ГЕОЛОГІЧНА ІНФОРМАТИКА

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ABOUT METHODS OF RANDOM FIELDS STATISTICAL SIMULATION ON THE SPHERE BY THE AIRCRAFT MAGNETOMETRY DATA

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There have been developed universal methods of statistical simulation (Monte Carlo methods) of geophysical data for generating random fields on the sphere on grids of required detail and regularity. Most of the geophysical research results are submitted in digital form, which accuracy depends on various random effects (including equipment measurement error). The map accuracy problem occurs when the data cannot be obtained with a given detail in some areas. It is proposed to apply statistical simulation methods of random fields realizations, to solve the problems of conditional maps, adding of data to achieve the necessary precision, and other similar problems in geophysics. Theorems on the mean-square approximation of homogeneous and isotropic random fields on the sphere have been proved by special partial sums. A spectral coefficients method was used to formulate algorithms of statistical simulation by means of these theorems. A new effective statistical technique has been devised to simulate random fields on the sphere for geophysical problems. Statistical simulation of random fields on the sphere based on spectral decomposition has been introduced in order to enhance map accuracy by the example of aeromagnetic survey data in the Ovruch depression. It is divided into deterministic and random components for data analysis. The deterministic component is proposed to approximate by cubic splines and the random component is proposed to modeling on the basis of random fields on the sphere by spectral decomposition. Model example – the aircraft magnetometry data. According to the algorithm we received random component implementations on the study area with twice detail for each profile. When checking their adequacy we made the conclusions that the relevant random components histogram has Gaussian distribution. The built variogram of these implementations has the best approximation by theoretical variogram which is connected to the Bessel type correlation function. The final stage was the imposing array of random components on the spline approximation of real data. As a result, we received more detailed implementation for the geomagnetic observation data in the selected area.

Keywords: Statistical simulation, spectral decomposition, spline interpolation, conditional maps.

Introduction. The problems of random fields statistical simulation on the sphere with given probability characteristics arise solving the actual geophysics problems. In this case a special care is necessary for reduction of calculations, amount of which rapidly grows together with the dimension of the argument of the random field. Different approaches related to the solving of problems of random fields statistical simulation where described in a lot of papers.

It is proposed in the papers (*Вижва та ін., 2010; Vyzhva and Vyzhva, 2016*) to apply methods of statistical simulation of realizations of random fields on the plane, to solve the problems of conditional maps, adding of data to achieve the necessary precision, and other such problems in geophysics. The approximations theorems and built on their base algorithms of statistical simulation of Gaussian homogeneous and isotropic random fields on the plane using the spectral representation are considered. Model example is the aircraft magnetometry data. It is divided into deterministic and random components for data analysis. The deterministic component is proposed to approximate by cubic splines and the stationary random component is proposed to modeling on the basis of spectral decomposition of random fields on the plane. But the magnetometry data was investigated on the great square, because we consider it on the part of the sphere. It is proposed the stationary random component to modeling on the basis of spectral decomposition of random fields on the sphere in this paper. Using the above method makes it possible to supplement the missing magnetometry data in the study area with greater accuracy than in the paper (*Вижва та ін., 2010; Vyzhva and Vyzhva, 2016*) with the 2-D method.

In this paper the algorithm of statistical simulation of Gaussian isotropic random fields on the sphere using the basic spectral representation (*Вижва та Ядренко, 2000*) is considered.

Random field statistical simulation based on spectral representation was introduced in order to enhance map accuracy by the example of aeromagnetic survey data in the Ovruch depression.

Methods of statistical simulation of random field on the sphere based on representation it by stochastic sums was considered in papers (*Yadrenko, 1993; Chiles and Delfiner,1999; Prigarin, 2005; Vyzhva, 1997; Vyzhva, 2003; Вижва та Ядренко, 2000*) and other.

The spectral representation of isotropic random fields on the sphere and approximation theorems. We consider a real-valued isotropic random field $\xi(r,\theta,\varphi)$ on the sphere $S_3(\boldsymbol{r})$ on 3-D space (r,θ,ϕ – spherical coordinates). It is known, that square-mean continuous real-valued isotropic random field $\xi(r,\theta,\varphi)$, what is narrowing on the sphere with radius r on 3-D Euclidean space R^3, admit the spectral decomposition

$$\xi(r,\theta,\phi) = c_3 \sum_{m=o}^{\infty} \sum_{l=-m}^{m} \varsigma_m^l(r)\; S_m^l(\theta,\phi),$$

where $c_3 = \sqrt{2}\pi$, $\varsigma_m^l(r) = \int_0^\infty \frac{J_{m+\frac{1}{2}}(\lambda r)}{(\lambda r)^{\frac{1}{2}}} Z_m^l(d\lambda)$, and

$\{Z_m^l(.)\}$ is a sequence of orthogonal random measures on Borel subsets from the interval $[0,+\infty)$, i. e.

$$E\, Z_m^l(S_1)\, Z_{m'}^{l''}(S_2) = \delta_l^{l'}\, \delta_m^{m'}\, \Phi(S_1 \cap S_2),$$

for any Borel subsets S_1 and S_2,

where $\delta_m^{m'}$ is Kronecker symbol, $\Phi(\lambda)$ is the bounded nondecreasing function so-called spectral function and spherical harmonics $S_m^l(x)$ are

$$S_m^l(\theta,\phi) = \tilde{c}_{m,l}\, P_m^{|l|}(\cos\theta)\; e^{i\,l\,\phi},$$

where $P_m^l(x)$ is associated Legendre functions degree m,

$$\tilde{c}_{m,l} = \frac{1}{2}\sqrt{\frac{\nu_l}{\pi}\frac{(m-l)!}{(m+l)!}(2m+1)}, \quad (1)$$

$$\nu_l = \begin{cases} 1, & l \neq 0, \\ 2, & l = 0. \end{cases} \quad (2)$$

The correlation function $B(\rho)$ of the isotropic random field $\xi(r,\theta,\phi)$ on 3-D Euclidean space R^3 may by presented (*Vyzhva and Fedorenko, 2013*) as an integral

$$B(\rho) = \sqrt{\frac{\pi}{2}}\int_0^\infty \frac{J_{\frac{1}{2}}(\lambda\rho)}{\sqrt{\lambda\rho}}\, d\Phi(\lambda), \quad (3)$$

where $\Phi(\lambda)$ is spectral function, ρ is distance between the points $x, y \in R^3$ ($x = (r_1, \theta_1, \phi_1)$, $y = (r_2, \theta_2, \phi_2)$): $\rho = \sqrt{r_1^2 + r_2^2 - 2 r_1 r_2 \cos\psi}$, and $\cos\psi$ – angular distance between vectors $x, y \in R^3$: $\cos\psi = \cos\theta_1 \cos\theta_2 + \sin\theta_1 \sin\theta_2 \cos(\phi_1 - \phi_2)$.

The variances of $\varsigma_m^l(r)$ we obtain as

$b_m(r) = Var\, \varsigma_m^l(r) = E\left|\varsigma_m^l(r)\right|^2$, l = 1, 2,..., h (m, 3).

Than we have the formulas for coefficients $b_m(r)$

$$b_m(r) = \int_0^\infty \frac{J^2_{m+\frac{1}{2}}(\lambda r)}{\lambda r}\Phi(d\lambda), m = 0,1,\dots. \quad (4)$$

We will call the coefficients $b_m(r)$ as spectral coefficients.

These coefficients are defined by the correlation function $B(\rho)$ in the way:

$$b_m(r) = 2\pi\int_0^\pi B(\rho)\, P_m(\cos\psi)\, \sin\psi\, d\psi. \quad (5)$$

The variance of random field $\xi(r,\theta,\phi)$ we obtain by this as

$$E\xi^2(r,\theta,\phi) = Var\xi(r,\theta,\phi) = \pi/2\sum_{m=0}^\infty (2m+1) b_m(r). \quad (6)$$

However, there is used the spectral decomposition of this random field by solution problems of statistical simulation of realizations of random fields on the sphere in 3-D space, on this figurate real-valued random variables. Let us adduce that decomposition. The following statement is true.

Theorem 1. Let $\xi(r,\theta,\phi)$ is a mean square continuous realvalued isotropic random field on the sphere $\boldsymbol{S_3}(r)$ in 3-D space (r – radius sphere) with zero mean. Then this random field admits (*Вижва, 2011*) the following spectral decomposition:

$$\xi(r,\theta,\phi) = = \sum_{m=0}^\infty\sum_{l=0}^m \tilde{c}_{m,l}\, P_m^l(\cos\theta)\,[\varsigma_{m,1}^l(r)\cos l\phi + \varsigma_{m,2}^l(r)\sin l\phi], \quad (7)$$

where $\{\varsigma_{m,k}^l(r)\}$ (k = 1, 2),

$\varsigma_{m,k}^l(r) = \int_0^\infty \frac{J_{m+\frac{1}{2}}(\lambda r)}{(\lambda r)^{\frac{1}{2}}} Z_{m,k}^l(d\lambda)$, – random values sequences, satisfying the next conditions:

1) $M\,\varsigma_{m,k}^l(r) = 0$; (8)

2) $M\,\varsigma_{m,k}^l(r)\,\varsigma_{m',k'}^{l'}(r) = \delta_l^{l'}\,\delta_m^{m'}\,\delta_k^{k'}\, b_m(r)$, (9)

where $\delta_p^{p'}$ – Kronecker symbol, $\tilde{c}_{m,l}$ – constants sequences are calculated by the formula (1), a $b_m(r)$ – the spectral coefficients (4).

Remark.

If we consider this theorem for the Gaussian isotropic random fields $\xi(r,\theta,\phi)$ on the sphere, then random values sequences $\{\varsigma_{m,k}^l(r)\}$ in decomposition (7) are interdependent independent Gaussian random values.

A procedure of the statistical simulation of random fields on the sphere. The statistical simulation of realizations of random fields on the sphere $\boldsymbol{S_3}(r)$ on the basis of spectral decomposition (7) is considered.

Approximation model is constructed by using the partial sums of series (7)

$$\xi_N(r,\theta,\phi) = \sum_{m=0}^N\sum_{l=0}^m c_{ml}\, P_m^l(\cos\theta)\left[\zeta_{m,1}^l(r)\cos l\phi + \zeta_{m,2}^l(r)\sin l\phi\right], N \in \mathrm{N} \quad (10)$$

The mean square approximation of random field $\xi(\rho,\theta,\varphi)$ by model (10) is

$$M\left|\xi(r,\theta,\phi) - \xi_N(r,\theta,\phi)\right|^2 \le \pi\int_0^\infty\left\{\sum_{m=N+1}^\infty\left(m+\frac{1}{2}\right)\frac{J^2_{m+1/2}(\lambda r)}{\lambda r}\right\}d\Phi(\lambda) = \pi/2\sum_{m=N+1}^\infty (2m+1)\int_0^\infty\frac{J^2_{m+1/2}(\lambda r)}{\lambda r}\,d\Phi(\lambda).$$

$$E\left|\xi(r,\theta,\varphi) - \xi_N(r,\theta,\varphi)\right|^2 \le 2\pi\sum_{m=N+1}^\infty\left(m+\frac{1}{2}\right)b_m(r) \quad (11)$$

We need this mean square approximation in the convenient form for the constructing statistical simulation of realizations of isotropic random fields on the sphere algorithm. These estimates were received in the following theorems.

We denote the following:

$$\mu_k = \int_0^{+\infty}\lambda^k\Phi(d\lambda)., \text{k=0, 1, 2, } \dots . \quad (12)$$

Theorem 2. Let a mean square continuous realvalued isotropic random field $\xi(r,\theta,\phi)$ on the sphere $\boldsymbol{S_3}(r)$ in 3-D space with zero mean(r – radius sphere). If $\mu_3 < +\infty$, then the mean square approximation of this random field by model (10) is such that

$$M\left[\xi(r,\theta,\phi) - \xi_N(r,\theta,\phi)\right]^2 \le \frac{5\pi r^3}{2N^2}\mu_3, \quad (13)$$

where

$$\mu_3 = \int_0^\infty \lambda^3\Phi(d\lambda). \quad (14)$$

Proof: We estimate the inequality (13) by using (*Вижва, 2011*) and (*Yadrenko and Gamaliy, 1998*) stated below.

Lemma 1. (*Vyzhva and Fedorenko, 2016*) We have the next inegualites for the Bessel functions of the first kind :

$$\sum_{m=M+1}^{\infty} J^2{}_{\theta+m}(z) \le \frac{z^2}{M},\ \theta \in [0,1),$$

and

$$\sum_{m=M+1}^{\infty}\left(m+\frac{1}{2}\right)J^2{}_{\frac{1}{2}+m}(z) \le \frac{5z^4}{4M^2}.$$

We obtain, applying the previous lemma, the next evaluation for mean square approximation (11):

$$2\pi\sum_{m=N+1}^{\infty}\left(m+\frac{1}{2}\right)b_m(r) =$$

$$= 2\pi\int_0^{+\infty}\sum_{m=N+1}^{\infty}\left(m+\frac{1}{2}\right)\frac{J^2{}_{\frac{1}{2}+m}(\lambda r)}{\lambda r}\Phi(d\lambda) \le$$

$$2\pi\int_0^{+\infty}\frac{5(\lambda r)^4}{4N^2\lambda r}\Phi(d\lambda) \le \frac{5\pi r^3}{2N^2}\mu_3, \qquad (15)$$

where μ_3 is (14).

We can conclude, that the statement of Theorem 2 holds true on the base Lemma 1.

Further we consider another estimate of the mean square approximation of random fields on the sphere $S_3(r)$, where the spectral function $\Phi(\lambda)$ satisfies the following condition

$$\mu_{2N+2} = \int_0^{+\infty}\lambda^{2N+2}\Phi(d\lambda) < +\infty.$$

Theorem 3. Let a mean square continuous realvalued isotropic random field $\xi(r,\theta,\phi)$ on the sphere $S_3(r)$ in 3-D space with zero mean (*r* – radius sphere). If $\mu_{2N+2} < +\infty$, then the mean square approximation of this random field by model $\xi_N(r,\theta,\phi)$ (10) is such ineguality

$$M\left|\xi(r,\theta,\phi)-\xi_N(r,\theta,\phi)\right|^2 \le \frac{2^{N+2}r^{2N+2}(N+1)!}{(2N+3)!}\mu_{2N+2}. \quad (16)$$

where

$$\mu_{2N+2} = \int_0^{+\infty}\lambda^{2N+2}\Phi(d\lambda). \qquad (17)$$

Proof: We estimate the inequality (15) by using (*Вижва та Федоренко, 2013*) and the lemma 2.

Let us give some properties of integral expressing (4) for spectral coefficients $b_m(r), m = 0,1,\ldots$ by means of the results in the book Watson (*Watson, 1949*) and we proved next lemma.

Lemma 2. If $b_m(r), m = 0,1,\ldots$ is evaluated by formula (4), then the following properties of it hold true

$$\sum_{m=0}^{\infty}(2m+1)b_m(r) = \frac{2}{\pi}\mu_0 \qquad (18)$$

$$\sum_{m=s}^{\infty}(2m+1)b_m(r) \le \frac{2^{s+1}r^{2s}s!}{(2s+1)!\pi}\mu_{2s}. \qquad (19)$$

Proof. We derive, applying results Watson (Watson, 1949), p. 42 formula 2.6 (1), such us

$$J^2_{\frac{1}{2}+m}(\lambda\rho) = \frac{2}{\pi}\int_0^{\frac{\pi}{2}}J_{2m+1}(2\lambda\rho\sin\theta)\,d\theta$$

and p. 44, formula 2.7 (1) by n=1, such us

$$(\frac{1}{2}z)^n = \sum_{m=0}^{\infty}(2m+n)\frac{(m+n-1)!}{m!}J_{2m+n}(z)$$

the equality (17) as

$$\sum_{m=0}^{\infty}(2m+1)b_m(\rho) =$$

$$= \int_0^{+\infty}(\lambda\rho)^{-1}\sum_{m=0}^{\infty}(2m+1)J^2_{\frac{1}{2}+m}(\lambda\rho)\Phi(d\lambda) =$$

$$= \frac{2}{\pi}\int_0^{+\infty}(\lambda\rho)^{-1}\int_0^{\frac{\pi}{2}}\sum_{m=0}^{\infty}(2m+1)J_{2m+1}(2\lambda\rho\sin\theta)\,d\theta\Phi(d\lambda) =$$

$$= \frac{2}{\pi}\int_0^{+\infty}(\lambda\rho)^{-1}\int_0^{\frac{\pi}{2}}(\lambda\rho\sin\theta)\,d\theta\Phi(d\lambda) = \frac{2}{\pi}\mu_0.$$

Then we obtain the inequality(18), by using formula 2.6 (1) Watson (*Watson, 1949*), p. 42, as follows:

$$\sum_{m=s}^{\infty}(2m+1)b_m(0,r) =$$

$$= \int_0^{+\infty}(\lambda\rho)^{-1}\sum_{m=s}^{\infty}(2m+1)J^2_{\frac{1}{2}+m}(\lambda\rho)\Phi(d\lambda) =$$

$$= \frac{2}{\pi}\int_0^{+\infty}(\lambda\rho)^{-1}\int_0^{\frac{\pi}{2}}\sum_{m=s}^{\infty}(2m+1)J_{2m+1}(2\lambda\rho\sin\theta)\,d\theta\Phi(d\lambda). \quad (20)$$

Then we use $(\tilde{m}+2s)! \ge \tilde{m}!(2s)! = 2^s s!\tilde{m}!,\ \forall m,s \ge 0,$ 2.7 (1) Watson (*Watson, 1949*), p. 44, and the following inequality holds

$$\sum_{m=s}^{\infty}(2m+1)J_{2m+1}(2\lambda\rho\sin\theta) =$$

$$= \sum_{\tilde{m}=0}^{\infty}(2\tilde{m}+2s+1)J_{2\tilde{m}+2s+1}(2\lambda\rho\sin\theta) \le$$

$$\frac{1}{2^s s!}\sum_{\tilde{m}=0}^{\infty}(2\tilde{m}+2s+1)\frac{(\tilde{m}+2s+1-1)!}{\tilde{m}!}J_{2\tilde{m}+2s+1}(2\lambda\rho\sin\theta) \le$$

$$\le \frac{(\lambda\rho\sin\theta)^{2s+1}}{2^s s!}.$$

We use the latter result and formulas 3.621 (1) C,P° 8.384 (1) Gradshteyn and Ryzhik (*Градштейн и Рыжик, 1971*) and obtain inequality (19) as

$$\sum_{m=s}^{\infty}(2m+1)b_m(r) \le \frac{2}{\pi}\int_0^{+\infty}(\lambda\rho)^{-1}\int_0^{\frac{\pi}{2}}\frac{(\lambda\rho\sin\theta)^{2s+1}}{2^s s!}d\theta\Phi(d\lambda) =$$

$$= \frac{2\rho^{2s}}{2^s s!\pi}\int_0^{+\infty}\lambda^{2s}\int_0^{\frac{\pi}{2}}(\sin\theta)^{2s+2-1}d\theta\Phi(d\lambda) =$$

$$= \frac{2\rho^{2s}}{2^s s!\pi}2^{2s+2-2}B(s+1,s+1)\mu_{2s} =$$

$$= \frac{2^{s+1}\rho^{2s}}{\pi s!}\frac{\Gamma^2(s+1)}{\Gamma(2s+2)}\mu_{2s} = \frac{2^{s+1}\rho^{2s}s!}{(2s+1)!\pi}\mu_{2s},$$

where $B(s+1,s+1)$ – is Beta function.

Thus, the statement of Lemma 2 holds true.

Finally, we use inequalities (15), (19) and equality (18) and obtaine the mean square estimates for the approximation of a random field $\xi(r,\theta,\phi)$ by model (10).

The finding estimate we find from

$$E\left|\xi(r,\theta,\varphi)-\xi_N(r,\theta,\varphi)\right|^2 \le \pi \sum_{m=N+1}^{\infty}(2m+1)b_m(r) = $$

$$= \pi \sum_{m=N+1}^{\infty}(2m+1)\int_0^{\infty} \frac{J^2_{m+\frac{1}{2}}(\lambda r)}{\lambda r}\Phi(d\lambda).$$

We use Lemma 2 by s=N and the finding estimate of **Theorem 3** is written as (16). We can conclude, that the statement of Theorem 3 holds true on the base of Lemma 2.

Using the approximation theorems 2 and 3 the algorithm of the statistical simulation of realizations of isotropic random fields on the sphere may be formulated. We formulate based on the idea spectral decomposition of realvalued isotropic random field on the sphere procedure of such kind. The first one, of algorithm by Z.O. Vyzhva in (*Вижва та Ядренко, 2000*) is called "spectral coefficients" algorithm.

Below we describe the procedure for the statistical simulation of realizations of Gaussian isotropic random fields $\xi(r,\theta,\phi)$ on the sphere $\boldsymbol{S_3}(r)$ (*r* – fixed radius sphere), which was constructed on the basis of model (10) and estimates (13) and (16).

Algorithm.

1. Natural number N (border of summation) is chosen according to necessary $\varepsilon > 0$ accuracy of approximation the model (10) by means of one of the next inequalities (13) or (16) mentioned below:

$$\frac{5\pi r^3}{2N^2}\mu_3 \le \varepsilon, \qquad (21)$$

or

$$\frac{5^{N+2} r^{2N+2}(N+1)!}{(2N+3)!}\mu_{2N+2} \le \varepsilon. \qquad (22)$$

1. Calculate the spectral coefficients $b_m(r)$, $m = 0,1,...,N$ by formula (5).

2. Simulate the sequences of independent Gaussian random variables:

$$b_m(r),\ \left\{\varsigma^l_{m,k}(r)\right\},\ k=1,2;\ m=0,1,2...N;\ l=1,...,m;$$

that satisfy conditions (8) and (9).

3. Calculate the realization of the stochastic random field $\xi(r,\theta,\phi)$ by formula (10) in given point by means of substituting in it values from the previous items 1, 2 and 3, numbers N and sequences of Gaussian random variables.

4. Check whether the realization of the random field $\xi(r,\theta,\phi)$ generated in step 3 fits the data by testing the corresponding statistical characteristics (distribution, correlation function $B(\rho)$).

The statistical simulation of realizations of the Gaussian isotropic random fields on the sphere can be done by means of this algorithm. If the random field have another type of distribution, than we simulate the sequences of independent random variables in step 2 with corresponding distribution.

Statistical simulation methods of random fields by the aircraft magnetometry data. Most of the geophysical, meteorology, oceanography and other research results are submitted in digital form, which accuracy depends on various random effects (including equipment measurement error). The map accuracy problem occurs when the data cannot be obtained with a given detail in some areas. In such cases the methods of statistical modeling realizations of random fields are recommended (*Вижва та ін., 2010; Vyzhva et al., 2012*) to supplement data missing.

There has been introduced random field statistical simulation based on spectral representation on the sphere in order to enhance map accuracy by the example of aeromagnetic survey data in the Ovruch depression. The object of research is data aero magnetic survey of 1: 10 000 scale in the area 2500 × 2500 m^2 that was conducted during 1996–2002 years (Fig. 1). The full magnetic field intensity vector **T** was investigated. The work was carried out on 25 profiles with a distance of 100 meters between them (X from 0 to 2500 m and Y from 0 to 2500 m – 625 points).

Because the magnetometry data was investigated on the great square, we consider it on the part of the sphere. We translate the Cartesian coordinates (*x, y, z*) of the three-dimensional space, which are tied to the points of measurement, into spherical coordinates (r,θ,φ) (*r* – fixed radius sphere).

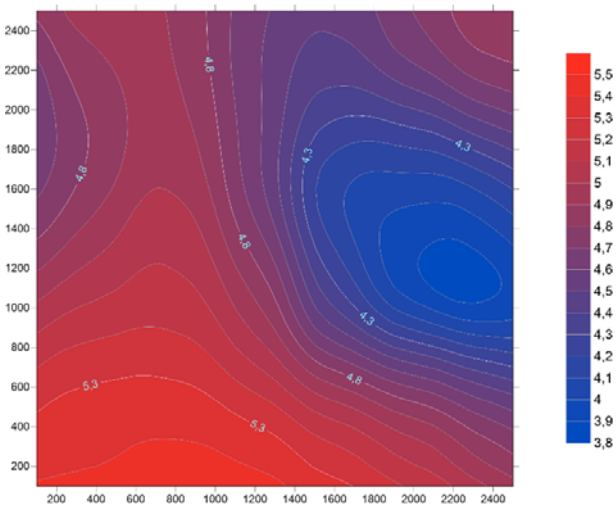

Fig. 1. The map of aeromagnetic survey data ΔT_{an} in the Ovruch depression

While constructing data graphs for each account, we noticed that it is expedient to distinguish deterministic and random components. Deterministic function can be selected in different ways. One determination method its analytical form (trend $f_i(r,\theta,\phi)$ (*i=1, 2, …, 25* – profile numbers) as a function of exponentially damped sinusoid or cosinusoid) was considered in (*Вижва та Федоренко, 2013*). But there is a more accurate way to select deterministic component – approximation by cubic spline data. The difference between spline approximation of data with gaps (e.g. due to one) for each profile and spline curve for all points is a random process that is frequently stationary for most profiles.

We introduce the notation for input data on the profile as a random field $\eta_i(r,\theta,\phi)$ (*i* – profile numbers) on the sphere. The stationary random component $\xi_i(r,\theta,\phi)$ (random fields) and trend $f_i(r,\theta,\phi)$ as determined cubic spline function were selected for each profile (*i=1, 2, …, 25*). Input data on the profiles is a random field

$$\eta_i(r,\ \theta,\phi) = f_i(r,\theta,\phi) + \xi_i(r,\theta,\phi), i = 7,...,20. \qquad (23)$$

Solid line on Figure 2 shows a deposited spline approximation $S_i^{(1)}(r,\theta,\phi)$, built by means of the MathCad software for PR1 (profile №1) data that are taken without spaces. Parameters defined by the data were determined for such spline. They ask each profile trend $f_i(r,\theta,\phi)$. Dashed line shows the spline approximation graph $S_i^{(2)}(r,\theta,\phi)$ of the first profile data with gaps due to one point of observation (i.e. for 50 points out of 100). Noise was obtained by calculating the following difference:

$$\xi_i(r,\theta,\phi) = S_i^{(1)}(r,\theta,\phi) - S_i^{(2)}(r,\theta,\phi),\ i = 7,8,...,20. \qquad (24)$$

From observations (values) of random component $\xi_i(r,\theta,\phi), i=7,...,20$ in all 13 profiles we created array that frequently represents isotropic random field $\xi(r,\theta, f)$ on the sphere $S_3(r)$ with zero mathematical expectation and approximately Gaussian distribution (Fig. 3).

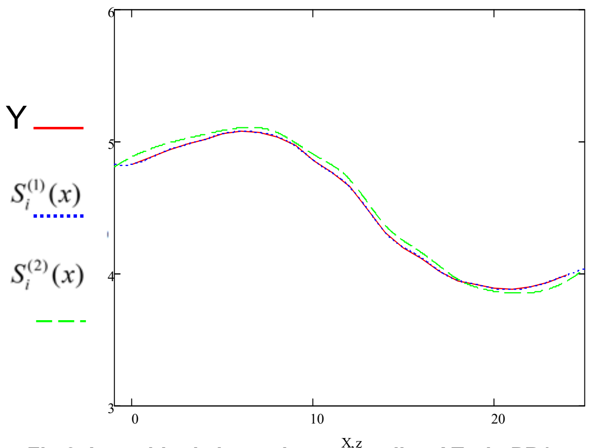

Fig.2. Logarithmic input data and spline ΔT_{an} in PR1

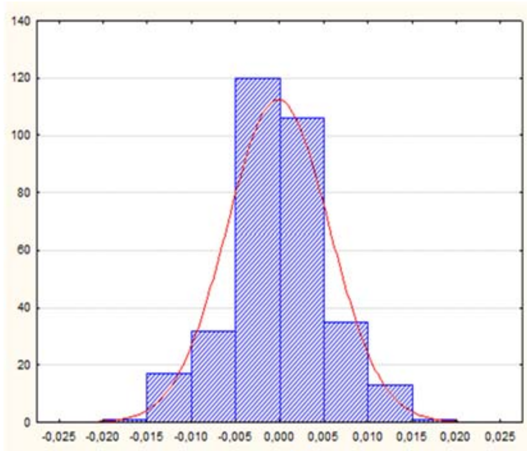

Fig.3. Observed values of random component in all 13 profiles (for PR7-PR20)

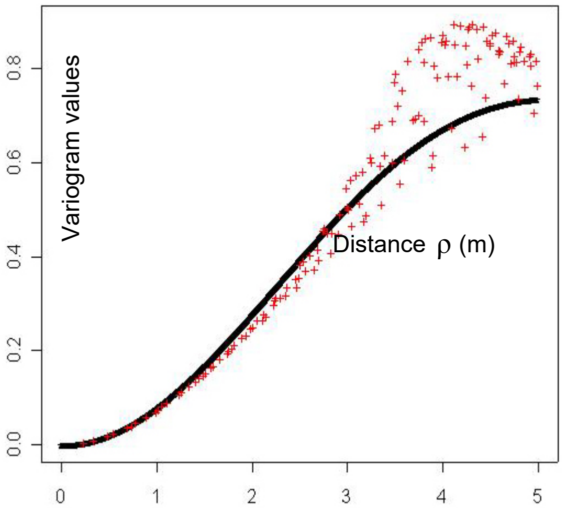

Fig. 4. Variogram of input data arrays ΔT_{an} for PR7-PR20, corresponding to Bessel type correlation function $B(\rho)=3\sqrt{\frac{\pi}{2}}\,\frac{J_{\frac{3}{2}}(a\rho)}{(a\rho)^{\frac{3}{2}}}$, **(a ≈4,2*10^{-3}).**

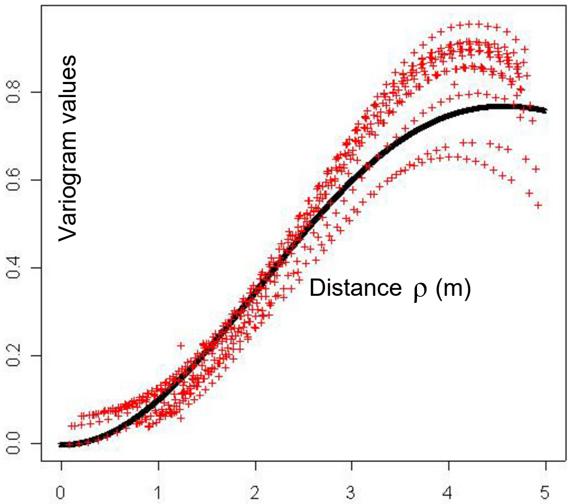

Fig. 5. Variogram of simulated data arrays ΔT_{an} for PR7-PR20, corresponding to Bessel type correlation function $B(\rho)=3\sqrt{\frac{\pi}{2}}\,\frac{J_{\frac{3}{2}}(a\rho)}{(a\rho)^{\frac{3}{2}}}$, **(a ≈4,2*10^{-3}).**

By fields of such properties we can apply the method of statistical simulation of random fields on the sphere based on their spectral expansions (*Vyzhva, 1997*), which allows finding the perfect image of entire observations field for their certain implementation values. So we generate additional random component data in the points where geomagnetic measurements were not carried out, for example, with double precision intervals of 50 compare to 100 meters or between profiles. We can impose this data on the spline curve trend $S_i^{(1)}(r,\theta,\phi), i=7,...,20$ for each profile and obtain more detailed aeromagnetic survey data. This method differs from the traditional interpolation method, which uses average value of neighboring measured points for calculation point. Our method takes into account the correlation between data points and their statistical distribution. Using the above method makes it possible to supplement the missing data in the study area with greater accuracy than in (*Vyzhva et al., 2012*) (the mean square deviation is 0, 225), taking into account their statistical nature.

The built variogram of these implementations $\xi_i(r,\theta,\phi), i=7,...,20$ has the best approximation (the mean square deviation is 0, 195) by theoretical variogram which is connected to the Bessel type correlation function (*Vyzhva, 1997*), p. 214 for parameter a ≈4,2*10^{-3}:

$$B(\rho)=3\sqrt{\frac{\pi}{2}}\,\frac{J_{\frac{3}{2}}(a\rho)}{(a\rho)^{\frac{3}{2}}},(a>0) \qquad (25)$$

where $J_{\frac{3}{2}}(a\rho)$ is the Bessel function of the first kind of order 3/2.

This confirms the adequacy of simulated implementations to the real research data.

The spectral coefficients, which correspond to the correlation function (25) of random field $\xi(r,\theta,\phi)$, are calculated (*Вижва, 2011*), p. 213 by the formula

$$b_m(r) = \frac{3\pi^2}{ar}\left[J^2_{m+\frac{1}{2}}(ar) - J_{m+\frac{1}{2}}(ar)\ J_{m-\frac{1}{2}}(ar) \right], \quad (26)$$

where $J_m(ar)$ is the Bessel function of the first kind of order m.

These spectral coefficients we used in proposed above algorithm. The statistical simulation of realizations of the Gaussian isotropic random fields $\xi_i(r,\theta,\phi), i = 7,...,20$ can be done by means of this algorithm.

Variograms of input and simulated data arrays ΔT_{an} for PR7-PR20, corresponding to Bessel type correlation function (25) at the value of the parameter $a \approx 4{,}2*10^{-3}$ are shown on Figure 4 and Figure 5 respectively. This confirms the adequacy of simulated implementations to the real research data.

According to the algorithm we received random component implementations on the study area with twice detail for each profile. The final stage was the imposing array of realizations $\xi_i(r,\theta,\phi), i = 7,...,20$ what we got by statistical simulation on the spline approximation of real data. As a result, we received more detailed implementation for the geomagnetic observation data in the selected area (Fig. 6).

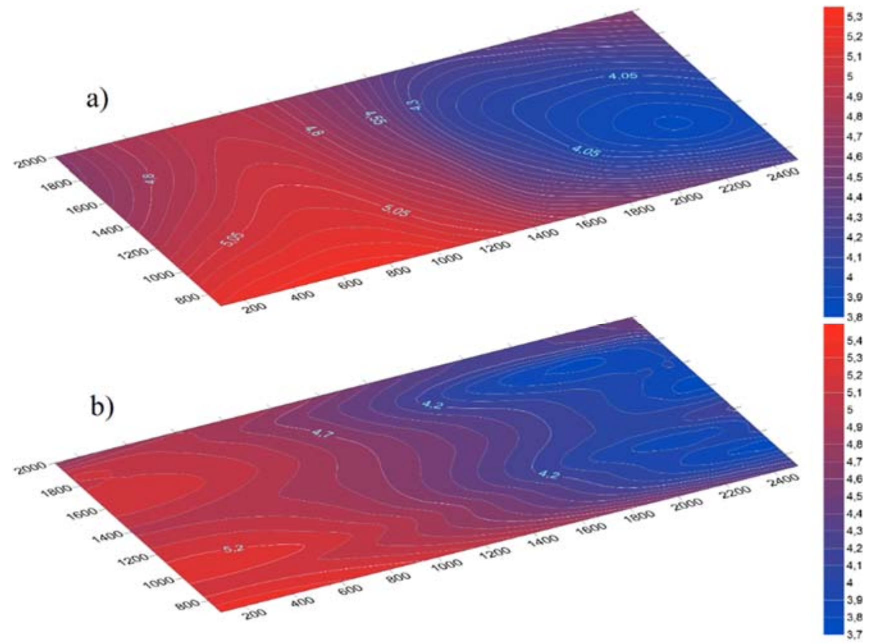

Fig.6. a) The map of aeromagnetic survey data ΔT_{an} (general map) M 1:10 000, (PR 7-20); b) the map of aeromagnetic survey data ΔT_{an} plus generated additional data in the points with double precision intervals in the Ovruch depression M 1:10 000

Conclusions. The statistical simulation method of random field on the sphere implementations makes it possible to supplement with a given detail the measurement results of magnetic field full vector on the great square territory. The built variogram of random component has the best approximation (the mean square deviation is 0, 195) than in (*Вижва та ін., 2010; Vyzhva and Vyzhva, 2016*) (the mean square deviation is 0, 225) by theoretical variogram which is connected to the Bessel type correlation.

Список використаних джерел

1. Вижва, З.О., Ядренко, М.Й. (2000). Статистичне моделювання ізотропних випадкових полів на сфері. Вісник Київського національного університету. Математика і Механіка, 5, 5-11.
2. Вижва, З.О., Вижва, С.А., Демидов, В.К. (2010). Статистичне моделювання випадкових полів на площині сплайновими апроксимаціями (на прикладі даних аеромагнітометрії). Вісник Київського національного університету. Геологія, 51, 31-36.
3. Вижва, З.О. (2011). Статистичне моделювання випадкових процесів та полів. Київ: Обрії.
4. Вижва, З.О., Федоренко, К.В. (2013). Про статистичне моделювання випадкових полів у тривимірному евклідовому просторі. Вісник Київського національного університету. Математика і Механіка, 30(2), 19-24.
5. Градштейн, И.С., Рыжик, И.М. (1971). Таблицы интегралов, сумм, рядов и произведений. Москва: Наука.
6. Пригарин, С. М. (2005). Методы численного моделирования случайных процессов и полей. Новосибирск: Изд-во ИВМ и МГ.
7. Chiles, J.P., Delfiner, P. (1999). Geostatistics: Modeling Spatial Uncertainty. New York, Toronto: John Wiley & Sons, Inc.
8. Vyzhva, Z.O., Vyzhva, S.A., Demidov, V. K. (2012). The statistical simulation of random processes and 2-D fields on aerial magnetometry. Visnyk of Taras Shevchenko National University of Kyiv. Geology, 56, 52-55.
9. Vyzhva, Z.O. (1997). On Approximation of Isotropic Random Fields on the Sphere and Statistical Simulation. Theory of Stochastic Processes, 3(19), 463-467.
10. Vyzhva, Z.O. (2003). About Approximation of 3-D Random Fields and Statistical Simulation. Random Operator and Stochastic Equation, 4, 3, 255-266.
11. Vyzhva, Z.O., Fedorenko, K.V. (2013). The Statistical Simulation of 3-D Random Fields by Means Kotelnikov-Shannon Decomposition. Theor. Probability and Math. Statist., 88, 17-31.
12. Vyzhva, Z.O., Fedorenko, K.V. (2016). About Statistical Simulation of 4D Random Fields by Means of Kotelnikov-Shannon Decomposition. Journal of Applied Mathematics and Statistics. Columbia International Publishing, 2, 2, 59-81.
13. Vyzhva, Z.O., Vyzhva, A.S. (2016). About methods of statistical simulation of random fields on the plane by the aircraft magnetometry data. Visnyk of Taras Shevchenko National University of Kyiv. Geology, 4 (75), 88-93.
14. Yadrenko, M.Y. (1993). Spectral theory of random fields. New York: Optimization Software Inc., Publications Division.
15. Yadrenko, M.Y., Gamaliy, O. (1998). The Statistical Simulation of Homogeneous and Isotropic Three-dimensional Random Fields and Estimate Simulation Error. Theor. Probability and Math. Statist., 59, 171-175.
16. Watson, G.N., (1945). A treatise on the theory of Bessel functions. 2nd ed. Cambridge: Cambridge University Press. (Russ. ed.: Watson, G.N., (1949). Teoriya besselevykh funktsiy. Moscow: Inostrannaya literatura Publ.).

References

1. Chiles, J.P., Delfiner, P. (1999). Geostatistics: Modeling Spatial Uncertainty. New York, Toronto: John Wiley & Sons, Inc.
2. Gradshteyn, I.S., Ryzhik, I.M. (1971). Tables of Integrals, Series and Products. Nauka: Moscow. [in Russian]
3. Prigarin, S. M. (2005). Numerical Modeling of Random Processes and Fields. Novosibirsk: Inst. of Comp. Math. and Math. Geoph. Publ. [in Russian]
4. Vyzhva, Z.O. (1997). On Approximation of Isotropic Random Fields on the Sphere and Statistical Simulation. Theory of Stochastic Processes, 3(19), 463-467.
5. Vyzhva, Z.O. (2003). About Approximation of 3-D Random Fields and Statistical Simulation. Random Operator and Stochastic Equation, 4, 3, 255-266.
6 Vyzhva, Z.O. (2011). The Statistical Simulation of Random Processes and Fields. Kyiv: Obrii. [in Ukrainian]
7. Vyzhva, Z.O., Fedorenko, K.V. (2013). About The Statistical Simulation of Random Felds on the 3D Euclid Space. Visnyk of Taras Shevchenko National University of Kyiv. Mathematics and Mechanics, 30(2), 19-24. [in Ukrainian]

8. Vyzhva, Z.O., Fedorenko, K.V. (2013). The Statistical Simulation of 3-D Random Fields by Means Kotelnikov-Shannon Decomposition. Theor. Probability and Math. Statist., 88, 17-31.
9. Vyzhva, Z.O., Fedorenko, K.V. (2016). About Statistical Simulation of 4D Random Fields by Means of Kotelnikov-Shannon Decomposition. Journal of Applied Mathematics and Statistics. Columbia International Publishing, 2, 2, 59-81.
10. Vyzhva, Z.O., Vyzhva, A.S. (2016). About methods of statistical simulation of random fields on the plane by the aircraft magnetometry data. Visnyk of Taras Shevchenko National University of Kyiv. Geology, 4 (75), 88-93.
11. Vyzhva, Z.O., Vyzhva, S.A., Demidov, V. K. (2012). The statistical simulation of random processes and 2-D fields on aerial magnetometry. Visnyk of Taras Shevchenko National University of Kyiv. Geology, 56, 52-55.
12. Vyzhva, Z.O., Vyzhva, S.A., Demidov, V.K. (2010). The statistical simulation of random fields on the plane by splain approximation (on aerial magnetometry data example). Visnyk of Taras Shevchenko National University of Kyiv. Geology, 51, 31-36. [in Ukrainian]
13. Vyzhva, Z.O., Yadrenko, M.Y. (2000). The Statistical Simulation of Isotropic Random Felds on the Sphere. Visn. Kyiv University. Mathematics and Mechanics., 5, 5-11. [in Ukrainian]
14. Watson, G.N., (1945). A treatise on the theory of Bessel functions. 2nd ed. Cambridge: Cambridge University Press. (Russ. ed.: Watson, G.N., (1949). Teoriya besselevykh funktsiy. Moscow: Inostrannaya literatura Publ.).
15. Yadrenko, M.Y. (1993). Spectral theory of random fields. New York: Optimization Software Inc., Publications Division.
16. Yadrenko, M.Y., Gamaliy, O. (1998). The Statistical Simulation of Homogeneous and Isotropic Three-dimensional Random Fields and Estimate Simulation Error. Theor. Probability and Math. Statist., 59, 171-175.

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ПРО МЕТОДИ СТАТИСТИЧНОГО МОДЕЛЮВАННЯ ВИПАДКОВИХ ПОЛІВ НА СФЕРІ ДЛЯ ДАНИХ АЕРОМАГНІТОМЕТРІЇ

Розроблено універсальні методи статистичного моделювання (методи Монте-Карло) геофізичних даних, які дають можливість розвязувати проблеми генерування реалізацій випадкових полів на сітці сфери будь-якої регулярності та детальності. У геофізиці більшість результатів досліджень подається у цифровій формі, точність якої залежить від різних випадкових впливів (у тому числі від похибки вимірювання апаратури). При цьому виникає проблема кондиційності карт у випадку, коли дані неможливо отримати із заданою детальністю на деяких ділянках. Для розв'язання проблем кондиційності карт, доповнення даними для досягнення необхідної точності та інших проблем подібного роду в геофізичних задачах пропонується застосовувати методи статистичного моделювання реалізацій випадкових полів. Використано теореми про оцінку середньоквадратичної апроксимації ізотропних випадкових полів на сфері частковими сумами рядів спеціального вигляду, за допомогою яких сформульовано алгоритми чисельного моделювання реалізацій таких випадкових полів методом спектральних коефіцієнтів. Розроблено нову ефективну методику застосування до розв'язання геофізичних задач методів статистичного моделювання випадкових полів на сфері. На прикладі даних аеромагнітної зйомки в районі Овруцької западини впроваджено статистичне моделювання реалізацій випадкових полів на основі спектрального розкладу у розв'язання проблем кондиційності карт шляхом доповнення даних до необхідної детальності. При аналізі даних по профілях їх розділено на детерміновану та випадкову складові. Детерміновану складову даних пропонується наближати кубічними сплайнами, ізотропну випадкову складову – моделювати на основі спектрального розкладу випадкових полів на сфері (модельний приклад – дані аеромагнітної зйомки). За наведеним алгоритмом було отримано реалізації випадкової складової на області дослідження з подвоєною детальністю по кожному профілю. При перевірці їх на адекватність зроблено висновки, що відповідна гістограма випадкової складової має гауссівський розподіл. Побудована варіограма цих реалізацій має найкраще наближення теоретичною варіограмою, яка пов'язана із кореляційною функцією бесселевого типу. Завершальним етапом роботи було накладення масиву випадкової складової на сплайнову апроксимацію реальних даних. У результаті цього отримано більш детальну реалізацію для даних геомагнітних спостережень у виділеній області. Отже, метод статистичного моделювання реалізацій випадкових полів на сфері дає можливість максимально адекватно доповнити, із заданою детальністю, даними результати вимірювань повного вектора напруженості магнітного поля.

Ключові слова: статистичне моделювання, спектральний розклад, сплайн-інтерполяція, кондиційність карт.

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О МЕТОДАХ СТАТИСТИЧЕСКОГО МОДЕЛИРОВАНИЯ СЛУЧАЙНЫХ ПОЛЕЙ НА СФЕРЕ ДЛЯ ДАННЫХ АЭРОМАГНИТОМЕТРИИ

Разработаны универсальные методы статистического моделирования (методы Монте-Карло) геофизичеких данных, которые дают возможность решить проблемы генерирования реализаций случайных полей на сфере на сетке любой детальности и регулярности. В геофизике большинство результатов исследований подаётся в цифровой форме, точность которой зависит от разных случайных влияний (в том числе от погрешности измерения аппаратуры). При этом возникает проблема кондиционности карт в случае, когда данные невозможно получить на некоторых участках. Для решения проблем кондиционности карт, дополнения данными для достижения необходимой точности и других проблем подобного рода в геофизических задачах предлагается применять методы статистического моделирования реализаций случайных полей. Использованы теоремы об оценке среднеквадратической аппроксимации изотропных случайных полей на сфере частичными суммами рядов специального вида, при помощи которых сформулированы алгоритмы численного моделирования реализаций таких случайных полей методом спектральных коэффициентов. Разработана новая эффективная методика применения методов статистического моделирования случайных полей на сфере при решении геофизических задач. На примере данных аэромагнитной съёмки в районе Овруцкой впадины разработана методика внедрения статистического моделирования случайных полей на сфере на основании спектрального розложения для решения проблем кондиционности карт дополнением данных необходимой детальности. При анализе данных по профилям, их разделено на детерминированную и случайную составляющие. Детерминированную составляющую предлагается аппроксимировать кубическими сплайнами, изотропную случайную составляющую – моделировать на основе спектрального разложения случайных полей на сфере (модельный пример – данные аэромагнитной съёмки). С помощью предложенного алгоритма были получены реализации случайной составляющей в области исследования с удвоенной детальностью по каждому профилю. При проверке их на адекватность сделаны выводы, что соответствующая гистограмма случайной составляющей имеет гауссовское распределение. Построенная вариограмма этих реализаций имеет наилучшее приближение теоретической вариограммой, которая соответствует корреляционной функции бесселевого типа. Заключительным этапом роботы было наложение массива случайной составляющей на сплайновую аппроксимацию реальных данных. В результате получена более детальная реализация для данных геомагнитных наблюдений в выделенной области. Таким образом, метод статистического моделирования реализаций случайных полей на сфере даёт возможность максимально адекватно дополнить, с заданной детальностью, данными результаты измерений полного вектора напряжённости магнитного поля.

Ключевые слова: статистическое моделирование, спектральное розложение, сплайн-интерполяция, кондиционность карт.