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Варіанти напіврешіток

Для скінченної нижньої напіврешітки, яка розглядається як напівгрупа відносно взяття точної нижньої грані двох елементів, встановлено критерій ізоморфності двох варіантів. Потім цей критерій використовується для опису варіантів скінченного ланцюга, булеану скінченної множини, решітки розбиттів скінченної множини і решітки підпросторів скінченного простору.

Ключові слова: варіант, напіврешітка, сендвіч операція.

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Variants of a semilattice

In the paper there is obtained a criterion of isomorphism for two variants of a finite lower semilattice, which is considered as a semigroup under the operation of taking of the infinum (i.e. a greatest lower bound). Then this criterion is being used to describe variants of a finite chain, the powerset of a finite set, the lattice of partitions of a finite set, and the lattice of subspaces of a finite space.

Key Words: variant, semilattice, sandwich-operation.

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Introduction

Let S be a semigroup and $a \in S$. For $x, y \in S$ let $x *_a y = xay$, then $*_a$ is an associative binary operation on S. The operation $*_a$ is usually called the sandwich-operation. The semigroup $(S, *_a)$ is called the variant of S, or, alternatively, the sandwich semigroup of S with respect to the sandwich element a.

The task of researching variants of semigroups was first raised in Liapin's famous monograph [1]. Although initially Liapin formulated problem only for semigroups of transformations, soon different authors began to explore options for many other classes of semigroups (see for example, [2] [3] [4] [5], and chapter 13 of the monograph [6] with the literature listed there).

In this paper, we study variants of a finite lower semilattice that is considered as a semigroup under the operation of taking of the infimum of two elements.

The main result of this paper is Theorem 1, which is establishing a criterion of isomorphism of two variants for such semigroups. Then this criterion is used to describe variants of several classical lattices.

1 Isomorphism of variants criterion

Let $(L, \leqslant)$ be a finite lower semilattice with zero 0, $a \in L$. The semigroup $(S, *_a)$ with the operation $x *_a y = x \wedge a \wedge y$ is said to be the variant of semilattice L with the sandwich element a.

For each element $x \in [0, a]$ define weight $\omega(x) = |\{y \in L \mid a \wedge y = x\}|$.

Theorem 1 (Criterion of isomorphism of variants of lower semilattice). *Two variants $(L, *_a)$ and $(L, *_b)$ of semilattice L are isomorphic if and only if there exist an isomorphism from the interval $[0, a]$ to the interval $[0, b]$ which saves the weights of all elements.*

Proof. Necessity. Let variants $(L, *_a)$ and $(L, *_b)$ be isomorphic, and

$$\varphi : (L, *_a) \to (L, *_b) \quad \text{and} \quad \psi : (L, *_b) \to (L, *_a)$$

be mutually inverse isomorphisms. Then the following relations hold:

$$\varphi(x \wedge a \wedge y) = \varphi(x) \wedge b \wedge \varphi(y), \quad \text{and} \\ \psi(x \wedge b \wedge y) = \psi(x) \wedge a \wedge \psi(y). \quad (1)$$

When $x = y = a$, the first of these relations turns into equality $\varphi(a \wedge a \wedge a) = \varphi(a) \wedge b \wedge \varphi(a)$, which implies $\varphi(a) = \varphi(a) \wedge b$. Hence, $\varphi(a) \leqslant b$.

Similarly, when $x = y = a$, from the second relation from (1) we get that $\psi(b) = \psi(b) \wedge a$ and $\psi(b) \leqslant a$.

Applying the isomorphism φ to both parts of the equality $\psi(b) = \psi(b) \wedge a$, we obtain:

$$b = \varphi(\psi(b)) = \varphi(\psi(b) \wedge a) = \varphi(\psi(b) \wedge a \wedge a) = \\ = \varphi(\psi(b)) \wedge b \wedge \varphi(a) = b \wedge b \wedge \varphi(a) = b \wedge \varphi(a).$$

Hence, $b \leqslant \varphi(a)$. Then, taking into account an inequality $\varphi(a) \leqslant b$, it follows that $b = \varphi(a)$. Since isomorphisms φ i ψ are mutually inverse, we also get $a = \psi(b)$.

Now we'll prove that the isomorphism φ maps each element from the interval $[0, a]$ into an element from the interval $[0, b]$. Indeed, let $x \in [0, a]$. Then $x = x \wedge a$. Applying isomorphism φ to both parts of this equality, we get:

$$\varphi(x) = \varphi(x \wedge a) = \varphi(x \wedge a \wedge x) = \varphi(x *_a x) = \\ = \varphi(x) *_b \varphi(x) = \varphi(x) \wedge b \wedge \varphi(x) = \varphi(x) \wedge b.$$

It proves that $\varphi(x) \leqslant b$, thus $\varphi(x) \in [0, b]$.

If $x, y \in [0, a]$, then $x \wedge a \wedge y = x \wedge y$. Furthermore, in this case $\varphi(x), \varphi(y) \in [0, b]$, therefore $\varphi(x) \wedge b \wedge \varphi(y) = \varphi(x) \wedge \varphi(y)$. But then if $x, y \in [0, a]$, the first relation from (1) gives us equality $\varphi(x \wedge y) = \varphi(x) \wedge \varphi(y)$. Thus the restriction $\widetilde{\varphi}$ of isomorphism φ to interval $[0, a]$ is a homomorphism from the partially ordered set $[0, a]$ to the partially ordered set $[0, b]$. Since for $\widetilde{\varphi}$ there exists a mutually inverse homomorphism $\widetilde{\psi}$ induced by the mapping ψ, $\widetilde{\varphi}$ is an isomorphism of intervals $[0, a]$ and $[0, b]$.

It is left to prove that isomorphism $\widetilde{\varphi}$ saves the weights of all elements. Fix an element $x \in [0, a]$ and consider an arbitrary element $y \in L$ such that the equality $y \wedge a = x$ holds. Then

$$\widetilde{\varphi}(x) = \varphi(x) = \varphi(y \wedge a) = \varphi(y \wedge a \wedge a) = \\ = \varphi(y *_a a) = \varphi(y) *_b \varphi(a) = \\ = \varphi(y) \wedge b \wedge b = \varphi(y) \wedge b.$$

Therefore, the isomorphism φ maps every element of set $\Omega(x) = \{y \in L \mid a \wedge y = x\}$ into an element of set $\Omega(\widetilde{\varphi}(x)) = \{z \in L \mid z \wedge b = \widetilde{\varphi}(x)\}$. Similarly we check that inverse isomorphism ψ maps the set $\Omega(\widetilde{\varphi}(x))$ into the set $\Omega(x)$. But then $|\Omega(\widetilde{\varphi}(x))| = |\Omega(x)|$, i.e. $\omega(\widetilde{\varphi}(x)) = \omega(x)$.

Sufficiency. Let $\widetilde{\varphi}$ be an isomorphism from interval $[0, a]$ to interval $[0, b]$, which saves the weights of elements. We'll show that it can be expanded to an isomorphism $\varphi : (L, *_a) \to (L, *_b)$ of the corresponding variants.

For each element $x \in [0, a]$ consider the set $\Omega(x) = \{y \in L \mid a \wedge y = x\}$. The sets $\Omega(x)$, $x \in [0, a]$ are generating a partition of semilattice L. Indeed, since for any $y \in L$ we have $a \wedge y \in [0, a]$, then $\bigcup\limits_{x \in [0,a]} \Omega(x) = L$. On the other hand, sets $\Omega(x)$ are pairwise disjoint, because for any $x_1, x_2 \in [0, a]$ $y \in \Omega(x_1) \cap \Omega(x_2)$ implies $x_1 = a \wedge y = x_2$. Similarly one can show that sets $\Omega(\widetilde{\varphi}(x))$, $x \in [0, a]$ are also generating a partition of semilattice L.

The fact that isomorphism $\widetilde{\varphi}$ saves the weights of elements implies that for each $x \in [0, a]$ sets $\Omega(x)$ and $\Omega(\widetilde{\varphi}(x))$ have the same cardinality. For each element $x \in [0, a]$ fix an arbitrary bijection $\psi'_x : (\Omega(x) \setminus \{x\}) \to (\Omega(\widetilde{\varphi}(x)) \setminus \{\widetilde{\varphi}(x)\})$ and expand it to bijection $\psi_x : \Omega(x) \to \Omega(\widetilde{\varphi}(x))$, putting $\psi_x = \widetilde{\varphi}(x)$. Consider mapping

$$\varphi : (L, *_a) \to (L, *_b), \quad \text{where } \varphi(z) = \psi_x(z), \\ \text{if } z \in \Omega(x) \setminus \{x\}. \quad (2)$$

Obviously, the mapping φ is a bijection. To complete the proof it suffices to show that φ is a homomorphism of semigroups. Take any two elements $u \in \Omega(x)$, $v \in \Omega(y)$. Then element

$$u *_a v = u \wedge a \wedge v = u \wedge a \wedge a \wedge v = x \wedge y$$

belongs to the interval $[0, a]$, therefore $\varphi(u *_a v) = \widetilde{\varphi}(x \wedge y)$. On the other hand, for the elements u and v we have:

$$\varphi(u) *_b \varphi(v) = \varphi(u) \wedge b \wedge \varphi(v) = \\ = \varphi(u) \wedge b \wedge b \wedge \varphi(v) = \widetilde{\varphi}(x) \wedge \widetilde{\varphi}(y).$$

But $\widetilde{\varphi}$ is an isomorphism from $[0, a]$ to $[0, b]$, so $\widetilde{\varphi}(x \wedge y) = \widetilde{\varphi}(x) \wedge \widetilde{\varphi}(y)$. Thus $\varphi(u *_a v) = \varphi(u) *_b \varphi(v)$ and this completes the proof of the theorem.

2 Variants of some lattices

Consider the applications of theorem 1 to some classical lattices.

1. Finite chain.

Proposition 1. *All variants of a finite chain are pairwise non-isomorphic .*

Proof. If a and b are two different elements of a finite chain L, then one of the intervals $[0,a]$ and $[0,b]$ is strictly contained in another. Therefore they have different cardinality and are not isomorphic. From the theorem 1 it immediately follows that the variants $(L,*_a)$ and $(L,*_b)$ are also not isomorphic.

Corollary 1. *Number of pairwise non-isomorphic variants of a chain with length n is equal to $n+1$.*

2. Power set of a finite set.

The set $\mathfrak{B}_n$ of all subsets of the set $\{1,2,\dots,n\}$ (i.e. set $\mathfrak{B}_n$, ordered by inclusion) called power set of a finite set.

Proposition 2. *Let A and B be two elements of the power set $\mathfrak{B}_n$. Variants $(\mathfrak{B}_n,*_A)$ and $(\mathfrak{B}_n,*_B)$ are isomorphic if and only if sets A and B have the same cardinality.*

Proof. If the set $A\in\mathfrak{B}_n$ has the form $A=\{a_1,\dots,a_k\}$, than interval $[\varnothing,A]$ is isomorphic to the power set $\mathfrak{B}_k$ and has 2^k elements. Let now $X=\{a_{i_1},\dots,a_{i_m}\}$ be any element from $[\varnothing,A]$. Set $Y\in\mathfrak{B}_n$ satisfies equality $Y\cap A=X$ if and only if it has the form $Y=X\cup Z$, where Z $\{1,2,\dots,n\}\setminus A$.

So the weight $\omega(X)$ of element X is equal to 2^{n-k}. Therefore all the elements of interval $[\varnothing,A]$ have the same weight, which is fully determined by the cardinality of the set A.

Thus intervals $[\varnothing,A]$ and $[\varnothing,B]$ are isomorphic if and only if $|A|=|B|$. Moreover, any isomorphism between these intervals saves the weights. Reference to the theorem 1 ends the proof of the statement.

Corollary 2. *The number of non-isomorphic variants of power set $\mathfrak{B}_n$ is equal to $n+1$.*

Proof. This follows from the proposition 2 together with the fact that cardinality of the set $A\in\mathfrak{B}_n$ may change from 0 to n.

3. Lattice of subspaces of a finite dimensional vector space.

Let $\mathcal{L}(n,q)$ be the lattice of subspaces of the n–dimensional vector space $\mathbb{F}_q^n$ over the finite field $\mathbb{F}_q$ of dimension q, ordered by inclusion.

Proposition 3. *Variants $(\mathcal{L}(n,q),*_U)$ and $(\mathcal{L}(n,q),*_W)$ of lattice $\mathcal{L}(n,q)$ are isomorphic if and only if subspaces U and W have the same dimension.*

Proof. Let $\mathbb{F}_q^n\supseteq U\supseteq X$, and let subspaces U and X have dimensions m and k respectively. Then subspace $Y\subseteq\mathbb{F}_q^n$ satisfies condition $Y\cap U=X$ if and only if quotient space Y/X will have a zero crossing with $(m-k)$–dimensional quotient space U/X in $(n-k)$–dimensional quotient space $\mathbb{F}_q^n/X$. Then quotient spaces U/X and Y/X have a zero crossing if and only if the union of their bases is a linearly independent system. Fixed base $\boldsymbol{e}_1,\dots,\boldsymbol{e}_{m-k}$ of quotient space U/X can be extended to a linearly independent system by $\boldsymbol{e}_1,\dots,\boldsymbol{e}_{m-k},\boldsymbol{v}_{m-k+1},\dots,\boldsymbol{v}_{m-k+i}$

$$(q^{n-k}-q^{m-k})(q^{n-k}-q^{m-k+1})\cdots(q^{n-k}-q^{m-k+i-1})$$

ways, and $(q^i-1)(q^i-q)\cdots(q^i-q^{i-1})$ different sets $\boldsymbol{v}_{m-k+1},\dots,\boldsymbol{v}_{m-k+i}$ will generate the same i–dimensional space Y/X. Therefore the total number of subspaces Y that satisfy the condition $Y\cap U=X$, is equal to

$$\sum_{i=0}^{n-m}\frac{(q^{n-k}-q^{m-k})(q^{n-k}-q^{m-k+1})\cdots}{(q^i-1)(q^i-q)\cdots(q^i-q^{i-1})}\times$$

$$\frac{\cdots(q^{n-k}-q^{m-k+i-1})}{(q^i-1)(q^i-q)\cdots(q^i-q^{i-1})}. \quad (3)$$

Thus the weight of the element X from interval $[0,U]$, which is given by expression (3), depends only on the dimensions of the subspaces U and X.

Interval $[0,U]$ of the lattice $\mathcal{L}(n,q)$ is isomorphic to the lattice $\mathcal{L}(n,\dim U)$. Therefore intervals $[0,U]$ and $[0,W]$ are isomorphic if and only if $\dim U=\dim W$. But if $\dim U=\dim W$ then subspaces U and W are isomorphic, and each isomorphism from U to W induces an isomorphism from $[0,U]$ to $[0,W]$ which save dimensions of subspaces and their weights. Therefore, according to the theorem 1, variants $(\mathcal{L}(n,q),*_U)$ and $(\mathcal{L}(n,q),*_W)$ are isomorphic if and only if $\dim U=\dim W$.

Corollary 3. *The number of non-isomorphic variants of lattice $\mathcal{L}(n,q)$ is equal to $n+1$.*

4. The lattice of partitions of a finite set.

Let Part_n be lattice of all partitions of a set $\{1,2,\dots,n\}$, and partition ρ is said to be less than partition τ if and only if each block of the partition ρ is contained in one of the blocks of the partition τ. Partition τ has type $\langle l_1,l_2,\dots,l_n\rangle$ (or

$1^{l_1}2^{l_2}\dots n^{l_n}$) if it contains l_1 blocks of length 1, l_2 blocks of length 2, …, l_n blocks of length n. It is clear that $l_1+2l_2+\dots+nl_n=n$.

We need several lemmas, proofs of which are obvious.

Lemma 1. *Partition ρ is coatom of lattice Part_n if and only if it is a two-blocks partition. In particular, lattice Part_n has exactly $2^{n-1}-1$ coatoms.*

Lemma 2. *Let ρ be a partition of the set $\{1,2,\dots,n\}$ into blocks $M_1,\dots,M_k$ of cardinalities $m_1,\dots,m_k$, respectively. Then the interval $[0,\rho]$ is isomorphic to the Cartesian product $\mathrm{Part}_{m_1}\times\dots\times\mathrm{Part}_{m_k}$.*

Denote the number of coatoms of the lattice L by $\kappa(L)$.

Lemma 3. *If a finite lattice L is the Cartesian product $L=L_1\times\dots\times L_m$ of lattices L_1, …, L_m, then element $b\in L$ is coatom if and only if it has a form $b=(1_1,\dots,1_{i-1},b_i,1_{i+1}\dots,1_m)$, where 1_j is a unit of lattice L_j, and b_i is a coatom of lattice L_i. In particular, $\kappa(L)=\kappa(L_1)+\dots+\kappa(L_m)$.*

Theorem 2. *Variants $(\mathrm{Part}_n,*_\rho)$ and $(\mathrm{Part}_n,*_\tau)$ of lattice Part_n are isomorphic if and only if partitions ρ and τ have the same type.*

Proof. First, we'll prove that intervals $[0,\rho]$ and $[0,\tau]$ are isomorphic if and only if partitions ρ and τ have the same type. The sufficiency of the condition is obvious; we will prove necessity by induction on the power of the interval. The statement is obvious if cardinality of interval is equal to 1 or 2. It gives the base for the induction.

Let intervals $[0,\rho]$ and $[0,\tau]$ be isomorphic and assume that the statement is already proved for intervals of all smaller dimensions. Then we can choose coatoms $\mu\in[0,\rho]$ and $\nu\in[0,\tau]$ such that the intervals $[0,\mu]$ i $[0,\nu]$ are also isomorphic. By the lemma 2 this intervals have the following form:

$$[0,\mu]\simeq\mathrm{Part}_{m_1}\times\dots\times\mathrm{Part}_{m_p},\quad [0,\nu]\simeq\mathrm{Part}_{n_1}\times\dots\times\mathrm{Part}_{n_q}. \tag{4}$$

By induction assumption, partitions μ and ν must have the same type so $p=q$ and after renumbering of factors we may assume that $m_1=n_1$, …, $m_p=n_p$. By the lemma 2, partition ρ is formed by joining two blocks of partition μ into one, and partition τ is formed by joining two blocks of partition ν into one.

Without loss of generality we can assume that one of two cases holds:

I. $\rho\simeq\mathrm{Part}_{m+k}\times\mathrm{Part}_l\times\mathrm{Part}_{m_4}\times\cdots\times\mathrm{Part}_{m_p}$,
$\tau\simeq\mathrm{Part}_{m+l}\times\mathrm{Part}_k\times\mathrm{Part}_{m_4}\times\cdots\times\mathrm{Part}_{m_p}$;

II. $\rho\simeq\mathrm{Part}_{m+r}\times\mathrm{Part}_k\times\mathrm{Part}_l\times\mathrm{Part}_{m_5}\times\cdots\times\mathrm{Part}_{m_p}$,
$\tau\simeq\mathrm{Part}_{k+l}\times\mathrm{Part}_m\times\mathrm{Part}_r\times\mathrm{Part}_{m_4}\times\cdots\times\mathrm{Part}_{m_p}$.

Consider the first case. Without loss of generality we can assume that $k\geqslant l$.

Since the intervals $[0,\mu]$ and $[0,\nu]$ are isomorphic, they contain the same number of coatoms. From the lemma 3, it follows that the number of coatoms of interval $[0,\rho]$, nonunit part of which lies in $\mathrm{Part}_{m+k}\times\mathrm{Part}_l$, must be equal to the number of coatoms of the interval $[0,\tau]$, nonunit part of which lies in $\mathrm{Part}_{m+l}\times\mathrm{Part}_k$, and this numbers are equal to

$$(2^{m+k-1}-1)+(2^{l-1}-1)\quad\text{and}$$

$$(2^{m+l-1}-1)+(2^{k-1}-1),$$

respectively. Hence

$$2^{m+k}+2^l=2^{m+l}+2^k. \tag{5}$$

If $m+l>k$ it leads to the equality

$$2^l(2^{m+k-l}+1)=2^k(2^{m+l-k}+1),$$

and $m+k-l>0$ i $m+l-k>0$. Then $l=k$, and partitions ρ and τ have the same type.

Inequality $m+l\leqslant k$ is not possible because then number 2 will be included with the exponent l in decomposition of the left part of the equality (5), and with the exponent at least $m+l$ in decomposition of the right part.

Thus in the first case partitions ρ and τ have the same type.

Now consider the second case. Similarly to the case I from the counting coatoms in intervals $[0,\rho]$ and $[0,\tau]$ we obtain the equality

$$2^{m+r}+2^k+2^l=2^{k+l}+2^m+2^r. \tag{6}$$

Count the number of atoms in each of the intervals $[0,\rho]$ and $[0,\tau]$. Obviously, the partition is atom if and only if all its blocks have exactly one element, except the one block which contains two elements. And both elements of this block must belong to the same block of the partition ρ (for the interval $[0,\rho]$) or partition τ (for the interval $[0,\tau]$). It gives equality

$$\binom{m+r}{2}+\binom{k}{2}+\binom{l}{2}=\binom{k+l}{2}+\binom{m}{2}+\binom{r}{2}.$$

Hence we obtain:

$$mr = kl. \tag{7}$$

Without loss of generality, we can assume that $m \geqslant r$ i $k \geqslant l$. If $m = k$ then $r = l$ and partitions ρ and τ have the same type. Suppose now that $m \neq k$. We can assume that $m > k$. Then (7) implies $r < l$.

If $k = l$ then (7) implies that $r < k$. Now from equality (6) we obtain:

$$2^{k+1}(2^{m+r-k-1}+1) = 2^r(2^{2k-r}+2^{m-r}+1),$$

and $m+r-k-1 > 0$, $2k-r > 0$, $m-r > 0$. Therefore $k+1 = r$, which contradicts with the inequality $r < k$. Hence the case $k = l$ is impossible.

It is left to consider the case $m > k > l > r$. In this case equality (6) can be rewritten as

$$2^l(2^{m+r-l}+2^{k-l}+1) = 2^r(2^{k+l-r}+2^{m-r}+1),$$

and $m+r-l > 0$, $k-l > 0$, $2^{k+l-r} > 0$, $m-r > 0$. But then $l = r$ which again leads to a contradiction.

Thus in all cases the assumption that $m \neq k$ leads to a contradiction. Hence, intervals $[0,\rho]$ and $[0,\tau]$ are isomorphic if and only if the partitions ρ and τ have the same type.

Now let partitions

$$\rho = a_1 \dots a_k | b_1 \dots b_l | \dots | d_1 \dots d_m \quad \text{and}$$
$$\tau = a'_1 \dots a'_k | b'_1 \dots b'_l | \dots | d'_1 \dots d'_m$$

have the same type. Consider the permutation

$$\pi = \begin{pmatrix} a_1 & \dots & a_k & b_1 & \dots & b_l & \dots & d_1 & \dots & d_m \\ a'_1 & \dots & a'_k & b'_1 & \dots & b'_l & \dots & d'_1 & \dots & d'_m \end{pmatrix}.$$

Then mapping $\varphi_\pi : \mathrm{Part}_n \to \mathrm{Part}_n$, which sends a partition $x_1 \dots x_p | \dots | y_1 \dots y_q$ into partition $\pi(x_1)\dots\pi(x_p)|\dots|\pi(y_1)\dots\pi(y_q)$, is an automorphism of lattice Part_n which maps the interval $[0,\rho]$ into the interval $[0,\tau]$. Since in this case expression $\delta \wedge \rho = \mu$ go into expression $\varphi_\pi(\delta) \wedge \varphi_\pi(\rho) = \varphi_\pi(\mu)$, the restriction of the automorphism φ_π on interval $[0,\rho]$ save the weights of elements. From the theorem 1 now follows that variants $(\mathrm{Part}_n, *_\rho)$ and $(\mathrm{Part}_n, *_\tau)$ are isomorphic.

Corollary 4. *Lattice* Part_n *has exactly* $p(n)$ *pairwise non-isomorphic variants, where* $p(n)$ *is the number of unordered partitions of number* n *into the sum of natural summands.*

Proof. The statement follows from theorem 2 and the fact that if we put in the corresponding to partition ρ the sum of power of it blocks, we will obtain a one-to-one correspondence between the types of partition of the set $\{1,2,\dots,n\}$ and partitions of number n in the sum of natural summands.

3 Conclusions

We obtained a criterion of isomorphism for two variants of a finite lower semilattice, which is considered as a semigroup under the operation of taking of the infinum. We have applied this criterion to describe variants of a few classical semilattices such as: a finite chain, the powerset of a finite set, the lattice of partitions of a finite set, and the lattice of subspaces of a finite space.

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