ГЕОЛОГІЧНА ІНФОРМАТИКА

UDC 550.3 (519.21)

Z. Vyzhva, Dr. Sci. (Phys.-Math.), Assoc. Prof.
E-mail: zoya_vyzhva@ukr.net,
K. Fedorenko, Postgraduate Student,
E-mail: slims_mentol@mail.ru
A. Vyzhva, Postgraduate Student,
E-mail: motomustanger@ukr.net
Institute of Geology
Taras Shevchenko National University of Kyiv
90 Vasylkivska Str., Kyiv, 03022 Ukraine

ABOUT ALGORITHMS OF STATISTICAL SIMULATION OF SEISMIC NOISE IN THE OBSERVATION PROFILE FOR DETERMINATION THE FREQUENCY CHARACTERISTICS OF GEOLOGICAL ENVIRONMENT

(Рекомендовано членом редакційної колегії д-ром фіз.-мат. наук, проф. Б.П. Масловим)

The article is devoted to the theory and methods of random process and field statistical simulation on the basis of their spectral decomposition and modified Kotelnikov-Shennon interpolation sums, as well as using these methods in environmental geophysical monitoring. The problem of statistical simulation of seismic noise in the observation profile is under consideration for introduction into seismological researches for determination of the frequency characteristics of geological environment. Statistical model and numerical algorithm of simulation realizations of such random fields are built on the basis of modified Kotelnikov-Shennon interpolation decompositions for generating the adequate realizations of seismic noise. Real-valued random fields ξ(t, x), $t \in R$, $x \in R$, that are homogeneous with respect to time and homogeneous isotropic with respect to spatial variable are studied. The problem of approximation of such random fields by random fields with a bounded spectrum is considered. An analogue of the Kotelnikov–Shannon theorem for random fields with a bounded spectrum is presented. Estimates of the mean-square approximation of random fields in the space $R \times R$ by a model constructed with the help of the spectral decomposition and interpolation of Kotelnikov–Shannon formula are obtained. Some procedures for the statistical simulation of realizations of Gaussian random fields with a bounded spectrum that are homogeneous with respect to time and homogeneous isotropic with respect to spatial variable are developed. Teorems on the mean-square approximation of homogeneous in time and homogeneous isotropic with respect to the other variable random fields by special partial sums have been proved. A simulation method was used to formulate an algorithm of numerical simulation by means of these theorems. The spectral analysis methods of generated seismic noise realizations are considered. Universal methods of statistical simulation (Monte Carlo methods) of multi parameters seismology data for generating of seismic noise in the observation profile of required detail and regularity have been developed.

Keywords: statistical simulation, seismic noise, random process

Introduction. Due to the rapid development of computer technology, methods of numerical simulation (the so called Monte Carlo methods) of stochastic processes and random fields have an expanding range of applications. They are applied, in particular, in such natural sciences as geology, geophysics, geoinformatics, seismology, meteorology, oceanography, electrical engineering, statistical radio physics, nuclear physics, and others. Using statistical simulation techniques and computers, one can generate realizations of stochastic processes and random field for which some necessary statistical data is known.

The statistical simulation of random functions on the basis of their spectral decomposition (M. Yadrenko, 1983) is very important to resolve these problems [13]. Modified Kotelnikov–Shannon interpolation decompositions for stochastic processes and 2D random fields have been studied by J. Higgins, 1996; Z. Vyzhva, 2003, 2011, 2012; A. Olenko, 2004 [2–4, 6, 10].

The problem of improving the procedures developed in the works [4, 5] is considered in the article. The statistical simulation realizations of random fields ξ(t,s) of seismic noise with a bounded spectrum, depending on the time t, and given to the spatial variable s on profile of regular or irregular step is presented. These algorithms are based on the modified Kotelnikov–Shannon interpolation decomposition for implementation in seismological researches on the needs of determining the frequency characteristics of the geological environment at construction sites. Models and procedures for numerical simulation realizations of random fields are built on the basis of the errors. Estimates of mean square approximation of such fields are defined. The simulated realizations are adequate to realizations of seismogram noise in the case of two variables.

Practically it is important to use the statistical simulation realizations of such random fields for the extract the seismic noise, that depends on one or more significant parameters, and external influence and to obtain corresponding estimates of the frequency characteristics of three-dimensional geological environment of the monitoring profile. These estimates should be considered in the construction of different objects in order to ensure the reliability of buildings.

Statistical simulation of seismic noise on monitoring profile with regular step

The method developed on the basis of modified Kotelnikov–Shannon interpolation decompositions for random fields in the two-dimensional observation area with a bounded spectrum on a regular grid of observations, is used for statistical simulation of the observed seismograms noises [2, 3].

If we consider the random field ξ(t,s) (--∞≤ t, s ≤+-∞) in the two-dimensional observation area with a bounded spectrum with a bounded spectrum of each variable, it is possible to build the model of such field based on the modified Kotelnikov–Shannon interpolation decompositions [2]. The variable s can be interpreted as the distance between the point of observation of the simulated noise of seismogram and the initial point of observation.

The representation of separable two-dimensional random fields as a decomposition in infinite interpolation series

The representation of two-dimensional random functions in this paragraph as a decomposition in infinite interpolation series in this paragraph conduces to the necessity of researching the approximation errors of these functions by finite series, because in practice we are often unable to measure all the parameters of some event with complete accuracy. The error of experimental result is the most closely connected to the theorems of simulation, for example, the so-called "aliasing errors" [10].

For the statistical simulation realizations of random fields on a plane with the uniform grid of interpolation it is also important to know dependence of the accuracy of mean square approximation of such fields by series on the number of harmonics of this series by using in practice its partial sum. Therefore, the following theorem is presented [2].

Theorem 1. If ξ(t,s) (-∞ < t, s < +∞) is a separable random field, Mξ(t,s)=0, then it can be represented as the integral:

$$\xi(t,s)=\int_{U^2} f_1(t,u)f_2(s,v)Z(du,dv), \qquad (1)$$

where U is a set of parameters u (a bounded domain of real numbers), $U_2=U\times U$, and $Z(du, dv)$ is a random function of sets on $U\times U$ that satisfy the following condition:

$$MZ(S_1,S_2)Z(G_1,G_2)=F(S_1,S_2,G_1,G_2),\quad \forall S_1,S_2,G_1,G_2\in B_2. \qquad (2)$$

Moreover, F is a complex function of sets, that is additive for all arguments and positive definite and so:

$$\iint_{U^2U^2}\left|F(d\lambda,d\mu)\right|<\infty$$

We assume, that the functions fi(t,u), i=1, 2 can be determined in the plane of complex variable according to t to integer functions of exponential type with finite indexes and the following restrictions are:

$$L_{fi}=\sup_{u\in U}\sup_{-\infty<t<\infty}\left|f_i(t,u)\right|<+\infty,\quad i=1,2. \qquad (3)$$

Let $q_i(u)$, i=1,2 are indexes of functions $f_i(t,u)$, i= 1,2 respectively and the conditions:

$$\vartheta_i=\sup_{u_i\in U} q_i(u_i)<\infty,\qquad i=1,2 \qquad (4)$$

Then the probability one the next decomposition of random field in series as follows:

$$\xi(t,s)=\sum_{k,m=-\infty}^{\infty}\xi\left(\frac{km}{\alpha},\frac{m\pi}{\beta}\right)\frac{\sin\alpha\left(t-\frac{k\pi}{\alpha}\right)}{\alpha\left(t-\frac{k\pi}{\alpha}\right)}\frac{\sin\beta\left(s-\frac{m\pi}{\beta}\right)}{\beta\left(s-\frac{m\pi}{\beta}\right)}, \qquad (5)$$

where α and β (α=π / Δt, β=π / Δs, Δt, Δs are sampling intervals in variables t and s respectively; α,> ϑ1 β > ϑ2, ϑi , i=1,2 are numbers that satisfy specified conditions (4).

The estimate is given as:

$$M\left|\xi(t,s)-\xi_N(t,s)\right|^2\le 16L_{f_1}^2L_{f_2}^2\left[L_{02}(s)\Psi_2(\beta,\vartheta_2,N)\left(4L_{01}(t)\Psi_1(\alpha,\vartheta_1,N)+1\right)+L_{01}(t)\Psi_1(\alpha,\vartheta_1,N)\right]^2\iint_{U^2U^2}\left|F(d\lambda,d\mu)\right|, \qquad (6)$$

where

$$\Psi_1(\alpha,\vartheta_1,N)=\frac{\alpha}{(\alpha-\vartheta_1)N}+2\pi e^{-(\alpha-\vartheta_1)\left(N+\frac{1}{2}\right)\frac{\pi}{\alpha}}, \qquad (7)$$

$$\Psi_2(\beta,\vartheta_2,N)=\frac{\beta}{(\beta-\vartheta_2)N}+2\pi e^{-(\beta-\vartheta_2)\left(N+\frac{1}{2}\right)\frac{\pi}{\beta}}, \qquad (8)$$

$L_{01}(t)=\left(\frac{2}{\pi}\right)^2\left|\sin\alpha t\right|$, $L_{02}(s)=\left(\frac{2}{\pi}\right)^2\left|\sin\beta s\right|$ are functions that finite in any bounded domain of variables t and s respectively, L_{fi} is defined as (3), f_i (I =1,2) are functions in decomposition (1) of random field $\xi(t,s)$ and $\xi_N(t,s)$ is the partial sum of series (5) that is

$$\xi_N(t,s)=\sum_{k,m=-N}^{N}\xi\left(\frac{k\pi}{\alpha},\frac{m\pi}{\beta}\right)\frac{\sin\alpha\left(t-\frac{k\pi}{\alpha}\right)}{\alpha\left(t-\frac{k\pi}{\alpha}\right)}\frac{\sin\beta\left(s-\frac{m\pi}{\beta}\right)}{\beta\left(s-\frac{m\pi}{\beta}\right)}. \qquad (9)$$

Models and procedures of statistical simulation of the two-dimensional random fields on a regular grid of interpolation

It is possible to build the statistical model of random field ξ(t,s) by means of interpolation decomposition (5) of this field in the plane with the uniform grid of interpolation. It has the form as (9) where N is a positive integer number.

The following procedure of the statistical simulation realizations of the random fields in a plane, which are set on a uniform grid of interpolations, is presented below. It is based on the model (9) using the estimate (6) of the mean square approximation of such fields by the partial sum of series (5).

Procedure 1.

1. We choose positive integer numbers N for the model (9) according to the prescribed accuracy ε > 0 by using the following inequality

$$16L_{f_1}^2L_{f_2}^2\left[L_{02}(s)\Psi_2(\beta,\vartheta_2,N)\left(4L_{01}(t)\Psi_1(\alpha,\vartheta_1,N)+1\right)+L_{01}(t)\Psi_1(\alpha,\vartheta_1,N)\right]^2 B(0)<\varepsilon, \qquad (10)$$

where $L_{01}(t)$, $L_{02}(s)$ are finite functions in any bounded domain of changing variables t and s respectively, L_{f_1} are functions as (3) and f_i (i=1,2) are functions in decomposition (1), a $B(0)=D\xi(t,s)$.

2. We generate values of the Gaussian random variables:

$$\{\xi_{k,m}\},\quad k,m=\overline{-N,N}, \qquad (11)$$

which have the following statistical characteristic:

$$M\xi_{k,m}=0,\qquad k,m=\overline{-N,N},$$
$$D\xi_{k,m}=B\left(\frac{k\pi}{\alpha},\frac{m\pi}{\beta},\frac{k\pi}{\alpha},\frac{m\pi}{\beta}\right),\quad k,m=\overline{-N,N}, \qquad (12)$$

where B(u,v), u,v ∈ R2 is a correlation matrix of random field $\xi(t,s)$.

3. We calculate the value of the expression (9) at a given point (t,s), by substituting the number N and values of Gaussian random variables (11).

4. We check whether the realization of the random field $\xi(t,s)$ generated in step 3 fits the field data by testing the corresponding statistical characteristics.

Thus the procedure of the statistical simulation realizations of the random fields in the plane, which are set on the uniform grid of interpolation, is defined. This procedure gives an opportunity to generate realizations of the random field on a plane with accuracy that depends on the selected number for simulating the interpolation points of uniform grid in observation area. The procedure is based on a model as the generalized Kotelnikov–Shannon series for the two-dimensional random fields and requires using such statistical information of field data, as the mathematical expectation and variance of each nodal point.

Before applying the proposed procedure, the data is verified on type of statistical distribution by means of constructing the histogram. Thus, the best approximation of the random field in the plane by the developed models will be when this field has Gaussian (normal), lognormal or approximately Gaussian distributions. The application of these procedures is possible for other types of distribution but it will be with lesser accuracy.

The developed procedure can be applied not only for the random fields, which are defined on a regular square grid of joints in a plane of size $N\times N$ but also in any rectangular grid that does not exceed this square in linear sizes. Thus the empty square joints can be considered as zero.

Another model ξN,M(t,s) (0≤ t ≤T, 0≤ s ≤S; (T, S are lengths of observation intervals in time and in the distances respectively) for Gaussian random field ξ(t,s) in two-dimensional observation area with the following properties:

$$\xi_{N,M}(t,s)=\sum_{k=-N}^{N}\sum_{m=-M}^{M}\varsigma_{k,m}\frac{\sin\alpha\left(t-\frac{k\pi}{\alpha}\right)}{\alpha\left(t-\frac{k\pi}{\alpha}\right)}\frac{\sin\beta\left(s-\frac{m\pi}{\beta}\right)}{\beta\left(s-\frac{m\pi}{\beta}\right)}, \quad (13)$$

where α, β are parameters which are determined by the Nyquist frequency in each variable; N,M, are some positive integer numbers, which are associated with the number of elements in the series of the model; and $\{\xi_{k,m}\}$, $k=-\overline{N,N}$; $m=-\overline{M,M}$ are sequences of the Gaussian random variables that have the following statistical characteristics:

$$\varsigma_{k,m}=0,\ \ k,m=-\overline{N,N};\ \ D_{\varsigma_{k,m}}=B\left(\tfrac{k\pi}{\alpha},\tfrac{m\pi}{\beta},\tfrac{k\pi}{\alpha},\tfrac{m\pi}{\beta}\right),\ \ k=-\overline{N,N};\ \ m=-\overline{M,M} \quad (14)$$

where $B(u,v),u,v\in R_2$ is a correlation matrix realization of random field $\xi(t,s)$.

By using results [6] we find the improved estimate of mean square approximation of model $\xi_{N,M}(t,s)$ of such random field $\xi(t,s)$ in the form of inequality as:

$$E\left|\xi(t,s)-\xi_{N,M}(t,s)\right|^2\le\frac{16}{\pi^4(2N-1)(2M-1)}B(0). \quad (15)$$

Then by using the estimate (15) in the following procedure, it is possible to determine the positive integer numbers N and M which are the number of elements in the series of model $\xi_{N,M}(t,s)$ according to a prescribed accuracy $\varepsilon(\varepsilon>0)$.

The procedure for the numerical simulation realizations of Gaussian random fields ξ(t,s) in a two-dimensional area with the rectangular grid of observations with a bounded spectrum in each variable, is built on the basis of model (13) and estimate (15), which is:

The pocedure 2.

We choose positive integer numbers N and M for the model (13) according to a prescribed accuracy ε > 0 by using the following inequality

$$\frac{16}{\pi^4(2N-1)(2M-1)}B(0)<\varepsilon. \quad (16)$$

where $B(0)=D\xi(t,s)$ is a variance of random field $\xi(t,s)$.

We generate values of the Gaussian random variables

$$\{\xi_{k,m}\},\quad k=-\overline{N,N};\quad m=-\overline{M,M}$$

with statistical characteristics (14).

We calculate the value of expression (13) at a given point (t,s) t∈[0, T], s∈[0, S] (T is the length of the time observation interval, S is the length of spatial observation interval), by substituting the numbers N and M and values of Gaussian random variables $\{\xi_{k,m}\}$, $k=-\overline{N,N}$; $m=-\overline{M,M}$, that will be the value of the generated realization of a given random field $\xi(t,s)$ at this point.

We check whether the realization of the random field $\xi(t,s)$ generated in step 3 on the given regular grid of points in a two-dimensional domain [0, T] × [0, S] fits the field data by testing the corresponding statistical characteristics.

Description of the subject of inquiry and its statistical simulation.

We considered seismograms of two observation points in Odessa: BUG3 and PNT1. 9 segments of noise from seismograms for each of these points were selected. Total recording time of information that was selected for analysis realization lasted 1.5 hours for each of the items. Full vector of seismic waves recorded on components: "East-West" – EW, "North-South" – NS, and "vertical" – Z.

By means of seismograph recording the chart of motion the earth's surface in the form of changes the amplitude over time is obtained.

The method of statistical simulation of random fields [2, 5] can also solve an important problem of simulating the imitated realization of output noise seismogram for the imaginary observation point, located between observation points BUG3 and PNT1. Amplitude and phase spectra of such realization of noise can be used to obtain the frequency characteristics of the geological environment at the construction site, which describes its ability to change (increase or decrease) the amplitude of the seismic waves during earthquakes [1, 4]. The numerical simulation of frequency characteristics of soil strata, in some cases, can significantly reduce the cost of works on seismic zoning of the construction sites by reducing the number of points of instrumental observations of earthquakes, explosions and microseisms.

In the works [3, 5] the results of simulation the realization of noise seismogram (the realization of random field $\xi(t,s)$ at the value of spatial coordinate sJ=1/2, t - time) are described for an imaginary observation point, that is located in the middle between the points BUG3 and PNT1 for the components of NS vibrations. For the calculation the model (9) of the random field $\xi(t,s)$ of noise seismogram is used, that is based on a partial sum of the modified Kotelnikov–Shannon series for the random fields with a bounded spectrum on a regular grid of observations [3, p. 281].

Statistical analysis of the generated realization of the random field of noise in seismogram confirms the adequacy of input data.

For graphic interpretation of the simulated random field of noise in seismogram the framed map was built in the program Surfer on the three obtained fragments and the above-mentioned realization of the random field. Each fragment contains 100 first samples of these realizations. Visualization of output data shows correspondence between these realizations.

Spectral analysis of generated noise in the flat observation area

Frequency characteristic estimates for the geological environment in the observation profile points with multidimensional observation area (under construction sites) can be obtained by calculating and constructing the amplitude and phase spectra of noise in seismogram observation points in that observation points, considering fixed space argument s except time [5]. Calculations of the amplitude and phase spectra can be made by direct method [1, p. 179], i.e. periodogram method.

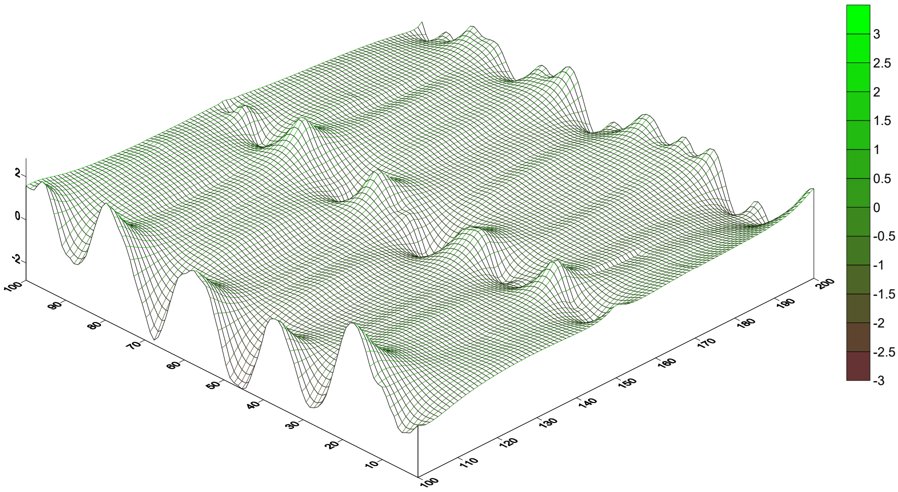

Fig. 1. The surface of realization of the random field of noise, which is built in points
$\xi(t_k, s_j)$, $s_j = 0{,}1/2{,}1$; $t_k = k \times 0{,}01$; $k = \overline{1,N}$; $N = 100$, **in a seismogram in the direction Z**

Fig. 2a and 2b show graphs of the Z component amplitude spectrum |S(ω)| for noise seismograms at observation points BUG3 and PNT1; Fig. 2c – the amplitude spectrum of simulated noise realization for the new observation point located equidistantly between points BUG3 and PNT1.

Graphs of the NS component amplitude spectrum |S(ω)| are shown on the Fig. 3, a, b, c.

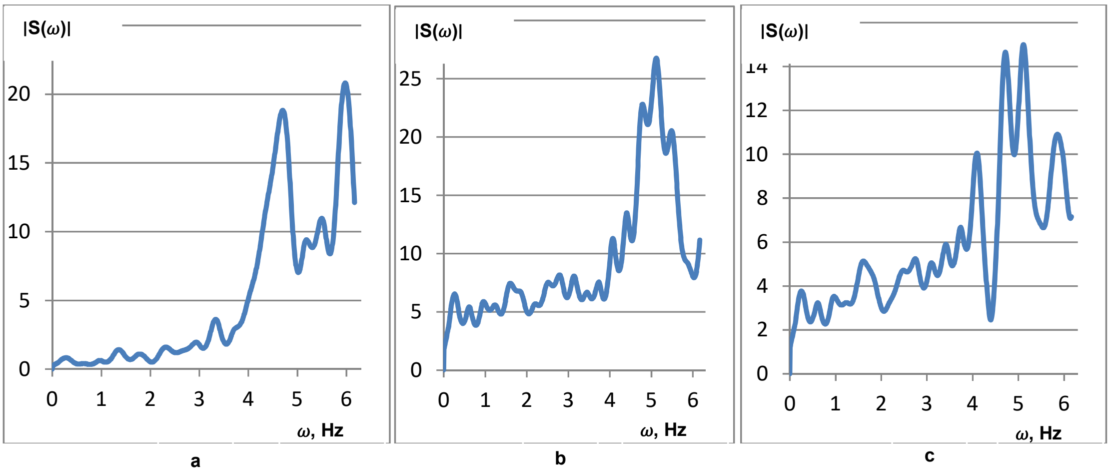

Fig. 2. Graphs of the Z component amplitude spectrum |S(ω)| of averaged input seismic noise data at observation points:
a – BUG3 and b – PNT1; c – the amplitude spectrum |S(ω)| of simulated noise realization
for the new equidistant observation point located between points BUG3 and PNT1

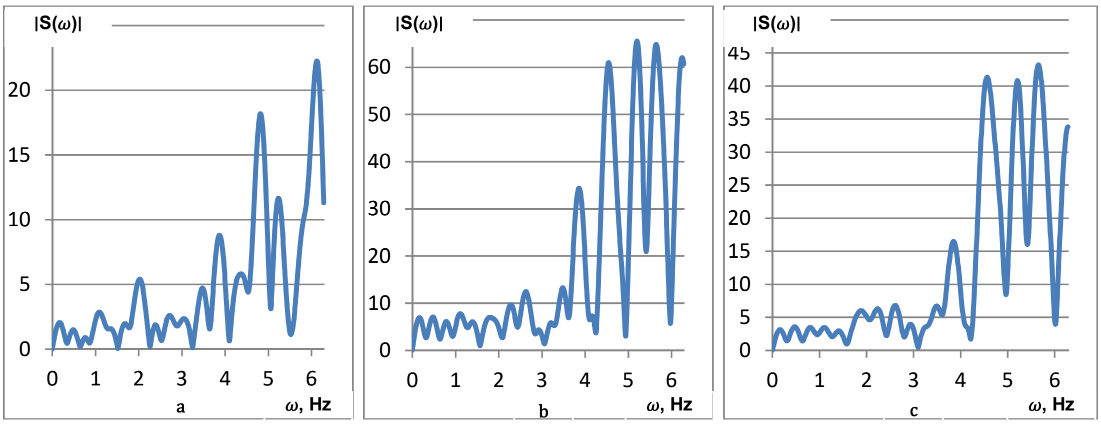

Fig. 3. Graphs of the amplitude spectrum |S(ω)| of averaged input seismic noise data for NS component at observation points:
a – BUG3 and b – PNT1; c – the amplitude spectrum |S(ω)| of simulated noise realization
for the new equidistant observation point located between points BUG3 and PNT1

Those spectral methods that use frequency as an independent parameter provide information about the structure and filtration properties of the upper crust layers, because any medium is a filter that due to resonance and reverberation effects increases the oscillation amplitude for some frequencies and reduces for the others [1, p. 270]. The ability to simulate the effects depends on amplitude and phase frequency characteristics of the geological environment for observation points under building sites and operating platforms, allows studying the geological section features and predicting places where significant increase in the seismic oscillation intensity is possible due to resonance effects and oscillation field interference nodes.

Among the many ways to eliminate the influence of various factors that affect the spectrum shape of seismic waves during earthquakes, explosions and microseism except that due to the influence of the upper crust section part, the way should be noted based on the use of the vertical |SZ(ω)| component spectra relations to the horizontal |SN (ω)| component. Spectra must be calculated for the same wave. This ratio is called the crust spectral ratio T(ω).

$$|S_Z(\omega)|/|S_N(\omega)| = T(\omega).$$

The ratio T(ω) is independent of the spectrum of incident seismic waves, but is determined entirely by the geological environment structure under the observation point.

Fig. 4a and 4b show graphs of the earth crust transmission ratio T(ω) for observation points BUG3 and PNT1 respectively. They were plotted as the Z to NS oscillation components ratio of amplitude spectrum |S(ω)| for initial seismic noise realization. Fig. 4c represents earth crust transmission ratio graph T(ω), that was built as the Z to NS oscillation components ratio of simulated noise seismogram smoothed amplitude spectrum for the new observation point located equidistantly between points BUG3 and PNT1.

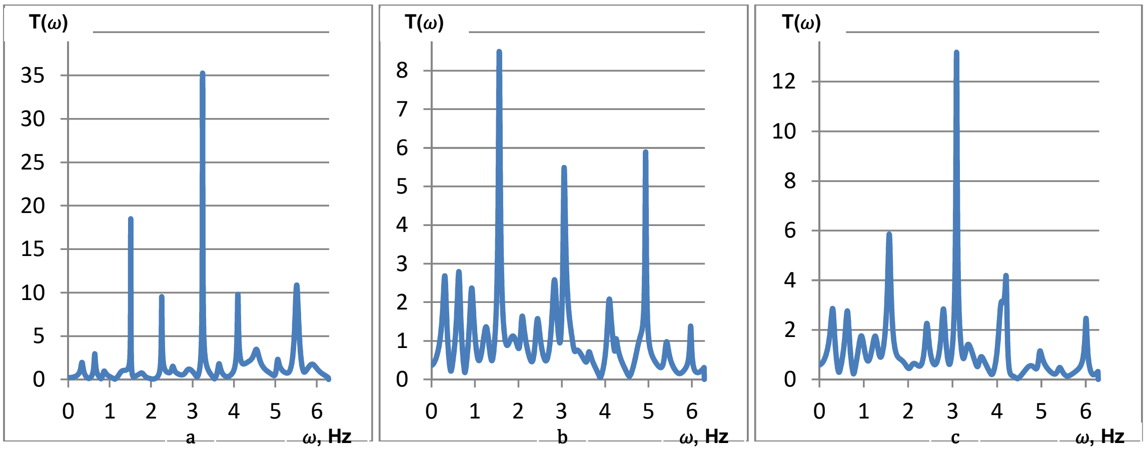

Fig. 4. Graphs of the earth crust transmission ratio T(ω) calculated as the Z to NS oscillation component ratio of amplitude spectrum |S(ω)| for initial seismic noise realization on observation points:
a – BUG3 and b – PNT1. c – earth crust transmission ratio graph T(ω), that was built as the Z to NS oscillation components ratio of simulated noise seismogram smoothed amplitude spectrum for the new observation point located equidistantly between points BUG3 and PNT1

Interpretation of crust transmission ratio for these observations was conducted by comparing them with theoretical ratio calculated for well-known models of the upper section part.

Horizontally layered, vertically inhomogeneous models of the geological environment are usually considered to minimize errors. The frequency characteristic calculations for more complex models are performed by finite-difference and finite-element methods [1].

Fig. 4a, 4б and 4в shows graph T(ω) of smoothed amplitude spectra transmission ratio for all observation points BUG3, PNT1 and imaginary observation points between those points that can be used to determine the increase of seismicity level on different parts of the building site, relative to the real observation point PNT1 of this paper.

A statistical simulation of seismic noise in observations profiles with an irregular step

It is necessary to notice that the model (9) and procedure 2 have one significant restriction, that lies in the fact that samples of data realizations of this random field $\xi(t,s)$ can be given on a uniform grid in both variables t and s. If for the time variable t this condition is mainly performed, and for the spatial variables, can satisfy this restriction in very rare cases. Therefore it is proposed in the space of variables (t,s) $t\in[0, T]$, $s\in[0, S]$ to simulate realizations of the random fields ξ(t,s) otherwise, by using the second approach for constructing the models and procedures.

The model [3], that based on Kotelnikov–Shannon decomposition [7] for homogeneous in time t and homogeneous isotropic with respect to x random fields $\xi(t,x)$ (0≤ t ≤T, 0≤ x ≤X) in unit cylinder in R×S2 with a bounded spectrum which concentrated on $[-\tilde{\alpha},\tilde{\alpha}]$, is constructed and summarized in the cylinder R×S(X)$_2$ with arbitrary radius X/2π. This model $\tilde{\xi}_{N,M(t,x)}$. has the representation of a series:

$$\tilde{\xi}_{N,M(t,x)} = \sum_{k=-N}^{N} \frac{\sin\alpha\left(t-\frac{n\pi}{\alpha}\right)}{\alpha\left(t-\frac{n\pi}{\alpha}\right)} \times \times\left\{\frac{1}{2}\varsigma_0\left(\frac{k\pi}{\alpha}\right)+\sum_{m=1}^{M}\left[\varsigma_k\left(\frac{k\pi}{\alpha}\right)\cos\frac{m\pi x}{X}+\eta_k\left(\frac{k\pi}{\alpha}\right)\sin\frac{m\pi x}{X}\right]\right\}, \qquad (17)$$

where α ($\alpha > \tilde{\alpha}$) is a parameter which is defined by the Nyquist frequency, $\left\{\varsigma_k\left(\frac{k\pi}{\alpha}\right)\right\},\left\{\eta_k\left(\frac{k\pi}{\alpha}\right)\right\}$, $k=\overline{-N,N}, m=\overline{0,M}$, are values of Gaussian random variables that satisfying the following conditions:

$$M\varsigma_k(t)\varsigma_r(s) = \delta_k^r b_k(t-s),$$

$$M\eta_k(t)\eta_r(s) = \delta_k^r b_k(t-s),$$
$$M\varsigma_k(t)\eta_r(s) = 0. \quad (18)$$

where $b_k(t-s)$ $(k = 0,1,2,...)$ are coefficients of decomposition in Fourier series the correlation function $B(t-s,|x_1-x_2|)$ of the random field $\xi(t,s)$ in the cylinder R×S(X)2, which is isotropic with respect to x and homogeneous in time t, which can be defined as follows:

$$b_k(t-s) = \int_{-\alpha}^{\alpha} e^{i\lambda(t-s)} F_k(d\lambda). \quad (19)$$

where Fk( ) is a sequence of nonrandom spectral measures on (–α, α).

From [2, 3] and [4] implies that the estimate of mean square approximation of random field $\xi(t,s)$ with a bounded spectrum on the cylinder R×S(X)2 which is homogeneous in time and isotropic with respect to x by the model (9) as follows:

$$M\left|\xi(t,x) - \tilde{\xi}_{N,M}(t,x)\right|^2 \le \frac{4B(0)}{\pi^2(2N-1)} K_p \left(\frac{X}{\pi}\right)^p \frac{M+2(p+1)}{M^p(p-1)}, \quad (p \ge 2). \quad (20)$$

where X is the length of space interval, p is the index of functions class Dp, that is to say, the function, that derivatives of order p–1 inclusive and derivative B(p-1)(φ) of order p–1 is absolutely continuous and derivate B(p)(φ) of order p is summarized and bounded;

$K_p = \max_{0 \le \varphi \le 2\pi} \left|B^{(p)}(t-s,|x_1-x_2|)\right|$ is a maximum of p-th derivative of the correlation function $B(t-s,|x_1-x_2|)$ of random field $\xi(t,s)$ in R×S(X)2 and $B(0)$ is a variance of random field $\xi(t,s)$.

It is necessary to notice that the cylindrical shape of variables area of the random fields means that the random field is a homogeneous in time t and periodic with respect to variable x (isotropy), that is to say, its correlation function with respect to the spatial variable x can continue periodically with a period equal to the interval of correlation.

We describe the constructed procedure for simulation realizations of the random field $\xi(t,s)$ homogeneous in time and isotropic with respect to x with a bounded spectrum in the cylinder R×S(X)2 , based on the model (17) and the estimate (20) of mean square approximation such as random fields, which are Gaussian distributed.

Procedure 3.

1) We choose positive integer numbers N and M for the model (17) according to a prescribed accuracy ε > 0 by using the following inequality

$$\frac{4B(0)}{\pi^2(2N-1)} K_p \left(\frac{X}{\pi}\right)^p \frac{M+2(p+1)}{M^p(p-1)} \le \varepsilon, \quad (p \ge 2).,$$

where K_p is a maximum of p-th derivative for the correlation function $B(t-s,|x_1-x_2|)$ of random field $\xi(t,s)$ on the cylinder R×S(X)2.

2) We generate values of the Gaussian random variables $\left\{\varsigma_k\left(\frac{k\pi}{\alpha}\right)\right\}, \left\{\eta_k\left(\frac{k\pi}{\alpha}\right)\right\}, \quad k = \overline{-N,N}, m = \overline{0,M},$ that satisfy the conditions (18).

3) We calculate the expression (17) at a given point (t,x) in R×S(X)2, by substituting the numbers N and M calculated in step 1 and values of Gaussian random variables calculated in step 2.

4) We check whether the realization of the random field $\xi(t,x)$ generated in step 3 fits the field data by testing the corresponding statistical characteristics.

By the constructed procedure it is possible to simulate realizations of the random fields, that are homogeneous with respect to t (time) and homogeneous isotropic (stationary) with respect to spatial coordinate x, and have a bounded spectrum. These fields can be arrays of noises in seismograms that are obtained simultaneously in the points of seismic observation located at some distance x from each other. The procedure 3 has some advantages over the procedure 2 that was proposed in [5], as samples of data realizations with respect to the spatial variable x can be given on the observation profile with an irregular step, but it is necessary, that the random field with respect to the variable x should be periodic.

Conclusion. The method of statistical simulation realizations of the random fields gives an opportunity to generate noises in seismograms by the constructed procedures, that depend on time t, and set on the spatial variable s on a profile with the regular or irregular step of placing observation points for evaluation of frequency characteristics of geological environment under these seismic stations and in closely spaced points on them.

References:

1. Bath, M. (1980). Spectral analysis in geophysics. Moskow: Nedra, 535 p. [in Russian].
2. Vyzhva, Z.O. (2003). The statistical simulation of random fields with regular interpolation greed on the flat. Dopovidi NAN Ukrainy – Reports of the National Academy of Sciences of Ukraine, 5, 7–12. [in Ukrainian].
3. Vyzhva, Z.O. (2011). The statistical simulation of random processes and fields. Kyiv: Obrii, 388 p. [in Ukrainian].
4. Vyzhva, Z. (2012). The statistical simulation of 2-D seismic noise for frequency characteristics of geology environment determination. Visnyk Kyivskoho Universytety. Geologiya - Visnyk of Taras Shevchenko National University of Kyiv: Geology, 59, 65–67. [in Ukrainian].
5. Kendzera, O., Vyzhva, Z., Fedorenko, K., Vyzhva, A. (2012). The frequency characteristics of under-building-site geology environment determination by using the statistical simulation of seismic noise by the example of Odessa city. The statistical simulation of 2-D seismic noise for frequency characteristics of geology environment determination. Visnyk Kyivskoho Universytety. Geologiya - Visnyk of Taras Shevchenko National University of Kyiv: Geology, 58, 57–61. [in Ukrainian].
6. Olenko, A.Ya. (2005). The estimation of error approximation"s on the multidimentional Kotelnikov-Shennon"s theorem. Visnyk Kyivskoho Universytety. Matematyka. Mekhanika – Visnyk of Taras Shevchenko National University of Kyiv: Mathematics. Mechanics, 13, 49–54. [in Ukrainian].
7. Prigarin, S.M. (2005). Numerical Modeling of Random Processes and Fields. G.A. Mikhailov (Ed. in Chief). Novosibirsk: Inst. of Comp. Math. and Math. Geoph. Publ., 259 p. [in Russian].
8. Chiles, J.P., Delfiner, P. (2009). Geostatistics: Modeling Spatial Uncertainty. New York: John Wiley & Sons, Inc., Toronto, 720 p.
9. Gneiting, T. (1997). Symmetric Positive Definite Functions with Applications in Spatial Statistics. Von der Universitat Bayeuth zur Erlangung des Grades eines Doktors der Naturwissenschaften (Dr. rer. nat.) genehmigte Abhandlung, 107 p.
10. Higgins, J.R. (1996). Sampling Theory in Fourier and Signal Analysis. Oxford, New York: Clarendon Press, 225 p.
11. Lantuéjoul, C. (2002). Geostatistical simulations: models and algorithm. Berlin: Springer, 256 p.
12. Mantoglov, A., Wilson, J.L. (1981). Simulation of random fields with the Turning bands method. Report / Ralph M. Parsons Laboratory Hydrology and Water Resources Systems, Department of Civil Engineering, Massachusetts Institute of Technology, 264, 199 p.
13. Yadrenko, M.I. (1983). The Spectral Theory of Random Fields. New York: Optimization Software Inc, 256 p.

Список використаних джерел:

1. Бат М. Спектральный анализ в геофизике / М. Бат ; пер. с англ. – М.: Недра, 1980. – 535 с.
2. Вижва З. О. Статистичне моделювання випадкових полів на площині з рівномірною решіткою інтерполяції / З. О. Вижва // Доповіді НАН України. – 2003. – № 5. – С. 7–12.
3. Вижва З. О. Статистичне моделювання випадкових процесів та полів / З. О. Вижва. – К.: Обрії, 2011. – 388 с.
4. Вижва З. Статистичне моделювання сейсмічного шуму у двовимірній області змінних для визначення частотних характеристик

геологічного середовища / З. Вижва // Вісн. Київ. ун-ту. Геологія. – 2012. – № 59. – С. 65–67.

5. Визначення частотних характеристик геологічного середовища під будівельними майданчиками з використанням статистичного моделювання сейсмічного шуму на прикладі спостережень в м. Одесі / О. Кендзера, З. Вижва, К. Федоренко, А. Вижва // Вісн. Київ. ун-ту. Геологія. – 2012. – № 58. – С. 57–61.

6. Оленко А. Я. Оцінка помилки інтерполяції в багатовимірній теоремі Котельникова-Шеннона / А. Я. Оленко // Вісник Київ. ун-ту. Математика. Механіка. – 2005. – № 13. – С. 49–54.

7. Пригарин С. М. Методы численного моделирования случайных процессов и полей / С. М. Пригарин. – Новосибирск: Изд-во ИВМ и МГ, 2005. – 259 с.

8. Chiles J. P. Geostatistics: Modeling Spatial Uncertainty / J. P. Chiles, P. Delfiner. – New York, Toronto: John Wiley & Sons, Inc., 2009. – 720 p.

9. Gneiting T. Symmetric Positive Definite Functions with Applications in Spatial Statistics : Von der Universitat Bayeuth zur Erlangung des Grades eines Doktors der Naturwissenschaften (Dr. rer. nat.) genehmigte Abhandlung / Gneiting T. –1997. – 107 p.

10. Higgins J. R. Sampling Theory in Fourier and Signal Analysis / J. R. Higgins. – Oxford, New York: Clarendon Press, 1996. – 225 p.

11. Lantuéjoul C. Geostatistical simulations: models and algorithm / C. Lantuéjoul. – Berlin : Springer, 2002. – 256 p.

12. Mantoglov A. Simulation of random fields with turning bands method : Report / A. Mantoglov, L. Wilson John ; Ralph M. Parsons Laboratory Hydrology and Water Resources Systems ; Department of Civil Engineering ; Massachusetts Institute of Technology. – 1981. – N 264. – 199 p.

13. Yadrenko M. I. The Spectral Theory of Random Fields / M. I. Yadrenko. – New York: Optimization Software Inc., 1983. – 256 p.

Надійшла до редколегії 08.08.16

З. Вижва, д-р фіз.-мат. наук, доц.,
E-mail: zoya_vyzhva@ukr.net,
К. Федоренко, асп.,
E-mail: slims_mentol@mail.ru,
А. Вижва, асп., E-mail: motomustanger@ukr.net
Київський національний університет імені Тараса Шевченка,
ННІ "Інститут геології", вул. Васильківська, 90, м. Київ, 03022, Україна

ПРО АЛГОРИТМИ СТАТИСТИЧНОГО МОДЕЛЮВАННЯ СЕЙСМІЧНОГО ШУМУ НА ПРОФІЛІ СПОСТЕРЕЖЕННЯ ДЛЯ ВИЗНАЧЕННЯ ЧАСТОТНИХ ХАРАКТЕРИСТИК ГЕОЛОГІЧНОГО СЕРЕДОВИЩА

Робота присвячена подальшій розробці теорії та методів статистичного моделювання випадкових процесів та полів на основі їх спектральних розкладів та модифікованих інтерполяційних рядів Котельникова-Шеннона, а також застосуванню таких методів у задачах геофізичного моніторингу навколишнього середовища. Розглянуто задачу статистичного моделювання випадкових полів сейсмічного шуму на профілі спостереження при впровадженні у сейсмологічні дослідження для визначення частотних характеристик геологічного середовища. Побудовано моделі та сформульовано алгоритми чисельного моделювання реалізацій таких випадкових полів на основі модифікованих інтерполяційних розкладів Котельникова-Шеннона для генерування адекватних реалізацій шуму сейсмограм. У статті також вивчаються дійснозначні випадкові поля $\xi(t, x)$, $t \in R$, $x \in R$ – однорідні за часом та однорідні ізотропні за просторовою змінною на профілі спостереження. Для випадкових полів з обмеженим спектром встановлено аналог теореми Котельникова-Шеннона. Наведено оцінки середньоквадратичного наближення таких випадкових полів моделлю, побудованою на основі спектрального розкладу та інтерполяційної формули Котельникова-Шеннона. Розроблено алгоритми статистичного моделювання реалізацій гауссівських однорідних за часом та однорідних ізотропних за просторовою змінною на профілі спостереження випадкових полів з обмеженим спектром. Наведено теореми про оцінки середньоквадратичної апроксимації однорідних за часом та однорідних ізотропних за n іншими змінними випадкових полів частковими сумами рядів спеціального вигляду, за допомогою яких сформульовано алгоритми чисельного моделювання реалізацій таких випадкових полів. Розглянуто способи проведення спектрального аналізу згенерованих реалізацій шуму сейсмограм. Розроблено універсальні методи статистичного моделювання (методи Монте-Карло) багатопараметричних сейсмологічних даних, які дають можливість вирішити проблеми генерування реалізацій шуму сейсмограм на профілі спостереження із кроком необхідної детальності та регулярності.

Ключові слова: статистичне моделювання, сейсмічний шум, випадкові процеси

З. Выжва, д-р физ.-мат. наук, доц.,
E-mail: zoya_vyzhva@ukr.net,
К. Федоренко, асп.,
E-mail: slims_mentol@mail.ru,
А. Выжва, асп.,
E-mail: motomustanger@ukr.net
Киевский национальный университет имени Тараса Шевченко,
УНИ "Институт геологии", ул. Васильковская, 90, г. Киев, 03022, Украина

ОБ АЛГОРИТМАХ СТАТИСТИЧЕСКОГО МОДЕЛИРОВАНИЯ СЕЙСМИЧЕСКОГО ШУМА НА ПРОФИЛЕ НАБЛЮДЕНИЯ ДЛЯ ОПРЕДЕЛЕНИЯ ЧАСТОТНЫХ ХАРАКТЕРИСТИК ГЕОЛОГИЧЕСКОЙ СРЕДЫ

Работа посвящена дальнейшей разработке теории и методов статистического моделирования случайных процессов и полей на основе их спектральных разложений и модифицированных интерполяционных рядов Котельникова-Шеннона, а также применению таких методов в задачах геофизического мониторинга окружающей среды. Рассмотрена задача статистического моделирования случайных полей на профиле наблюдения при внедрении в сейсмологические исследования для определения частотных характеристик геологической среды. Построены модели и сформулированы алгоритмы численного моделирования реализаций таких случайных полей на основании модифицированных интерполяционных разложений Котельникова-Шеннона для генерирования адекватных реализаций шума сейсмограмм. В статье изучаются действительнозначные случайные поля $\xi(t, x)$, $t \in R$, $x \in R$ – однородные по времени и однородные изотропные по пространственной переменной на профиле наблюдения. Для случайных полей с ограниченным спектром установлен аналог теоремы Котельникова-Шеннона. Приведены теоремы об оценках среднеквадратического приближения таких случайных полей моделью, которая построена на основе спектрального разложения и интерполяционной формулы Котельникова-Шеннона. Разработаны алгоритмы статистического моделирования реализаций гауссовских однородных по времени и однородных изотропных по пространственной переменной случайных полей с ограниченным спектром. Доказаны теоремы об оценке среднеквадратической аппроксимации однородных по времени и однородных изотропных по n другим переменным случайных полей частичными суммами рядов специального вида, при помощи которых сформулирован алгоритм численного моделирования реализаций таких случайных полей. Рассмотрены способы проведения спектрального анализа сгенерированных реализаций шума сейсмограмм. Разработаны универсальные методы статистического моделирования (методы Монте-Карло) многопараметрических сейсмологичеких данных, которые дают возможность решить проблемы генерирования реализаций шума сейсмограм на плоскости и в трехмерном пространстве на сетке необходимой детальности и регулярности.

Ключевые слова: статистическое моделирование, сейсмический шум, случайные процессы.