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## On some relation between sets of mappings of a set in factor-rings

*In the given paper some relations between sets of mappings of an abstract set in factor-rings of an associative-commutative ring are investigated. There are found conditions to be met by a given set of ideals, that made it possible to prove identities which form the base for elaboration of combinatorial schemes intended to compute the number of objects determined over associative-commutative rings.*
*Key Words: associative-commutative rings, mappings in factor-rings, combinatorial schemes.*

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## Про одне співвідношення між множинами відображень множини у фактор-кільця

*У статті досліджено деякі співвідношення між множинами відображень абстрактної множини у фактор-кільця асоціативно-комутативного кільця. Встановлено умови яким повинна задовольняти задана множина ідеалів що дають можливість встановити рівності які є основою для побудови комбінаторних схем що призначені для підрахунку об'єктів які визначено над асоціативно-комутативним кільцем.*
*Ключові слова: асоціативно-комутативні кільця, відображення у фактор-кільця, комбінаторні схеми.*

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### Introduction

Applications of algebraic models for information transformation settle down that elaboration of combinatorial schemes intended to compute the number of objects determined over rings is actual problem. Due to the approach generally accepted in Modern Combinatorics [1] it is naturally to present these combinatorial schemes via mappings of some abstract set in factor-rings of considered ring.

In [2,3] it was investigated some relation between sets of mappings of an abstract set $S$ in complete residue systems determined by pair-wise coprime elements $a_1,\ldots,a_m$ $(m\in\mathbf{N})$ of a Dedekind ring $K=(K,+,\cdot)$ and sets of mappings of the set $S$ in complete residue system determined by the element $\prod_{i=1}^{m} a_i$. Proposed scheme was determined as follows.

Selecting any single element in each coset of the quotient set $K/_{(a)}$ $(a\in K)$ (where $(a)$ is principal ideal of the ring $K$ generated by the element $a$), we get complete residue system $\mathrm{MOD}(a)$.

By $b<\mathrm{mod}\, a>$ $(a,b\in K)$ it is denoted the element $c\in\mathrm{MOD}(a)$, such that $b$ и $c$ are elements of the same coset of the quotient set $K/_{(a)}$.

Let
$F_{a_i}(S)=\{f \mid f: S\to \mathrm{MOD}(a_i)\}$ $(i=1,\ldots,m)$
and

$$F(S)=\{f \mid f: S\to \mathrm{MOD}(\prod_{i=1}^{m} a_i)\}.$$

For any fixed subsets $\hat{F}_{a_i}(S)\subseteq F_{a_i}(S)$ $(i=1,\ldots,m)$ we set

$$\tilde{F}_{a_i}(S)=\{f\in F(S) \mid f_{\mathrm{mod} a_i}\in \hat{F}_{a_i}(S)\},$$

where mapping $f_{\mathrm{mod} a_i}$ $(f\in F(S))$ is determined by identity

$$f_{\mathrm{mod} a_i}(s)=f(s)<\mathrm{mod}\, a_i> \quad (s\in S).$$

The following theorem holds.

**Theorem** [2,3]**.** For any set $S$ and pair-wise coprime elements $a_1,\ldots,a_m$ $(m\in\mathbf{N})$ of a Dedekind ring $K$ there holds the identity

$$|\mathop{\times}_{i=1}^{m}\hat{F}_{a_i}(S)| = |\bigcap_{i=1}^{m}\tilde{F}_{a_i}(S)|. \qquad (1)$$

If all sets $\hat{F}_{a_i}(S)$ $(i=1,\dots,m)$ are finite then identity (1) can be rewritten in the form

$$\prod_{i=1}^{m}|\hat{F}_{a_i}(S)|=|\bigcap_{i=1}^{m}\tilde{F}_{a_i}(S)|. \qquad (2)$$

It was established in [2,3] that identity (2) can be applied to combinatorial analysis of mathematical structures determined in terms of commutative-associative rings with unity or in terms of number-theoretic structures, used in applied problems of information transformation.

In the given paper it is investigated some general combinatorial scheme elaborated for associative-commutative ring (not necessary with unity), such that above described scheme is its special case. All algebraic notions that are not determined are the same as in [4-6].

The rest of the paper is organized in the following way. In chapter 1 necessary basic notions are determined and investigated. Main result is presented in chapter 2. The last chapter consists of some conclusion remarks.

## 1. Preliminary results

Let $K=(K,+,\cdot)$ be any associative-commutative ring and $I_K$ be the set of all ideals of the ring $K$. The ideal $I_1\cap I_2$ is the least common multiple of ideals $I_1$ and $I_2$. It is well known that inclusion

$$I_1I_2\subseteq I_1\cap I_2 \qquad (1)$$

holds for all $I_1,I_2\in I_K$.

**Proposition 1.** In any associative-commutative ring $K$ inclusion

$$I_1(I_2\cap I_3)\subseteq I_1I_2\cap I_1I_3 \qquad (2)$$

holds for all $I_1,I_2,I_3\in I_K$.

**Proof.** Let $K$ be any associative-commutative ring and $I_1,I_2,I_3\in I_K$.

Inclusion $I_2\cap I_3\subseteq I_2$ implies that inclusion $I_1(I_2\cap I_3)\subseteq I_1I_2$ holds, and inclusion $I_2\cap I_3\subseteq I_3$ implies that inclusion $I_1(I_2\cap I_3)\subseteq I_1I_3$ also holds. Inclusions $I_1(I_2\cap I_3)\subseteq I_1I_2$ and $I_1(I_2\cap I_3)\subseteq I_1I_3$ imply that inclusion (2) holds.

Q.E.D.

**Proposition 2.** In any associative-commutative ring $K$ for any integer $m\in\mathbf{N}$ $(m\geq 3)$ inclusion

$$\prod_{i=1}^{m}I_i\subseteq\bigcap_{i=1}^{m}I_i \qquad (3)$$

holds for all $I_1,\dots,I_m\in I_K$.

**Proof.** Let $K$ be any associative-commutative ring, $m\in\mathbf{N}$ $(m\geq 3)$ and $I_1,\dots,I_m\in I_K$.

We prove inclusion (3) by induction.

Let $m=3$. Inclusions (1) and (2) imply that

$$I_1I_2I_3\subseteq I_1(I_2\cap I_3)\subseteq I_1I_2\cap I_1I_3\subseteq$$
$$\subseteq(I_1\cap I_2)\cap(I_1\cap I_3)=I_1\cap I_2\cap I_3.$$

Suppose that inclusion (3) holds for all integers $m=3,\dots,n$. Then

$$\prod_{i=1}^{m}I_i\subseteq\bigcap_{i=1}^{m}I_i \quad (m=3,\dots,n) \qquad (4)$$

Let $m=n+1$. Inclusions (1) and (4) imply that

$$\prod_{i=1}^{m}I_i=\prod_{i=1}^{n+1}I_i=(\prod_{i=1}^{n}I_i)I_{n+1}\subseteq(\bigcap_{i=1}^{n}I_i)I_{n+1}\subseteq$$
$$\subseteq(\bigcap_{i=1}^{n}I_i)\cap I_{n+1}=\bigcap_{i=1}^{n+1}I_i=\bigcap_{i=1}^{m}I_i.$$

Q.E.D.

**Corollary 1.** In any associative-commutative ring $K$ for any integer $m\in\mathbf{N}$ $(m\geq 3)$ inclusion

$$\prod_{i=1}^{m}I_i\subseteq\bigcap_{i=1}^{m}I_i \quad (m\in\mathbf{N},m\geq 2) \qquad (5)$$

holds for all $I_1,\dots,I_m\in I_K$.

**Proof.** If $m=2$ then (5) is turned to (1). If $m\geq 3$ then (5) is turned to (3).

Q.E.D.

Quotient set $K/_I$ $(I\in I_K)$ treated as a partition of the set $K$ would be denoted by $\pi(K,I)$, i.e. $x\equiv y(\pi(K,I))$ if and only if $x\equiv y\,(\mathrm{mod}\,I)$.

**Remark 1.** Thus we get $\pi(K,I)=\{I+a\,|\,a\in K\}$ $(I\in I_K)$. It is worth to note that $\pi(K,I_1)\leq\pi(K,I_2)$ if and only if $I_1\subseteq I_2$.

We set

$$P_K=\{\pi(K,I)\,|\,I\in I_K\}.$$

The set $P_K$ can be characterized in the following way.

**Lemma 1.** In any associative-commutative ring $K$ for all $m\in\mathbf{N}$ inequality

$$\pi(K,\prod_{i=1}^{m}I_i)\leq\prod_{i=1}^{m}\pi(K,I_i) \qquad (6)$$

holds for all $I_1,\dots,I_m\in I_K$.

**Proof.** Let $K$ be any associative-commutative ring, $m\in\mathbf{N}$ and $I_1,\dots,I_m\in I_K$.

If $m=1$ then inequality (6) is turned to inequality $\pi(K,I_1)\leq\pi(K,I_1)$ and it holds for all $I_1\in I_K$

Let $m\geq 2$. For any $x,y\in K$ we get

$$x\equiv y(\pi(K,\prod_{i=1}^{m}I_i))\Leftrightarrow x\equiv y\,(\mathrm{mod}\prod_{i=1}^{m}I_i). \qquad (7)$$

Since $\prod_{i=1}^{m}I_i\subseteq\bigcap_{i=1}^{m}I_i$ then

$$x\equiv y\,(\mathrm{mod}\prod_{i=1}^{m}I_i)\Rightarrow x\equiv y\,(\mathrm{mod}\bigcap_{i=1}^{m}I_i)\Leftrightarrow$$
$$\Leftrightarrow x-y\in\bigcap_{i=1}^{m}I_i\Leftrightarrow(\forall i=1,\dots,m)(x-y\in I_i)\Leftrightarrow$$
$$\Leftrightarrow(\forall i=1,\dots,m)(x\equiv y\,(\mathrm{mod}\,I_i))\Leftrightarrow$$
$$\Leftrightarrow(\forall i=1,\dots,m)(x\equiv y\,(\pi(K,I_i)))\Leftrightarrow$$
$$\Leftrightarrow x\equiv y(\prod_{i=1}^{m}\pi(K,I)).\qquad(8)$$

Formulae (7) and (8) imply that inequality (6) holds.

Q.E.D.

**Corollary 2.** In any associative-commutative ring $K$ for all $m\in\mathbf{N}$ identity

$$\pi(K,\prod_{i=1}^{m}I_i)=\prod_{i=1}^{m}\pi(K,I_i)\qquad(9)$$

holds for all $I_1,\dots,I_m\in I_K$, such that $\prod_{i=1}^{m}I_i=\bigcap_{i=1}^{m}I_i$.

**Proof.** Let $K$ be any associative-commutative ring, $m\in\mathbf{N}$, $I_1,\dots,I_m\in I_K$, and $\prod_{i=1}^{m}I_i=\bigcap_{i=1}^{m}I_i$.

If $m=1$ then identity (9) is turned to identity $\pi(K,I_1)=\pi(K,I_1)$ and it holds for all $I_1\in I_K$.

Let $m\geq 2$. Identity $\prod_{i=1}^{m}I_i=\bigcap_{i=1}^{m}I_i$ implies that

$$x\equiv y\,(\mathrm{mod}\prod_{i=1}^{m}I_i)\Leftrightarrow x\equiv y\,(\mathrm{mod}\bigcap_{i=1}^{m}I_i).\qquad(10)$$

Substituting in (8) formula (10) instead of formula

$$x\equiv y\,(\mathrm{mod}\prod_{i=1}^{m}I_i)\Rightarrow x\equiv y\,(\mathrm{mod}\bigcap_{i=1}^{m}I_i),$$

we get that

$$x\equiv y\,(\mathrm{mod}\prod_{i=1}^{m}I_i)\Leftrightarrow x\equiv y\,(\prod_{i=1}^{m}\pi(K,I_i)).\qquad(11)$$

Formulae (7) and (11) imply that identity (9) holds.

Q.E.D.

**Theorem 1.** In any associative-commutative ring $K$ for any integer $m\in\mathbf{N}$ $(m\geq 2)$ identity

$$\pi(K,\prod_{i=1}^{m}I_i)=\{\bigcap_{i=1}^{m}B_i\,|B_i\in\pi(K,I_i)\,(i=1,\dots,m\}\qquad(12)$$

holds for all $I_1,\dots,I_m\in I_K$, such that $\prod_{i=1}^{r}I_i+I_{r+1}=K$ $(r=1,\dots,m-1)$ and $\prod_{i=1}^{h}I_i=\bigcap_{i=1}^{h}I_i$ $(h=2,\dots,m)$.

**Proof.** Let $K$ be any associative-commutative ring, $m\in\mathbf{N}$ $(m\geq 2)$, $I_1,\dots,I_m\in I_K$, $\prod_{i=1}^{r}I_i+I_{r+1}=K$ $(r=1,\dots,m-1)$ and $\prod_{i=1}^{h}I_i=\bigcap_{i=1}^{h}I_i$ $(h=2,\dots,m)$.

Identity (12) is equivalent to proposition that

$$\bigcap_{i=1}^{m}B_i\neq\varnothing\,(m\in\mathbf{N},m\geq 2)\qquad(13)$$

for all $B_i\in\pi(K,I_i)$ $(i=1,\dots,m)$.

We prove this proposition by induction.

Let $m=2$. Since $I_1I_2=I_1\cap I_2$, we get (see Corollary 2) that $\pi(K,I_1I_2)=\pi(K,I_1)\pi(K,I_2)$.

Let $B_i\in\pi(K,I_i)$ $(i=1,2)$, i.e. $B_i=I_i+a_i$ for some fixed elements $a_i\in K$. Identity $I_1+I_2=K$ implies that there exist elements $\alpha_i\in I_i$ $(i=1,2)$, such that $a_1-a_2=\alpha_2-\alpha_1$. Thus $a_1+\alpha_1=a_2+\alpha_2$. Since $\alpha_i\in I_i$ $(i=1,2)$ we get $a_i+\alpha_i\in B_i$.

Formulae $a_i+\alpha_i\in B_i$ $(i=1,2)$ and $a+\alpha_1=b+\alpha_2$ imply that $B_1\cap B_2\neq\varnothing$.

Suppose that considered proposition holds for all integers $m=2,\dots,n$.

Let $m=n+1$. Since $\prod_{i=1}^{m}I_i=\bigcap_{i=1}^{m}I_i$ we get (see Corollary 2) that

$$\pi(K,\prod_{i=1}^{m}I_i)=\pi(K,\prod_{i=1}^{n+1}I_i)=(\prod_{i=1}^{n}\pi(K,I_i))\pi(K,I_{n+1}).$$

Let $B\in\pi(K,\prod_{i=1}^{n}I_i)$ and $B_{n+1}\in\pi(K,I_{n+1})$, i.e. $B=\prod_{i=1}^{m}I_i+a_1$ and $B_{n+1}=I_{n+1}+a_2$ for some fixed elements $a_i\in K$ $(i=1,2)$. Identity $\prod_{i=1}^{n}I_i+I_{n+1}=K$ implies that there exist elements $\alpha_1\in\prod_{i=1}^{n}I_i$ and $\alpha_2\in I_{n+1}$, such that $a_1-a_2=\alpha_2-\alpha_1$. Thus $a_1+\alpha_1=a_2+\alpha_2$. Since $\alpha_1\in\prod_{i=1}^{n}I_i$ and $\alpha_2\in I_{n+1}$ we get that $a_1+\alpha_1\in B$ and $a_2+\alpha_2\in B_{n+1}$.

Formulae $a_1+\alpha_1\in B$, $a_2+\alpha_2\in B_{n+1}$ and $a_1+\alpha_1=a_2+\alpha_2$ imply that $B\cap B_{n+1}\neq\varnothing$.

Q.E.D.

**2. Main result.**

Let $S$ be any non-empty set, $K$ be an associative-commutative ring and $I_1,\dots,I_m\in I_K$ $(m\in\mathbf{N},m\geq 2)$ be such ideals that

$$\prod_{i=1}^{r} I_i + I_{r+1} = K \quad (r = 1,\ldots,m-1)$$

and

$$\prod_{i=1}^{h} I_i = \bigcap_{i=1}^{h} I_i \quad (h = 2,\ldots,m).$$

We set

$$F_{I_i}(S) = \{ f \mid f : S \to \pi(K, I_i)\} \quad (i = 1,\ldots,m)$$

and

$$F(S) = \{ f \mid f : S \to \pi(K, \prod_{i=1}^{m} I_i)\}.$$

**Remark 2.** Identity $\prod_{i=1}^{m} I_i = \bigcap_{i=1}^{m} I_i$ implies (see Corollary 2) that $\pi(K, \prod_{i=1}^{m} I_i) = \prod_{i=1}^{m} \pi(K, I_i)$. Thus we get

$$F(S) = \{ f \mid f : S \to \prod_{i=1}^{m} \pi(K, I_i)\}.$$

For any subsets $\hat{F}_{I_i}(S) \subseteq F_{I_i}(S)$ $(i = 1,\ldots,m)$ we set

$$\tilde{F}_{I_i}(S) = \{ f \in F(S) \mid f_{I_i} \in \hat{F}_{I_i}(S)\} \quad (i = 1,\ldots,m),$$

where mapping $f_{I_i}$ $(f \in F(S))$ is determined as follows: if $f(s) = B$ $(s \in S)$ (where $B \in \prod_{i=1}^{m} \pi(K, I_i)$) then $f_{I_i}(s) = B'$, where $B'$ is such single block of the partition $\pi(K, I_{I_i})$ that $B \subseteq B'$.

**Theorem 2.** In any associative-commutative ring $K$ for any non-empty set $S$ and any integer $m \in \mathbf{N}$ $(m \geq 2)$ identity

$$|\mathop{\times}_{i=1}^{m} \hat{F}_{I_i}(S)| = |\bigcap_{i=1}^{m} \tilde{F}_{I_i}(S)| \qquad (14)$$

holds for all $I_1,\ldots,I_m \in I_K$, such that $\prod_{i=1}^{r} I_i + I_{r+1} = K$ $(r = 1,\ldots,m-1)$ and $\prod_{i=1}^{h} I_i = \bigcap_{i=1}^{h} I_i$ $(h = 2,\ldots,m)$.

**Proof.** Let $K$ be any associative-commutative ring, $m \in \mathbf{N}$ $(m \geq 2)$, $I_1,\ldots,I_m \in I_K$, $\prod_{i=1}^{r} I_i + I_{r+1} = K$ $(r = 1,\ldots,m-1)$ and $\prod_{i=1}^{h} I_i = \bigcap_{i=1}^{h} I_i$ $(h = 2,\ldots,m)$.

To prove identity (14) it is sufficient to construct injections

$$\varphi : \hat{F}_{I_1}(S) \times \ldots \times \hat{F}_{I_m}(S) \to \bigcap_{i=1}^{m} \tilde{F}_{I_i}(S) \qquad (15)$$

and

$$\psi : \bigcap_{i=1}^{m} \tilde{F}_{I_i}(S) \to \hat{F}_{I_1}(S) \times \ldots \times \hat{F}_{I_m}(S). \qquad (16)$$

Injection $\varphi$ can be constructed as follows.

For any $\mathbf{f} = (f_1,\ldots,f_m) \in \hat{F}_{I_1}(S) \times \ldots \times \hat{F}_{I_m}(S)$ we set $\varphi(\mathbf{f}) = f$, where mapping $f$ is determined by identity

$$f(s) = \bigcap_{i=1}^{m} f_i(s) \quad (s \in S). \qquad (17)$$

Theorem 1 implies that $f \in F(S)$.

Firstly we prove that mapping (17) is some mapping of the form (15).

Indeed, identity (17) implies that $f_{I_i}(s) = f_i(s)$ $(i = 1,\ldots,m)$ for all $s \in S$, i.e. $f_{I_i} = f_i \in \hat{F}_{I_i}(S)$ for all $i = 1,\ldots,m$. Since $f_{I_i} \in \hat{F}_{I_i}(S)$ for all $i = 1,\ldots,m$ we get that $f \in \tilde{F}_{I_i}(S)$ for all $i = 1,\ldots,m$, i.e. $f \in \bigcap_{i=1}^{m} \tilde{F}_{I_i}(S)$.

Now we prove that mapping (17) is injection, i.e. if $\mathbf{f}_r = (f_1^{(r)},\ldots,f_m^{(r)}) \in \hat{F}_{I_1}(S) \times \ldots \times \hat{F}_{I_m}(S)$ $(r = 1,2)$ and $\mathbf{f}_1 \neq \mathbf{f}_2$ then $\varphi(\mathbf{f}_1) \neq \varphi(\mathbf{f}_2)$.

Since $\mathbf{f}_1 \neq \mathbf{f}_2$ then there exists $j \in \mathbf{N}_m$ such that $f_j^{(1)} \neq f_j^{(2)}$. This implies that there exists an element $s \in S$ such that $f_j^{(1)}(s) \neq f_j^{(2)}(s)$, i.e. $f_j^{(1)}(s)$ and $f_j^{(2)}(s)$ are different blocks of partition $\pi(K, I_j)$. Thus $\prod_{i=1}^{m} f_i^{(1)}(s)$ and $\prod_{i=1}^{m} f_i^{(2)}(s)$ are different blocks of partition $\prod_{i=1}^{m} \pi(K, I_i)$. Since

$$\varphi(\mathbf{f}_1)(s) = \prod_{i=1}^{m} f_i^{(1)}(s) \neq \prod_{i=1}^{m} f_i^{(2)}(s) = \varphi(\mathbf{f}_2)(s)$$

we get that $\varphi(\mathbf{f}_1) \neq \varphi(\mathbf{f}_2)$.

Injection $\psi$ can be constructed as follows.

For any $f \in \bigcap_{i=1}^{m} \tilde{F}_{I_i}(S)$ we set

$$\psi(f) = (f_{I_1},\ldots,f_{I_m}). \qquad (18)$$

Identity (18) implies that for any mapping $f \in \bigcap_{i=1}^{m} \tilde{F}_{I_i}(S)$ identity $f(s) \in \bigcap_{i=1}^{m} f_{I_i}(s)$ holds for any element $s \in S$.

Firstly we prove that mapping (18) is some mapping of the form (16).

Since $f \in \bigcap_{i=1}^{m} \tilde{F}_{I_i}(S)$ we get that $f \in \tilde{F}_{I_i}(S)$ for all $i = 1,\ldots,m$. This implies that $f_{I_i} \in \hat{F}_{I_i}(S)$ for all $i = 1,\ldots,m$, i.e.

$$\psi(f) = (f_{I_1},\ldots,f_{I_m}) \in \hat{F}_{I_1}(S) \times \ldots \times \hat{F}_{I_m}(S).$$

Now we prove that mapping (18) is injection, i.e. if $f^{(1)}, f^{(2)} \in \bigcap_{i=1}^{m} F$ and $f^{(1)} \neq f^{(2)}$ then

$$\psi(f^{(1)}) = (f^{(1)}_{I_1}, \ldots, f^{(1)}_{I_m}) \neq (f^{(2)}_{I_1}, \ldots, f^{(2)}_{I_m}) = \psi(f^{(2)}).$$

Since $f^{(1)} \neq f^{(2)}$ $(f^{(1)}, f^{(2)} \in \bigcap_{i=1}^{m} F)$ then there exists an element $s \in S$ such that $f^{(1)}(s) \neq f^{(2)}(s)$, i.e. $f^{(1)}(s)$ and $f^{(2)}(s)$ are elements of different blocks of partition $\prod_{i=1}^{m} \pi(K, I_i)$. Thus we get

$$\bigcap_{i=1}^{m} f^{(1)}_{I_1}(s) = f^{(1)}(s) \neq f^{(2)}(s) = \bigcap_{i=1}^{m} f^{(2)}_{I_1}(s).$$

This means that there exists $j \in \mathbf{N}_m$ such that $f^{(1)}_{I_j}(s) \neq f^{(2)}_{I_j}(s)$, i.e. $f^{(1)}_{I_j} \neq f^{(2)}_{I_j}$. Since $f^{(1)}_{I_j} \neq f^{(2)}_{I_j}$ then we get $\psi(f^{(1)}) \neq \psi(f^{(2)})$.

Q.E.D.

If all sets $\hat{F}_{I_i}(S)$ $(i = 1, \ldots, m)$ are finite then identity (14) can be rewritten in the form

$$\prod_{i=1}^{m} |\hat{F}_{I_i}(S)| = |\bigcap_{i=1}^{m} \tilde{F}_{I_i}(S)|. \qquad (19)$$

From proof of Theorem 2 it follows immediately that the following corollary holds.

**Corollary 3.** If all sets $\hat{F}_{I_i}(S)$ $(i = 1, \ldots, m)$ are finite then the mappings $\varphi$ and $\psi$ constructed in proof of Theorem 2 are bijections such that $\varphi^{-1} = \psi$ and $\psi^{-1} = \varphi$.

It is worth to note that just the identity (19) forms the base for elaboration of combinatorial schemes intended to compute the number of objects determined over associative-commutative rings. The following example confirms this factor.

**Example.** 1. Let $K$ be any finite associative-commutative ring with unity. Proper ideals $I_1, I_2 \in I_K$ are comaximal if $I_1 + I_2 = K$. Proper ideals $I_1, \ldots, I_m \in I_K$ are pair-wise comaximal if $I_i$ and $I_j$ are comaximal for all $i, j = 1, \ldots, m$ $(i \neq j)$. It is known that if ideals $I_1, \ldots, I_m \in I_K$ are pair-wise comaximal then identity $\prod_{i=1}^{m} I_i = \bigcap_{i=1}^{m} I_i$ holds. Thus identity (19) also holds. If $|S| = 1$ then identity (19) establishes that factor-rings $K / \prod_{i=1}^{m} I_i$ and $\prod_{i=1}^{m} K / I_i$ have the same cardinality. Moreover, bijections $\varphi$ and $\psi$ constructed in proof of Theorem 2 establish isomorphism of these factor-rings.

2. If $K$ is a Dedekind ring and $a_1, \ldots, a_m$ $(m \in \mathbf{N})$ are pair-wise coprime elements then principal ideals $(a_1), \ldots, (a_m)$ are pair-wise comaximal. Thus all results established ad hoc in [2,3] can be treated as special case of identities (14) and (19).

## Conclusions

In the given paper mappings of an abstract set in factor-rings of an associative-commutative ring were investigated. Found in the paper conditions to be met by a given set of ideals, made it possible to prove identities (14) and (19) which form the base for elaboration of combinatorial schemes intended to compute the number of objects determined over associative-commutative rings. In particular, all results established ad hoc in [2,3] can be treated as special case of these identities.

Analysis of possible applications of identities (14) and (19) for computing the number of objects determined over associative-commutative rings of this or the other type (in particular, over rings of principal ideals) forms some trend for future research. Analysis of identities (14) and (19) under these or the others additional conditions form another trend for future research.

## List of references

1. *Sachkov V.N.* Introduction to combinatorial methods of discrete mathematics. – Moscow: Nauka, 1982. – 384 p. (in Russian).
2. *Skobelev V.V.* Analysis of some mappings of sets in a Dedekind ring // Dopovidi NAS of Ukraine. – 2011. – № 3 – P. 45-49. (in Russian).
3. *Skobelev V.V.* On two sequences of sets of mappings of abstract sets in a Dedekind ring // Cybernetics and system analysis. – 2012. – № 2. – P. 105-112.
4. Zariski O., Samuel P. Commutative algebra. Vol. 1. – Moscow: Inostrannaja literatura, 1963. – 374 p. (in Russian).
5. Kurosh A.G. Lectures in general algebra. – Moscow: Nauka, 1973. – 400 p. (in Russian).
6. Van der Varden B.L. Algebra. – Moscow: Nauka, 1979. – 624 p. (in Russian).

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