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# Розсіяння поверхневого плазмон поляритону на двовимірній напівпровідниковій наносмужці. Ефективна сприйнятливість двовимірної наносмужки

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# Scattering of surface plasmon polariton by two-dimensional semiconductor nanostripe. Effective susceptibility of two-dimensional nanostripe

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*Запропоновано метод розрахунку інтенсивності локального поля навколо нескінченно довгої наносмужки на поверхні зі збудженим поверхневим плазмоном. Розрахунок проводиться за допомогою методу функцій Гріна в рамках концепції ефективної сприйнятливості, використовуючи наближення ближнього поля. Розглянуто метод аналітичного розрахунку ефективної сприйнятливості двовимірної наносмужки. Отримано розподіл локального поля навколо кремнієвої наносмужки з шириною та висотою 100 нм. Отриманий розподіл несиметричний і дуже неоднорідний.*

*Ключові слова: поверхневий плазмон поляритон, розсіяння поверхневих хвиль, ефективна сприйнятливість, напівпровідникова наносмужка*

*Local-field intensity calculation of infinity-long nanostripe on the surface with excited surface plasmons is proposed. Calculation is performed using Green's function method in the frame of concept of effective susceptibility using near-field approximation. The method of analytically calculation of effective susceptibility of two-dimension nanostripe was considered. The main characteristic of the proposed approach is maximal using analytical calculations. Obtained results are universal and could be able to calculate local-field intensity of any low-dimension rectangular nanostripe with any aspect ratio or material of nanostripe or surface. The local-field intensity distribution near infinitely-long nanostripe was numerically calculated. For calculations was chosen silicon nanostripe placed on the gold surface with linear dimensions: width – 100 nm, height – 100 nm. Obtained distribution is unsymmetrical and highly inhomogeneous with "hot spots" (local field enhancement) on the nanostripe surface.*

*Key Words: plasmon polaritons, surface waves scattering, effective susceptibility, semiconductor nanostripe*

Статтю представив д.ф.-м.н. Скришевський В.А.

**Introduction.** Surface plasmon polaritons (SPPs) are electromagnetic surface waves propagating along metal-dielectric interfaces with their intensity maximum in the surface and exponential decaying perpendicular to the surface [2]. In recent years SPP propagating features has been intensively studied. Theoretical research of SPP scattering [3, 5] shows how to calculate scattered field intensity far from the scatterer. But sometimes we need to know local-field distribution near the scatterer. For calculation SPP scattering by some nanoobjects we need to solve Maxwell's equations [1], but this is quite difficult task. Of course, we can use numerical methods such as Finite Difference Time Domain (FDTD) method [4], but it requires huge computational resources.

In this article theoretical study of SPP scattering by infinitely long nanostripe using Green's function method is presented. The main characteristic of the proposing approach is maximal using analytical calculations. Calculation is performed using concept of effective susceptibility [7-8], which is a characteristics of both nano-object and material and it is a linear response on external field.

The main goal of our work is to calculate effective susceptibility of infinitely-long nanostripe and obtain local-field intensity distribution near silicon nanostripe.

**Effective susceptibility of infinitely-long nanostripe**. The system under consideration represents an infinitely-long homogeneous nanostripe (Fig.1) placed along y-axis with susceptibility $\chi_p$ placed on the surface with dielectric constant $\varepsilon_1$ (in the region $z<0$) and upper semi space has dielectric constant $\varepsilon_2$ (in the region $z>0$). Of cause, considered nanostripe has determined length, but we assume that length of nanostripe much greater than wave length and SPP width. The response of that system on external field can be found using Lippmann-Schwinger equation in the frame of concept of effective susceptibility [7]:

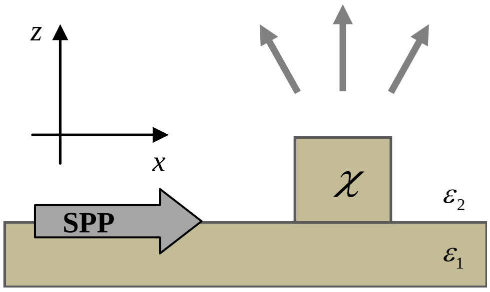


Fig.1. Schematic of the scattering system: an external SPP propagating along the dielectric-metal interface (x-axis) is scattered by a rectangular nanostripe.

$$E_i(\mathbf{R}) = E_i^0(\mathbf{R}) + k_0^2 \int_V d\mathbf{R}' G_{ij}(\mathbf{R},\mathbf{R}') \mathrm{X}_{jk}(\mathbf{R}') E_k^0(\mathbf{R}') \quad (1)$$

$$G_{ji}^D(\mathbf{R},\mathbf{R}') = \frac{1}{4\pi k_0^2 R^3}\left[-\delta_{ij} + 3\frac{R_i R_j}{R^2}\right] \quad (2)$$

$$G_{ji}^I(\mathbf{R},\mathbf{R}'') = \frac{\varepsilon_2-\varepsilon_1}{\varepsilon_2+\varepsilon_1}\frac{1}{4\pi k_0^2 R^3}\left[-\delta_{ij} + 3\frac{R_i R_j}{R^2}\right] \quad (3)$$

where $G_{ij}(\mathbf{R},\mathbf{R}')$ is the sum of direct (2) and indirect (3) part of Green's function with near-field approximation [8], $\mathbf{R}''=\{x',y',-z'\}$ and $R=\sqrt{(x-x')^2+(y-y')^2+(z+z')^2}$. And effective susceptibility is:

$$\mathrm{X}_{ij}(\mathbf{R}) = \frac{\chi_{ij}(\mathbf{R})}{\delta_{ij} - S_{ij}(\mathbf{R})} \quad (4)$$

$$S_{ij}(\mathbf{R}) = k_0^2 \int_V d\mathbf{R}' G_{ij}(\mathbf{R},\mathbf{R}') \chi_{jk}(\mathbf{R}') \quad (5)$$

Taking into account that nanostripe is infinitely-long along y-axis and homogeneous let us integrate eq. (5):

$$S_{ij}(\mathbf{R}) = \chi_{jk} k_0^2 \int_{s'} ds' \int_{-\infty}^{+\infty} dy' G_{ij}(\mathbf{R},\mathbf{R}') \quad (6)$$

$$S_{ij}(\mathbf{R}) = \chi_{jk} k_0^2 \int_{s} ds' G_{ij}^{2D}(x,z,x',z') \quad (7)$$

where $G_{ij}^{2D}(x,z,x',z')$ is Green's function (photon propagator) of two-dimensional system with an interface between two media and it is:

$$G_{ij}^{2D}(x,z,x',z') = \frac{1}{2\pi k_0^2} \times \begin{bmatrix} g_{11} & 0 & g_{13} \\ 0 & 0 & 0 \\ g_{31} & 0 & g_{33} \end{bmatrix} \quad (8)$$

$$g_{11} = \frac{(x-x_0)^2-(z-z_0)^2}{((x-x')^2+(z-z')^2)^2} + \kappa\frac{(x-x_0)^2-(z+z_0)^2}{((x-x')^2+(z+z')^2)^2} \quad (9)$$

$$g_{13} = \frac{2(x-x_0)(z-z_0)}{((x-x')^2+(z-z')^2)^2} - \kappa\frac{2(x-x_0)(z+z_0)}{((x-x')^2+(z+z')^2)^2} \quad (10)$$

$$g_{31} = \frac{2(x-x_0)(z-z_0)}{((x-x')^2+(z-z')^2)^2} + \kappa\frac{2(x-x_0)(z+z_0)}{((x-x')^2+(z+z')^2)^2} \quad (11)$$

$$g_{33} = -\frac{(x-x_0)^2-(z-z_0)^2}{((x-x')^2+(z-z')^2)^2} + \kappa\frac{(x-x_0)^2-(z+z_0)^2}{((x-x')^2+(z+z')^2)^2} \quad (12)$$

where $\kappa=(\varepsilon_2-\varepsilon_1)/(\varepsilon_2+\varepsilon_1)$. After all, effective susceptibility can be easily analytically calculated.

$$\mathrm{X}_{ij}(\mathbf{R}) = \frac{\chi_{ij}}{1-c_{11}\chi_{11}-c_{33}\chi_{33}-c_{13}c_{31}\chi_{11}\chi_{33}+c_{11}c_{33}\chi_{11}\chi_{33}} \times \times \begin{bmatrix} 1-c_{33}\chi_{33} & 0 & c_{13}\chi_{11} \\ 0 & 1 & 0 \\ c_{31}\chi_{33} & 0 & 1-c_{11}\chi_{11} \end{bmatrix} \quad (13)$$

$$c_{11}(x,z) = \frac{1}{2\pi}\left( \begin{array}{l} -\arctan\left(\frac{z-z_1}{x-x_1}\right) + \arctan\left(\frac{z-z_1}{x-x_2}\right) + \arctan\left(\frac{z-z_2}{x-x_1}\right) - \arctan\left(\frac{z-z_2}{x-x_2}\right) + \\ +\kappa\left(\arctan\left(\frac{z+z_1}{x-x_1}\right) - \arctan\left(\frac{z+z_1}{x-x_2}\right)\right) - \kappa\left(\arctan\left(\frac{z+z_2}{x-x_1}\right) - \arctan\left(\frac{z+z_2}{x-x_2}\right)\right) \end{array} \right) \quad (14)$$

$$c_{13}(x,z)=\frac{1}{4\pi}\begin{pmatrix}-\ln\left((x-x_1)^2+(z-z_1)^2\right)+\ln\left((x-x_2)^2+(z-z_1)^2\right)+\ln\left((x-x_1)^2+(z-z_2)^2\right)-\ln\left((x-x_2)^2+(z-z_2)^2\right)-\\-\kappa\left(\ln\left((x-x_1)^2+(z+z_1)^2\right)-\ln\left((x-x_2)^2+(z+z_1)^2\right)-\ln\left((x-x_1)^2+(z+z_2)^2\right)+\ln\left((x-x_2)^2+(z+z_2)^2\right)\right)\end{pmatrix} \quad (15)$$

$$c_{31}(x,z)=\frac{1}{4\pi}\begin{pmatrix}-\ln\left((x-x_1)^2+(z-z_1)^2\right)+\ln\left((x-x_2)^2+(z-z_1)^2\right)+\ln\left((x-x_1)^2+(z-z_2)^2\right)-\ln\left((x-x_2)^2+(z-z_2)^2\right)+\\+\kappa\left(\ln\left((x-x_1)^2+(z+z_1)^2\right)-\ln\left((x-x_2)^2+(z+z_1)^2\right)-\ln\left((x-x_1)^2+(z+z_2)^2\right)+\ln\left((x-x_2)^2+(z+z_2)^2\right)\right)\end{pmatrix} \quad (16)$$

$$c_{33}(x,z)=\frac{1}{2\pi}\begin{pmatrix}\arctan\left(\frac{z-z_1}{x-x_1}\right)-\arctan\left(\frac{z-z_1}{x-x_2}\right)-\arctan\left(\frac{z-z_2}{x-x_1}\right)+\arctan\left(\frac{z-z_2}{x-x_2}\right)+\\+\kappa\left(\arctan\left(\frac{z+z_1}{x-x_1}\right)-\arctan\left(\frac{z+z_1}{x-x_2}\right)\right)-\kappa\left(\arctan\left(\frac{z+z_2}{x-x_1}\right)-\arctan\left(\frac{z+z_2}{x-x_2}\right)\right)\end{pmatrix} \quad (17)$$

Here, $x_1, x_2, z_1, z_2$ are coordinates of the sides of the nanostripe (Fig.2.). In addition, it should be noted that $X_{yy}=\chi_{yy}$ which corresponds to a bulk material.

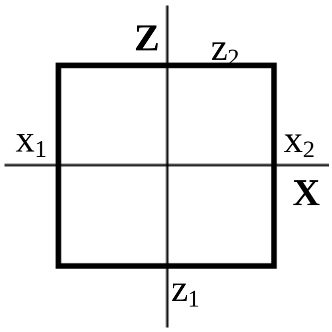


Fig.2. The coordinates of the sides of the nanostrpipe.

**Numerical calculation.** Let us consider the case when external field is surface plasmon polariton (SPP) that propagating along x-direction and perpendicular to the nanostripe that leads to SPP's scattering. Using eq. (1) we can easily calculate response of nanostripe on the SPP. In this case external field is [6]:

$$E_i^0(\mathbf{R})=\exp(ik_{spp}x-ik_{spp}Z_s z)\times\left[Z_s,0,1\right] \quad (18)$$

where SPP's wave number is $k_{spp}=(\omega/c)\sqrt{\varepsilon_1\varepsilon_2/(\varepsilon_1+\varepsilon_2)}$, $Z_s=-i\sqrt{\varepsilon_2/(-\varepsilon_1)}$. Let us calculate local-field intensity distribution at the nanostripe. For example, SPP propagating along gold-air interface with $\varepsilon_1=-20.13+0.29i$ (for $\lambda=632nm$) and $\varepsilon_2=1$ respectively. For stripe were used axial dimensions: $h_x=h_z=100nm$, $h_y$ much greater than wave length and SPP width, and material is silicon $\chi_p=14.045+0.008i$. The results of numerical calculations of local-field intensity are presented on Fig.3.

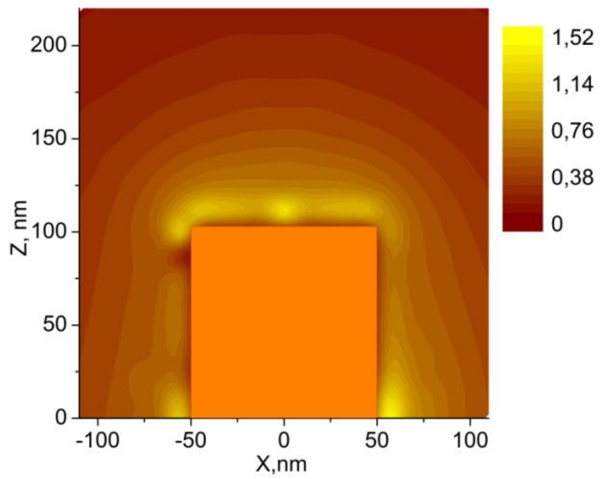


Fig.3. The coordinates of the sides of the nanostrpipe.

One can see the local-field intensity distribution is very inhomogeneous with "hot spots" – local-field enhancement on the stripe surface. It should be noted, local field distribution is unsymmetrical. The local field is lower on the front side to the surface wave then on the opposite side

**Conclusions.** The method of analytical calculation of effective susceptibility of infinity-long rectangular nanostripe and local-field intensity distribution is proposed. Developed approach is universal and could be able to find local-field distribution for any low-dimension nanostripe with any aspect ratio or material of the nanostripe or surface. The local-field intensity distribution near infinitely-long nanostripe was numerically calculated. For calculations was chosen silicon nanostripe placed on the gold surface with linear dimensions: width – 100 nm, height – 100 nm. Obtained distribution is unsymmetrical and highly inhomogeneous with "hot spots" (local field enhancement) on the nanostripe surface.

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