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Алгебра Ауслендера для пар ідемпотентних матриць з подвійним сендвіч-співвідношенням

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The Auslander algebra for the pairs of idempotent matrices with the double sandwich relation

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У роботі отримано канонічну форму для пари ідемпотентних матриць A, B з додатковим співвідношенням $ABA = BAB$ (яке називається подвійним сендвіч-співвідношенням) та обчислено алгебру Ауслендера для таких пар матриць.

Ключові слова: напівгрупа, ідемпотентна матриця, подвійне сендвіч-співвідношення, матричне зображення, ендоморфізм, алгебра Ауслендера.

One of the important notion of the area of modern representation theory that study categories of representations of various algebraic objects is Auslander algebra (which has been studied for decades by many mathematicians). This algebra is defined for an algebra of finite type (i. e. such that has, up to isomorphism, only finitely many indecomposable modules) as the endomorphism algebra of a direct sum of all indecomposable modules, one from each isomorphism class. We use instead of the module language the equivalent language of matrix representations. Since there is a natural correspondence between the representations of a semigroup and the representations of its semigroup algebra over any field, we can say about the Auslander algebra of each semigroup of finite representation type.

In the first part of this paper we obtain a canonical form of representation of the semigroup generated by two idempotent elements a, b with additional relation $aba = bab$ (which we call the double sandwich relation), or in other term, a canonical form of pairs idempotent matrices such that $ABA = BAB$. We prove that this semigroup has, up to equivalence, only finite number of indecomposable representations.

In the second part of this paper we calculate the Auslander algebra in this situation.

Key Words: semigroup, idempotent matrix, double sandwich relation, matrix representation, endomorphism, Auslander algebra.

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1 Introduction

Throughout this paper, K denotes a field.

Let $S = S_{dsr}(2, 2)$ denotes the semigroup with the generators $0, a, b$ and the defining relations

1) $0^2 = 0,\ 0a = a0 = 0,\ 0b = b0 = 0$;

2) $a^2 = a,\ b^2 = b$;

3) $aba = bab$.

We call the relation 3) the *double sandwich relation.*

By the general definition, a *matrix representation of the semigroup S over K* is a homomorphisms $T : S \to M_n(K)$, where $M_n(K)$ is the semigroup (with respect to the multiplication) of all $n \times n$ matrices over K (n is called the *dimension* of the representation T). We can assume, essentially without loss of generality, that $T(0) = 0$. Then T is uniquely determined by the pair of matrices $R(T) = (A, B)$ with $A^2 = A$, $B^2 = B$, $ABA = BAB$ (here $A = T(a), B = T(b)$). We identify the representation T with the pairs of matrices $R(T)$.

Representation $R = \{A, B\}$ and $R' = \{A', B'\}$ of the semigroup S are said to be *equivalent* if $A' =$

CAC^{-1} and $B' = CBC^{-1}$ for some invertible matrix C. A matrix representation R is said to be *decomposable* if it is equivalent to a direct sum of two representations, and *indecomposable* otherwise. For matrix representations of S, the Krull-Schmidt theorem (on the uniqueness of decomposition into a direct sum of indecomposables) holds.

We identify the problem on matrix representation of the semigroup $S_{dsr}(2,2)$ and the problem on pairs idempotent matrices with double sandwich relation using both notions depending on the particular situation.

The first aim of this paper it to classify (up to equivalence) the indecomposable representations of the semigroup $S_{dsr}(2,2)$. The second part is devoted to the study of the Auslander algebra of this semigroup.

The Auslander algebra (the endomorphism algebra of a direct sum of the representatives of the indecomposable representations) is introduced for algebras (in particular, for semigroup algebras, or equivalently, semigroups themselves) only in the case of finite representation type, i. e. with only finitely many (up to equivalence) indecomposable representations.

We emphasize that the semigroup $S(2,2)$ with the generators a, b and the defining relations $a^2 = a, b^2 = b$ is of infinite representation type; its representations can be classified by one of the method of [1] using a result of [2]. Since the semigroup $S_{dsr}(2,2)$ is of finite representation type (see below Theorem 2.1), we can say about its Auslander algebra.

2 Classification theorem

By E we denote any identity matrix of size $m \times m$ with $m \geqslant 0$.

Theorem 2.1. *Any matrix representation of the semigroup $S_{dsr}(2,2)$ is equivalent to a matrix representation of the form*

$$a \to A = \begin{pmatrix} E & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & E & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & E & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & E & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$b \to B = \begin{pmatrix} E & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & E & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & E & E & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & E & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & E & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Proof. We carry out the proof on the language of admissible elementary transformations, and following the traditions of the Roiter theory of matrix problems we denote, as rule, the result of applying such transformations to some matrices by the same symbols. When we partition into blocks some matrices of a matrix representation, we always assume (often by default) that all the other matrices are partitioned analogously. The same is assumed with respect to parts of matrices and these matrices themselves, and in the other analogously situations.

Let $R = \{A, B\}$ be a matrix representation of the semigroup $S = S_{dsr}(2,2)$ over K. We reduce A to a normal Jordan form, namely

$$A = \begin{pmatrix} E & 0 \\ 0 & 0 \end{pmatrix}$$

and partition B into blocks in the same way as A:

$$B = \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix}.$$

First, we use the equality $ABA = BAB$:

$$ABA = \begin{pmatrix} E & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix} \begin{pmatrix} E & 0 \\ 0 & 0 \end{pmatrix} =$$

$$= \begin{pmatrix} B_{11} & 0 \\ 0 & 0 \end{pmatrix};$$

$$BAB = \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix} \begin{pmatrix} E & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix} =$$

$$= \begin{pmatrix} B_{11} & 0 \\ B_{21} & 0 \end{pmatrix} \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix} =$$

$$= \begin{pmatrix} B_{11}^2 & B_{11}B_{12} \\ B_{21}B_{11} & B_{21}B_{12} \end{pmatrix};$$

consequently

$$B_{11}^2 = B_{11} \quad (1)$$

$$B_{11}B_{12} = 0 \quad (2)$$

$$B_{21}B_{11} = 0 \quad (3)$$

$$B_{21}B_{12} = 0 \tag{4}$$

Now we use the equality $B^2 = B$:

$$B_{11}^2 + B_{12}B_{21} = B_{11} \tag{5}$$

$$B_{11}B_{12} + B_{12}B_{22} = B_{12} \tag{6}$$

$$B_{21}B_{11} + B_{22}B_{21} = B_{21} \tag{7}$$

$$B_{21}B_{12} + B_{22}^2 = B_{22} \tag{8}$$

Taking into account relations (1)-(4) we write the equalities (5)-(8) in the following form:

$$B_{12}B_{21} = 0 \tag{9}$$

$$B_{12}B_{22} = B_{12} \tag{10}$$

$$B_{22}B_{21} = B_{21} \tag{11}$$

$$B_{22}^2 = B_{22} \tag{12}$$

Since the matrices B_{11} and B_{22} are idempotent (see (1) and (12)) they can be reduced, by admissible transfornations to be similarity trasformations with A and B (inside the 1st and 2nd horizontal and vertical bands), to the following forms:

$$B_{11} = \begin{pmatrix} E & 0 \\ 0 & 0 \end{pmatrix} \text{ and } B_{22} = \begin{pmatrix} E & 0 \\ 0 & 0 \end{pmatrix}.$$

In accordance with the partitions of B_{11} and B_{22} we partition B_{12} and B_{21} into the new blocks:

$$B_{12} = \begin{pmatrix} C_1 & C_2 \\ C_3 & C_4 \end{pmatrix} \text{ and } B_{21} = \begin{pmatrix} D_1 & D_2 \\ D_3 & D_4 \end{pmatrix}.$$

Then we have

$$\begin{pmatrix} C_1 & C_2 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

from equality (2),

$$\begin{pmatrix} D_1 & 0 \\ D_3 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

from equality (3),

$$\begin{pmatrix} C_1 & 0 \\ C_3 & 0 \end{pmatrix} = \begin{pmatrix} C_1 & C_2 \\ C_3 & C_4 \end{pmatrix}$$

from equality (10),

$$\begin{pmatrix} D_1 & D_2 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} D_1 & D_2 \\ D_3 & D_4 \end{pmatrix}$$

from equality (11).

So $C_1 = 0$, $C_2 = 0$, $C_4 = 0$ i $D_1 = 0$, $D_3 = 0$, $D_4 = 0$, and consequently we have

$$\begin{pmatrix} 0 & D_2 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ C_3 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

from equality (4) and

$$\begin{pmatrix} 0 & 0 \\ C_3 & 0 \end{pmatrix} \begin{pmatrix} 0 & D_2 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

from equality (9), whence $D_2C_3 = 0$, $C_3D_2 = 0$.

Thus our matrix representation $R = \{A, B\}$ consists of matrices of the following form:

$$A = \begin{pmatrix} E & 0 & 0 & 0 \\ 0 & E & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, B = \begin{pmatrix} E & 0 & 0 & 0 \\ 0 & 0 & C & 0 \\ 0 & D & E & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{13}$$

where $DC = 0$, $CD = 0$.

Now find out when the matrix representation $R = \{A, B\}$ is equivalent to another matrix representation $\overline{R} = \{\overline{A}, \overline{B}\}$ with the matrices $\overline{A}, \overline{B}$ to be the same form as A, B:

$$\overline{A} = \begin{pmatrix} E & 0 & 0 & 0 \\ 0 & E & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \overline{B} = \begin{pmatrix} E & 0 & 0 & 0 \\ 0 & 0 & \overline{C} & 0 \\ 0 & \overline{D} & E & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

Let X be an invertible matrix such that $\overline{A} = XAX^{-1}$, $\overline{B} = XBX^{-1}$, or equivalently,

$$\overline{A}X = XA, \quad \overline{B}X = XB.$$

First, we use the equality $\overline{A}X = XA$ (with the matrix X to be partitioned into blocks in accordance with the partitions of B_{11} and B_{22}):

$$\begin{pmatrix} E & 0 & 0 & 0 \\ 0 & E & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} X_{11} & X_{12} & X_{13} & X_{14} \\ X_{21} & X_{22} & X_{23} & X_{24} \\ X_{31} & X_{32} & X_{33} & X_{34} \\ X_{41} & X_{42} & X_{43} & X_{44} \end{pmatrix} =$$

$$= \begin{pmatrix} X_{11} & X_{12} & X_{13} & X_{14} \\ X_{21} & X_{22} & X_{23} & X_{24} \\ X_{31} & X_{32} & X_{33} & X_{34} \\ X_{41} & X_{42} & X_{43} & X_{44} \end{pmatrix} \begin{pmatrix} E & 0 & 0 & 0 \\ 0 & E & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

As result, we have

$$\begin{pmatrix} X_{11} & X_{12} & 0 & 0 \\ X_{21} & X_{22} & 0 & 0 \\ X_{31} & X_{32} & 0 & 0 \\ X_{41} & X_{42} & 0 & 0 \end{pmatrix} = \begin{pmatrix} X_{11} & X_{12} & X_{13} & X_{14} \\ X_{21} & X_{22} & X_{23} & X_{24} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

whence

$$X = \begin{pmatrix} X_{11} & X_{12} & 0 & 0 \\ X_{21} & X_{22} & 0 & 0 \\ 0 & 0 & X_{33} & X_{34} \\ 0 & 0 & X_{34} & X_{44} \end{pmatrix}.$$

Now we use the equality $\overline{B}X = XB$ (with X to be of the new form):

$$\begin{pmatrix} E & 0 & 0 & 0 \\ 0 & 0 & \overline{C} & 0 \\ 0 & \overline{D} & E & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} X_{11} & X_{12} & 0 & 0 \\ X_{21} & X_{22} & 0 & 0 \\ 0 & 0 & X_{33} & X_{34} \\ 0 & 0 & X_{34} & X_{44} \end{pmatrix} =$$

$$= \begin{pmatrix} X_{11} & X_{12} & 0 & 0 \\ X_{21} & X_{22} & 0 & 0 \\ 0 & 0 & X_{33} & X_{34} \\ 0 & 0 & X_{34} & X_{44} \end{pmatrix} \begin{pmatrix} E & 0 & 0 & 0 \\ 0 & 0 & C & 0 \\ 0 & D & E & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

As result, we have

$$\overline{C}X_{33} = X_{22}C, \quad \overline{D}X_{22} = X_{33}D,$$

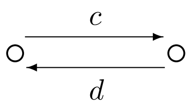
$$X_{12} = 0,\ X_{21} = 0,\ X_{34} = 0,\ X_{43} = 0.$$

Thus, the matrix X is block-diagonal, and the matrix representations $R = \{A, B\}$ and $\overline{R} = \{\overline{A}, \overline{B}\}$ (of the form (13)) are equivalent if and only if the following equalities hold:

$$\overline{C} = X_{22}CX_{33}^{-1} \tag{14}$$

$$\overline{D} = X_{33}DX_{22}^{-1} \tag{15}$$

In other terms this means that $R = \{A, B\}$ and $\overline{R} = \{\overline{A}, \overline{B}\}$ are equivalent if and only if the pairs of matrices (C, D) and $(\overline{C}, \overline{D})$ are equivalent as matrix representations of the quiver



with relations $cd = 0$, $dc = 0$.

It is easy to see (and is well known) that any matrix representation (C, D) of this quiver is equivalent to those of the following form:

$$C = \begin{pmatrix} 0 & 0 & E \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad D = \begin{pmatrix} 0 & 0 & E \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

(the horizontal and vertical partitions of D are the same as the vertical and horizontal partitions of C, respectively). Substituting these matrices in (13) we obtain the matrices A, B of the form indicated in the theorem, and the proof is complete.

We have the following corollary.

Наслідок 1. *The indecomposable matrix representations of the semigroup $S = S_{dsr}(2, 2)$ over the field K are exhausted, up to equivalence, by the following (pairwise non-equivalent) ones:*

1) $a \to 0, \quad b \to 0;$
2) $a \to 1, \quad b \to 0;$
3) $a \to 0, \quad b \to 1;$
4) $a \to 1, \quad b \to 1;$
5) $a \to \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad b \to \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix};$
6) $a \to \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad b \to \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}.$

Indeed, it follows from Theorem 1 that any matrix representation of $S_{dsr}(2, 2)$ is equivalent to a direct sum of representations of the form 1)−6). On the other hand, all representation 1)−6) are indecomposable: 1) − 4) as one-dimensional representations and 5), 6) as representations with the trivial algebra of endomorphisms.

3 Calculation of the Auslander algebra

Using Theorem 2.1 we describe the Auslander algebra for the pairs of idempotent matrices A, B with the double sandwich relation $ABA = BAB$ over the field K. Since by this theorem the pair of matrices A, B with all blocks E to be of size 1×1, i. e. the pair of matices

$$A_0 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$B_0 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

is (up to the same permutation of rows and columns) a direct sum of all indecomposable, pairwise non-similar, pairs of idempotent matrices with the double sandwich relation, the (matrix) Auslander algebra in this case consists of all the matrices X such that $A_0X = XA_0, B_0X = XB_0$.

First, we consider the equality $A_0X = XA_0$:

$$\begin{pmatrix} x_{11} & x_{12} & x_{13} & x_{14} & x_{15} & x_{16} & x_{17} & x_{18} \\ x_{21} & x_{22} & x_{23} & x_{24} & x_{25} & x_{26} & x_{27} & x_{28} \\ x_{31} & x_{32} & x_{33} & x_{34} & x_{35} & x_{36} & x_{37} & x_{38} \\ x_{41} & x_{42} & x_{43} & x_{44} & x_{45} & x_{46} & x_{47} & x_{48} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} =$$

$$= \begin{pmatrix} x_{11} & x_{12} & x_{13} & x_{14} & 0 & 0 & 0 & 0 \\ x_{21} & x_{22} & x_{23} & x_{24} & 0 & 0 & 0 & 0 \\ x_{31} & x_{32} & x_{33} & x_{34} & 0 & 0 & 0 & 0 \\ x_{41} & x_{42} & x_{43} & x_{44} & 0 & 0 & 0 & 0 \\ x_{51} & x_{52} & x_{53} & x_{54} & 0 & 0 & 0 & 0 \\ x_{61} & x_{62} & x_{63} & x_{64} & 0 & 0 & 0 & 0 \\ x_{71} & x_{72} & x_{73} & x_{74} & 0 & 0 & 0 & 0 \\ x_{81} & x_{82} & x_{83} & x_{84} & 0 & 0 & 0 & 0 \end{pmatrix},$$

whence the matrix X has the form

$$X = \begin{pmatrix} x_{11} & x_{12} & x_{13} & x_{14} & 0 & 0 & 0 & 0 \\ x_{21} & x_{22} & x_{23} & x_{24} & 0 & 0 & 0 & 0 \\ x_{31} & x_{32} & x_{33} & x_{34} & 0 & 0 & 0 & 0 \\ x_{41} & x_{42} & x_{43} & x_{44} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & x_{55} & x_{56} & x_{57} & x_{58} \\ 0 & 0 & 0 & 0 & x_{65} & x_{66} & x_{67} & x_{68} \\ 0 & 0 & 0 & 0 & x_{75} & x_{76} & x_{77} & x_{78} \\ 0 & 0 & 0 & 0 & x_{85} & x_{86} & x_{87} & x_{88} \end{pmatrix}.$$

Now, using the above form of X, we consider the equality $B_0X = XB_0$:

$$\begin{pmatrix} x_{11} & x_{12} & x_{13} & x_{14} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & x_{75} & x_{76} & x_{77} & x_{78} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ x_{41} & x_{42} & x_{43} & x_{44} & x_{55} & x_{56} & x_{57} & x_{58} \\ 0 & 0 & 0 & 0 & x_{65} & x_{66} & x_{67} & x_{68} \\ 0 & 0 & 0 & 0 & x_{75} & x_{76} & x_{77} & x_{78} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} =$$

$$= \begin{pmatrix} x_{11} & 0 & 0 & 0 & 0 & 0 & x_{12} & 0 \\ x_{21} & 0 & 0 & 0 & 0 & 0 & x_{22} & 0 \\ x_{31} & 0 & 0 & 0 & 0 & 0 & x_{32} & 0 \\ x_{41} & 0 & 0 & 0 & 0 & 0 & x_{42} & 0 \\ 0 & 0 & 0 & x_{55} & x_{55} & x_{56} & x_{57} & 0 \\ 0 & 0 & 0 & x_{65} & x_{65} & x_{66} & x_{67} & 0 \\ 0 & 0 & 0 & x_{75} & x_{75} & x_{76} & x_{77} & 0 \\ 0 & 0 & 0 & x_{85} & x_{85} & x_{86} & x_{87} & 0 \end{pmatrix}.$$

So it is easy to see that

$$X = \begin{pmatrix} x_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & x_{22} & x_{23} & x_{24} & 0 & 0 & 0 & 0 \\ 0 & 0 & x_{33} & x_{34} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_{44} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & x_{44} & x_{56} & x_{57} & 0 \\ 0 & 0 & 0 & 0 & 0 & x_{66} & x_{67} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & x_{22} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & x_{88} \end{pmatrix}. \quad (16)$$

Thus we proved the following theorem.

Theorem 3.1. *The matrix Auslander algebra for the pairs of idempotent matrices A, B over K with the double sandwich relation $ABA = BAB$ consists of all matrices of the form (16).*

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