{1}{\lambda_1}\ln\left(\frac{\Delta}{\delta}\right)$.

2.2. Robust estimates of correlation dimension

Analysis of nonlinear dynamics plays an important role in science. Especially low-dimensional chaos has been found in various natural and technical systems, e.g., space weather, EEG signals during sleep, or the cardiovascular system [17-28]. One of the important invariant measures to characterize a time series generated by nonlinear dynamics is the correlation dimension *D*. Following in the lines of Grassberger and Procaccia the fractal dimension of the attractor can be estimated from a time series by using the power law behavior of the correlation sum.

Suppose we have a scalar time series $x_1, x_2, \dots, x_{N_f}$

We make time-delay reconstruction of the phase space with the reconstructed vectors:

$$V_N = (x_n, x_{n=\tau}, \dots, x_{n-(d-1)\tau}), \qquad (1)$$

where τ is time-delay, d is embedding dimension, and $n = (d-1)\tau + 1, \dots, N_{f.}$ Following the embedding theorem there generically exist a function $F : R^d \to R$ such that

$$x_{n+1} = F(V_n), \qquad (2)$$

if d is sufficiently large. Then problem is how to choose the τ and d, i.e. time-delay and embedding dimension , such that the above equation exists.

To choose the embedding dimension d_e, we minimize the average absolute prediction errors E, i.e.

$$d_e = \arg\min\{E(d)\} : 1 \le d \le D_{\max},$$

where $D_{\max}$ is the maximum dimension we define in searching for the minimum value of $E(d)$.

2.3. Experimental data

The time series that characterize the solar wind consist from multidimensional components. These components increase the dimensionality of the model input; as a result the complexity of the identification problem significantly increases. It would be shown, that the identification problem can be reduced to the solving of the corresponding mathematical programming problem with constraints. Such approach substantially differs from the widely used statistical methods, where the great number of input variables is exploited. This paper focus on a novel approach to reconstructing guaranteed prediction interval, so called "prediction tube".

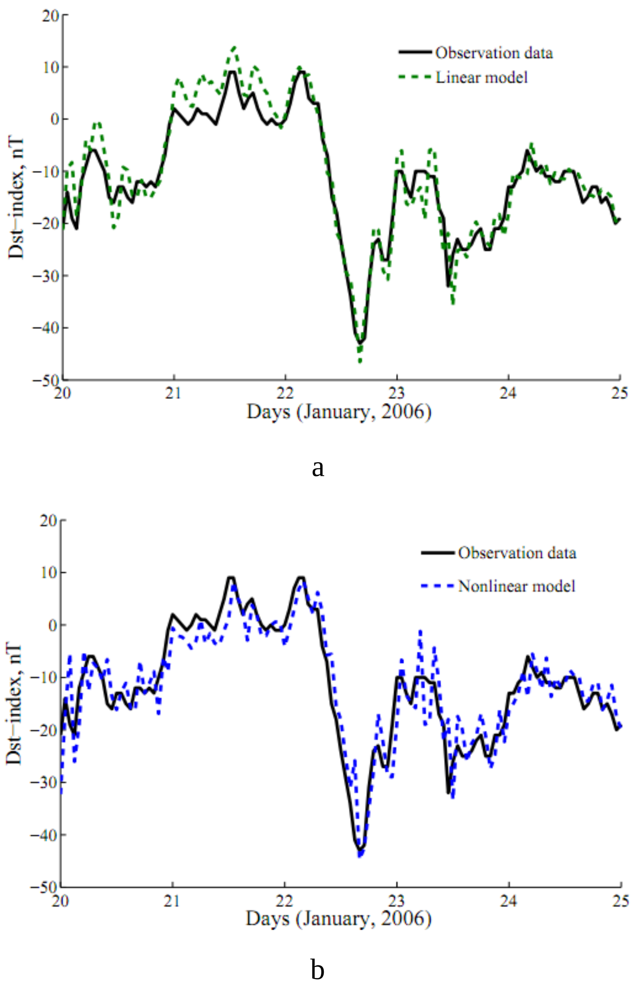
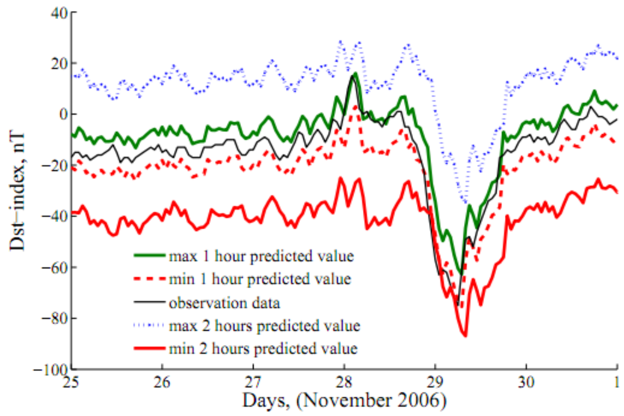
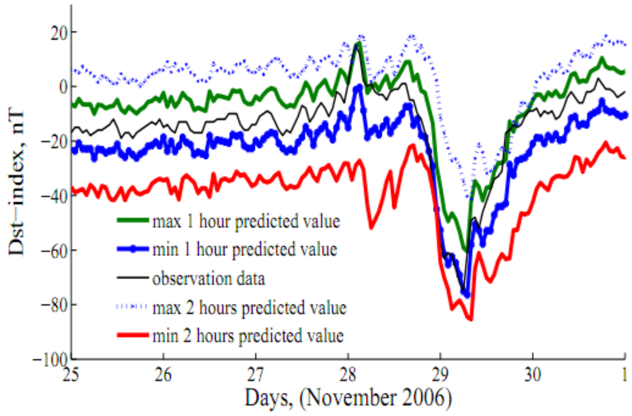


Figure 1. Illustration of *Dst*-index modelling: a – linear model (3); b – nonlinear model (4)



a



b

Figure 2. *Dst*-index observation data and results of guarantee prediction for one and two hours: a – linear model (3); b – nonlinear model (4)

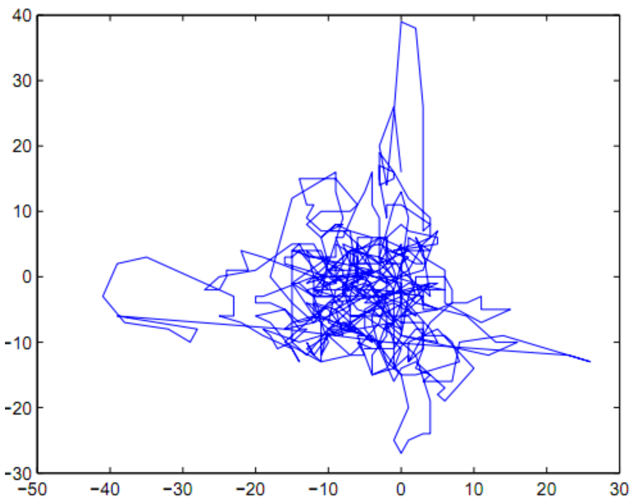


Figure 3. Phase space of *Dst*-index for 01.2012 (744 values)

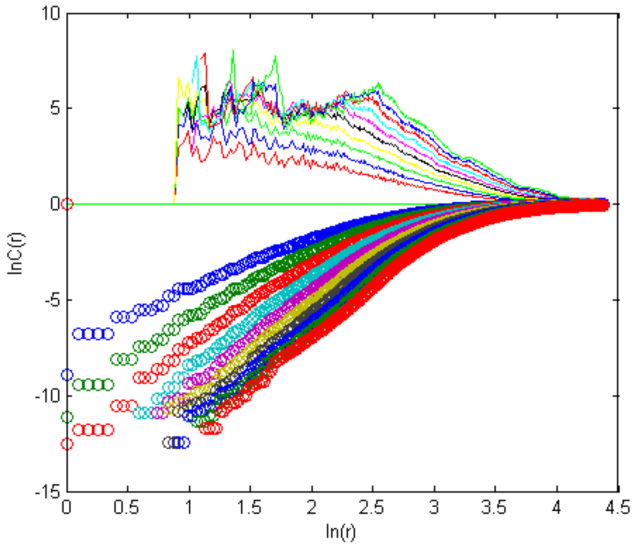


Figure 4. Combination of two plots: circles shows the dependency between correlation function of the distance between points; lines show derivatives of this function. Average values of these derivatives are:

- n=2: 0,9933735
- n=3: 1,3245144
- n=4: 1,7477829
- n=5: 2,1931131
- n=6: 2,6238707
- n=7: 3,0793864

The correlation dimension is equal 3,2.

3 Results

Two models, linear and nonlinear were reconstructed with the constraints of lags $n_y = 24$, for Dst -index guaranty prediction. First model was identified only from most significant linear regressors $\beta = 1$

$$\begin{aligned} y(k) &= 1.25y(k-1) - 0.35y(k-2) + 2.5u(k) + \\ &+ 0.15y(k-3) + 0.01y(k-6) - 2.74u(k-1) + \\ &+ 0.95u(k-2) - 0.2u(k-4) + 0.14u(k-8) \end{aligned} \quad (3)$$

And the second one was identifier using linear and nonlinear nodes $\beta = 0$

$$
\begin{aligned}
y(k) &= 2.5u(k) - 3.13u(k-1) + 0.65u(k-2) \\
&- 0.13u(k-4) + 0.19u(k-8) + 0.11u(k-12) \\
&- 0.03u(k-14) + 0.07u(k-25) + 1.17y(k-1) \\
&- 0.41y(k-2) + 0.12y(k-3) + 0.04y(k-6) \\
&+ 0.02y(k-12) - 0.004y(k-17) \\
&- 0.03u(k-3)y(k-9) + 0.01u(k-7)y(k-12) \\
&+ 0.01y(k-15)u(k-1) + 0.03u(k-15)u(k-6) \\
&- 0.003y(k-13)y(k-17) \\
&+ 0.03y(k-20)u(k-4) \\
&- 0.05y(k-21)u(k-4) - 0.35u(k-1)u)(k-1) \\
&+ 0.03u(k-16)y(k-2) \\
&- 0.022y(k-12)u(k-2) \\
&+ 0.002y(k-15)y(k-12) - 0.2u(k-1)u(k-3).
\end{aligned} \tag{4}
$$

The models (3) and (4) structure and parameters were numerically determined using the identification algorithm and data set of measurements for $\mathrm{N} = 2000$ hours. The parameter β was used for models accuracy test. On the figure 2.a the results of Dst-index behavior modeling using models (3) and (4) are shown together with observations. The analysis of $u(k)$ behavior on the 150 hours time interval before time moment k was made for determination of the the maximum changes of the $u(k)$ and value of the δ. On the figure 2.b are shown the results of Dst-index prediction if interval of estimation using models (3) and (4).

Let us conduct the analysis of the interval d_q change when $q = 1$ for the simplest case, on the example of the linear model (3). We can show that

$$
\begin{aligned}
\underline{y}(k+1) &= \min[1.255y(k) - 0.355y(k-1) \\
&+ 0.153y(k-2) + 0.013y(k-5) + 2.5(u(k) \\
&- \Delta u(k)) - 2.74u(k) + 0.95u(k-1) \\
&- 0.2u(k-3) + 0.14u(k-7)],
\end{aligned} \tag{5}
$$

$$
\begin{aligned}
\overline{y}(k+1) &= \max[1.255y(k) - 0.355y(k-1) \\
&+ 0.153y(k-2) + 0.013y(k-5) + 2.5(u(k) \\
&- \Delta u(k)) - 2.74u(k) + 0.95u(k-1) \\
&- 0.2u(k-3) + 0.14u(k-7)],
\end{aligned} \tag{6}
$$

and then conduct the numerical modeling of the $\{\overline{y}(k+1); \underline{y}(k+1)\}$ values change subject to $\Delta u_j(k)$. It is obviously that $\overline{y}(k+1)$ reaches it's maximum at $\Delta u_j(k) = \delta$, and $\underline{y}(k+1)$ reaches the minimum at $\Delta u_j(k) = -\delta$. Then

$$\underline{y}(k+1) = f(y,u) - 2.5\delta,$$

$$\overline{y}(k+1) = f(y,u) + 2.5\delta,$$

where

$$
\begin{aligned}
f(y,u) &= 1.255y(k) - 0.355y(k-1) \\
&+ 0.153y(k-2) + 0.013y(k-5) + 2.5(u(k) \\
&- \Delta u(k)) - 2.74u(k) + 0.95u(k-1) \\
&- 0.2u(k-3) + 0.14u(k-7).
\end{aligned} \tag{7}
$$

Let us represent the change of the interval d_1

$$d_1 = \left|\underline{y}(k+1) - \overline{y}(k+1)\right| \le 5\delta + 2\varepsilon,$$

where ε is the model accuracy error.

Now let us consider the dependence of d_q from δ during the estimation of $\tilde{y}(k+q), q = 2$. For this purpose let us rewrite (5) and (6) for $k+2$ time moment

$$\underline{y}(k+2) = f'(y,u) - 5.3975\delta,$$

$$\underline{y}(k+2) = f'(y,u) - 5.3975\delta,$$

where

$$
\begin{aligned}
f'(y,u) &= 1.22y(k) - 0.29y(k-1) + 0.19y(k-2) \\
&+ 0.01y(k-4) + 0.02y(k-5) + 0.89u(k) \\
&+ 1.19u(k-1) - 0.2u(k-2) - 0.25u(k-3) \\
&+ 0.14(k-6) + 0.18u(k-7).
\end{aligned}
$$

As result the interval d_2 of the value $\tilde{y}(k+2)$ can be represented as

$$d_2 = \left|\underline{y}(k+2) - \overline{y}(k+2)\right| \le 10.795\delta + 4.51\varepsilon.$$

Obviously, when q increases, the d_q value will also increase. Also it can be seen that using nonlinear model in the case $q > 2$ leads to quicker increase of d_q value. However, by using the methods of optimization it is possible to conduct the adaptive calculation of value δ when internal d_q will be minimal. In further research, we will examine the problem of prediction interval estimation solving the minimax mathematical programming problem.

4 Conclusions

This paper concentrates on the following problems: (a) empirical time series prediction with nonlinear autoregressive moving-average models; (b) dynamical-information approach to prediction of space weather; (c) estimation of Lyapunov exponents of time series from geomagnetic indexes; (d) dynamic probabilistic risk analysis of satellite devices with complex characterizations for damages using a physical model of elements and a predictable level of ionizing radiation and space weather; (e) spatio-temporal nonlinear and bilinear modelling the amplitude and location of the disturbance as a function of space, as well as its time evolution; (f) optimization techniques to identification of dynamical models using geomagnetic indexes. The local and global Lyapunov exponents based on infinitesimal uncertainty dynamics are considered to reflect an optimal predictability. An error analysis in bilinear dynamics is also used to develop criteria necessary for progress evaluation in space weather. Numerical results of *Dst*-index prediction are shown.

A new approach to geomagnetic *Dst*-index prediction using satellite observation data of solar wind parameters has been described. It is based on the reconstruction of a nonlinear discrete dynamical input-output system with several input variables [29-32]. A model structure, as well as its parameters are chosen by solving the constraint mathematical programming problem. An advantages of this approach are: 1) automatical selection of the appropriate input variables; 2) using an optimization method based on genetic algorithms for simultaneous search of the optimal model structure and its parameters. The problem of a guaranty prediction is also considered. The dependence between the prediction interval and the input model parameters changes has been discovered. It is shown that the prediction error is nonlinearly rising with the increase of the prediction time. The numerical calculations shown the possibility of forecasting horizon extension by using optimization methods. Also the possibility of real time prediction of *Dst*-index has been implemented.

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