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Електронні стани на найнижчому рівні Ландау у невпорядкованому діраківському напівметалі

Розглянуто електронні стани на найнижчому рівні Ландау у невпорядкованому діраківському напівметалі в другому порядку теорії збурень по потенціалу домішок. Отриманий результат порівняно з точним розв'язком і обговорено необхідність врахування процесів розсіяння між різними вейлівськими точками для того, щоб провідність була скінченною.

Ключові слова: найнижчий рівень Ландау, невпорядкований діраківський напівметал

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Electron states in the lowest Landau level in disordered Dirac semimetal

The electron states in the lowest Landau level in disordered Dirac semimetal are studied in the second order of perturbation theory in impurities potential. The results are compared with the exact solution and the necessity of impurity scattering between different Weyl nodes to produce a finite conductivity is discussed.

Key words: lowest Landau level, disordered Dirac semimetal

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Recently there was a significant interest in condensed matter community to the role of quantum anomalies in transport and electromagnetic properties of Dirac and Weyl semimetals, whose low energy quasiparticles are described by the Dirac and Weyl equation, respectively. Historically, the first example of a metal whose low effective energy includes three dimensional Dirac fermions is yielded by bismuth Ref.[1]. Its band structure features pockets of holes near T point in the Brillouin zone and pockets of electrons near the three equivalent L points which are described by (3+1)-dimensional Dirac equation with non-zero mass.

It is interesting that the magnitude of this mass can be tuned by doping bismuth with antimony. As Sb concentration in $Bi_{(1-x)}Sb_x$ alloy increases, the mass decreases and closes at $x \approx 0.03 - 0.04$ realizing a truly massless point. As x is further increased, the gap reopens and for $x > 0.07$ the material becomes an inverted-band bulk insulator with topological surface states (see, e.g., Ref.[2]).

Recently, applying magnetic fields, a negative magnetoresistivity [3] was observed in $Bi_{0.97}Sb_{0.03}$. Negative magnetoresistivity connected with the chiral anomaly was long ago [4] argued to be a fingerprint of the existence of a Weyl semimetal phase, where the single Dirac point splits into two Weyl nodes with opposite chirality and the distance in momentum space between these points is proportional to the applied magnetic field. It is well known that only the fermion states on the lowest Landau level (LLL) produce the chiral anomaly (see, e.g., Ref.[5]). According to Ref.[4], one cannot neglect scattering processes involving two Weyl nodes. As shown in [4], if the internode scattering rate for the electron states in the LLL is small, this leads to a negative magnetoresistivity.

It is clear that in order to theoretically describe negative magnetoresistivity, it is necessary to calculate the width of the LLL states due to the scattering on random impurities. The standard method to perform such a calculation is the impurity (or "cross") diagram technique [6,7]. According to this technique, the interaction Hamiltonian of electrons with impurities is given by

$$H_{int} = \int d^3\boldsymbol{r}\, U(\boldsymbol{r})\psi^+(\boldsymbol{r})\psi(\boldsymbol{r}), \qquad (1)$$

where $U(\boldsymbol{r}) = \Sigma_j u(\boldsymbol{r}-\boldsymbol{r}_j)$ and summation is performed over impurities situated at random positions. The simplest interaction potential with an impurity is the short-range one

$$u(\boldsymbol{r}-\boldsymbol{r}_j) = u_0\delta^3(\boldsymbol{r}-\boldsymbol{r}_j). \qquad (2)$$

Further, the averaging over the positions of impurities proceeds according to the rule [6,7]

$$< f(\boldsymbol{r}_j) >= \frac{1}{V}\int f(\boldsymbol{r}_j) d^3\boldsymbol{r}_j, \qquad (3)$$

where V is the volume of the system. Using Eq.(3), we easily find the following average of the impurity correlation function:

$$< u(\boldsymbol{r}-\boldsymbol{r}_j)u(\boldsymbol{r}'-\boldsymbol{r}_j) >$$
$$= u_0^2 n_c \delta^3(\boldsymbol{r}-\boldsymbol{r}'), \qquad (4)$$

where $n_c = N/V$ is the concentration of impurities. Obviously, Eq.(4) can be interpreted as an effective impurity propagator in the impurity diagram technique for electrons in disordered material. In momentum space, it equals

$$D_U(\omega,\boldsymbol{k}) = 2\pi u_0^2 n_c \delta(\omega). \qquad (5)$$

Two features of the effective propagator are worth noticing. At first, it is purely real in contrast to usual propagators due to the exchange of a particle in quantum field theories. This is a very important fact and its one immediate consequence as we will see below is that the scattering on impurities leads to an imaginary contribution in the electron self-energy producing a finite width for electron states. At second, the effective propagator (5) depends on frequency only through the Dirac δ-function, therefore, virtual particles in the impurity diagram technique have energies that are completely defined by the energies of external particles. Of course, this result is a trivial consequence of the fact that the electron scattering elastically on an impurity can change its momentum, however, it cannot change the energy. Using Eq.(5), we find the following second order contribution to the electron self-energy due to scattering on an impurity:

$$\Sigma(\Omega,\boldsymbol{p}) = -i\int \frac{d\omega d^3k}{(2\pi)^4} S(\omega,\boldsymbol{k}) D_U(\Omega-\omega,\boldsymbol{p}-\boldsymbol{k})$$

$$= -iu_0^2 n_c \int \frac{d^3k}{(2\pi)^3} S(\Omega,\boldsymbol{k}). \qquad (6)$$

Let us consider how impurity scattering affects the electron states in a magnetic field. Since the effective propagator (4) is local in coordinate space, the Schwinger phase is irrelevant for the second order contribution to the electron self-energy. Then using the translation invariant part of the electron propagator in a magnetic field given, for example, by Eq.(A21) in Ref.[8], we find that Eq.(6) implies the following contribution:

$$\Sigma = -iu_0^2 n_c \Sigma_{n=0}^{\infty} \int \frac{d^3k}{(2\pi)^3} \overline{S_n}(\Omega,\boldsymbol{k}), \qquad (7)$$

where the summation is performed over Landau levels n and $\overline{S_n}(\Omega,\boldsymbol{k})$ is the translation invariant part of the electron propagator on the nth Landau level. Since we are interested in the present paper in the electron states in the lowest Landau level, we retain in Eq.(7) only the LLL contribution. Using

$$\overline{S_0}(\Omega,\boldsymbol{k}) \quad = ie^{-k^2l^2} \frac{\Omega - k^3\gamma^3\gamma^0}{\Omega^2 - k_3^2}$$
$$\times \quad [1 - i\gamma^1\gamma^2 sgn(eB)]\gamma^0, \qquad (8)$$

where $l = |eB|^{-1/2}$ is the magnetic length, we find that the electron self-energy in the LLL due to scattering on impurities equals

$$\Sigma_{LLL} = u_0^2 n_c \int \frac{d^3k}{(2\pi)^3} e^{-k^2l^2} \frac{\Omega - k^3\gamma^3\gamma^0}{\Omega^2 - k_3^2}$$
$$\times [1 - i\gamma^1\gamma^2 sgn(eB)]\gamma^0. \qquad (9)$$

Integrating over the transverse components of momentum and using the chiral representation of the Dirac matrices, it is not difficult to show that Eq.(9) can be rewritten as follows:

$$\Sigma_{LLL} = u_0^2 n_c |eB| \Sigma_{\chi=\pm} \int \frac{dk^3}{8\pi^2} \frac{1}{\Omega + \chi k^3 + i\delta sgn(\Omega)}$$
$$\times [1 - i\gamma^1\gamma^2 sgn(eB)]\gamma^0, \quad \delta \to +0. \qquad (10)$$

The structure of Eq.(10) reflects the well-known fact that LLL fermion propagator reduces to the propagator of two chiral edge states in an effective (1+1)-dimensional theory (see, e.g., Eq.(36) in Ref.[9]). Integrating in Eq.(10), we obtain

$$\Sigma_{LLL} = -i\frac{u_0^2 n_c |eB| sgn(\Omega)}{4\pi}$$
$$\times [1 - i\gamma^1\gamma^2 sgn(eB)]\gamma^0. \qquad (11)$$

Eq.(11) means that LLL states acquire due to scattering on impurities non-zero width $\Gamma = u_0^2 n_c |eB|/(4\pi)$. However, it is not clear how this result is consistent with the well-known result [10] that current-carrying chiral edge states are immune to scattering on impurities for a moderate amount of disorder (for a discussion of impurity scattering for helical edge states see, e.g., Ref.[11]).

In order to clarify the situation, we will calculate the exact Green`s function for chiral edge states. The Hamiltonian density of chiral edge electrons interacting with impurities is given by

$$H = \left(i\tau_3\partial_x + \Sigma_j u_0 \delta(x - x_j)\right)$$

$$= \Sigma_{\chi=\pm}(i\chi\partial_x + \Sigma_j u_0\delta(x - x_j))(1 + \chi\tau_3)/2\,. \quad (12)$$

Since Hamiltonian (12) is a diagonal matrix, it suffices to find the Green`s function at fixed chirality. The corresponding Green`s function in mixed frequency and coordinate representation is determined by

$$G_\omega^\chi(x,y) = < x\left|\frac{1}{\omega - H_\chi}\right| y >, \quad (13)$$

where $H_\chi = i\chi\partial_x + \Sigma_j u_0 \delta(x - x_j)$ is the Hamiltonian for the chiral edge state at fixed chirality. Obviously, without loss of generality we can consider the state with chirality $\chi = +$. Since H_+ defines a completely integrable system, we easily find its complete set of eigenfunctions given by

$$\psi_p(x) = (2\pi)^{-1/2}$$

$$\times \exp\left[-ipx + i\Sigma_j u_0 \int_{-\infty}^{x} dz\, \delta\left(z - x_j\right)\right]. \quad (14)$$

Then the Green`s function equals

$$G_\omega^+(x,y) = \int \frac{dp}{(2\pi)(\omega - p)} \exp[-ip(x - y)$$

$$+ i\Sigma_j u_0 \left(\theta\left(x - x_j\right) - \theta\left(y - x_j\right)\right)]. \quad (15)$$

The exact Green`s function (15) immediately implies that the processes of scattering on impurities do not affect conductivity and current-carrying chiral edge states are indeed immune to such processes.

Further, it is interesting to find the averaged exact Green`s function. This is not difficult to do using Eq.(3). Averaging in Eq.(15), we obtain

$$< G_\omega^+(x,y) = \frac{1}{L^N} \int \frac{dp}{(2\pi)(\omega - p)} e^{-ip(x-y)}$$

$$\times [L - |x - y| + |x - y|\, e^{iu_0 sgn(y-x)}]^N, \quad (16)$$

where L is the spatial size of the system. As it goes to infinity, the Green`s function (16) tends to

$$< G_\omega^+(x,y) = \int \frac{dp}{(2\pi)(\omega - p)} e^{-ip(x-y)} \exp[-2n_c^{(1)}$$

$$\times |x - y| sin^2 \frac{u_0}{2} - in_c^{(1)}(x - y)\sin(u_0)], \quad (17)$$

where $n_c^{(1)} = N/L$ is the one-dimensional concentration of impurities.

Let us discuss the physical meaning of the averaged Green`s function (17). It is known [6,7] that the averaged Green`s function in the first order of perturbation theory in impurity potential produces only a correction to the chemical potential. Clearly, the averaged Green`s function (17) is in agreement with this conclusion. On the other hand, the second order contribution of perturbation theory in impurity potential should produce [6,7] nonzero width of the states which this function describes. Obviously, expanding the averaged Green`s function (17) in the second order in impurity potential we find that the corresponding contribution is real. Therefore, this second order contribution indeed generates a nonzero width for the states which this Green`s function describes.

It is instructive to calculate the second order contribution in impurity potential using the exact Green`s function (15) and compare it with the second order contribution of perturbation theory to the self-energy of the LLL states given by Eq.(11). Expanding the Green`s function (15) in the second order in impurity potential, we obtain

$$G_\omega^{(2)}(x,y) = -u_0^2 \int \frac{dp}{(4\pi)(\omega - p)} e^{-ip(x-y)}$$

$$\times [\Sigma_j \left(\theta(x - x_j) - \theta(y - x_j)\right)]^2. \qquad (18)$$

As usual in the impurity diagram technique [6,7], only scattering on the same impurity should be retained in the second order contribution. Then we find

$$G_\omega^{(2)}(x,y) = -u_0^2 N \int \frac{dp}{(4\pi)(\omega - p)} e^{-ip(x-y)}$$

$$\times [\left(\theta(x - x_j) - \theta(y - x_j)\right)]^2. \qquad (19)$$

Averaging over impurity position in Eq.(19), we obtain

$$< G_\omega^{(2)}(x-y) >= -u_0^2 n_c^{(1)}$$

$$\times \int \frac{dp}{(4\pi)(\omega-p)} e^{-ip(x-y)} \, |x-y|. \qquad (20)$$

Of course, this contribution completely agrees with Eq.(17) in the leading in $n_c^{(1)}$ contribution and is consistent with the self-energy of LLL states given by Eq.(11). Thus, we conclude that although there is a non-zero width of chiral edge states in the averaged Green`s function due to the impurity scatterings within the same Weyl node only scattering processes that involve impurity scatterings between different Weyl nodes can provide finite conductivity.

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